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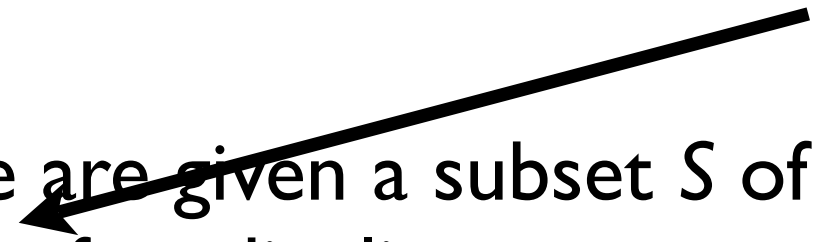
Note: von Neumann's notation: $u = \{ 0, 1, 2, ..., u - 1 \}$



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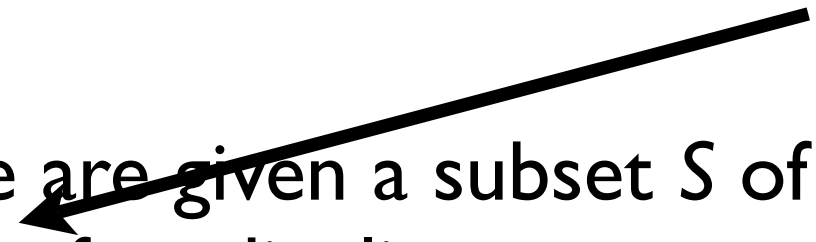

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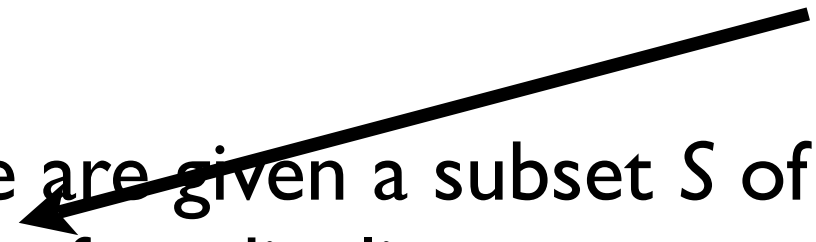

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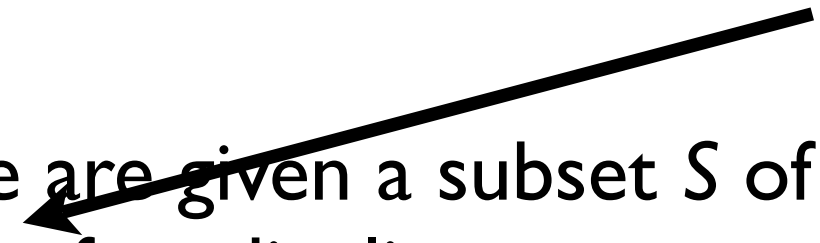

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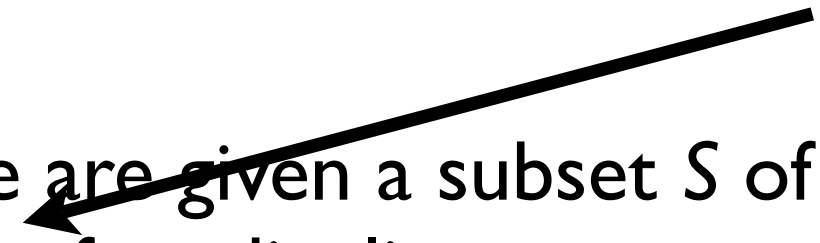

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- $$s(i) = \sum_{i<j} |d^{-1}(j)|$$
- Then, $h(x)=s(d(x))+g_{d(x)}(x)$ is a monotone minimal perfect hash function on S

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- For each bucket, compute the LCP
- Lemma: all LCPs are distinct
- We can identify a bucket by its LCP
- We can compute the LCP of the bucket of a key storing a map from S to $w = \log u$

000I00I000000
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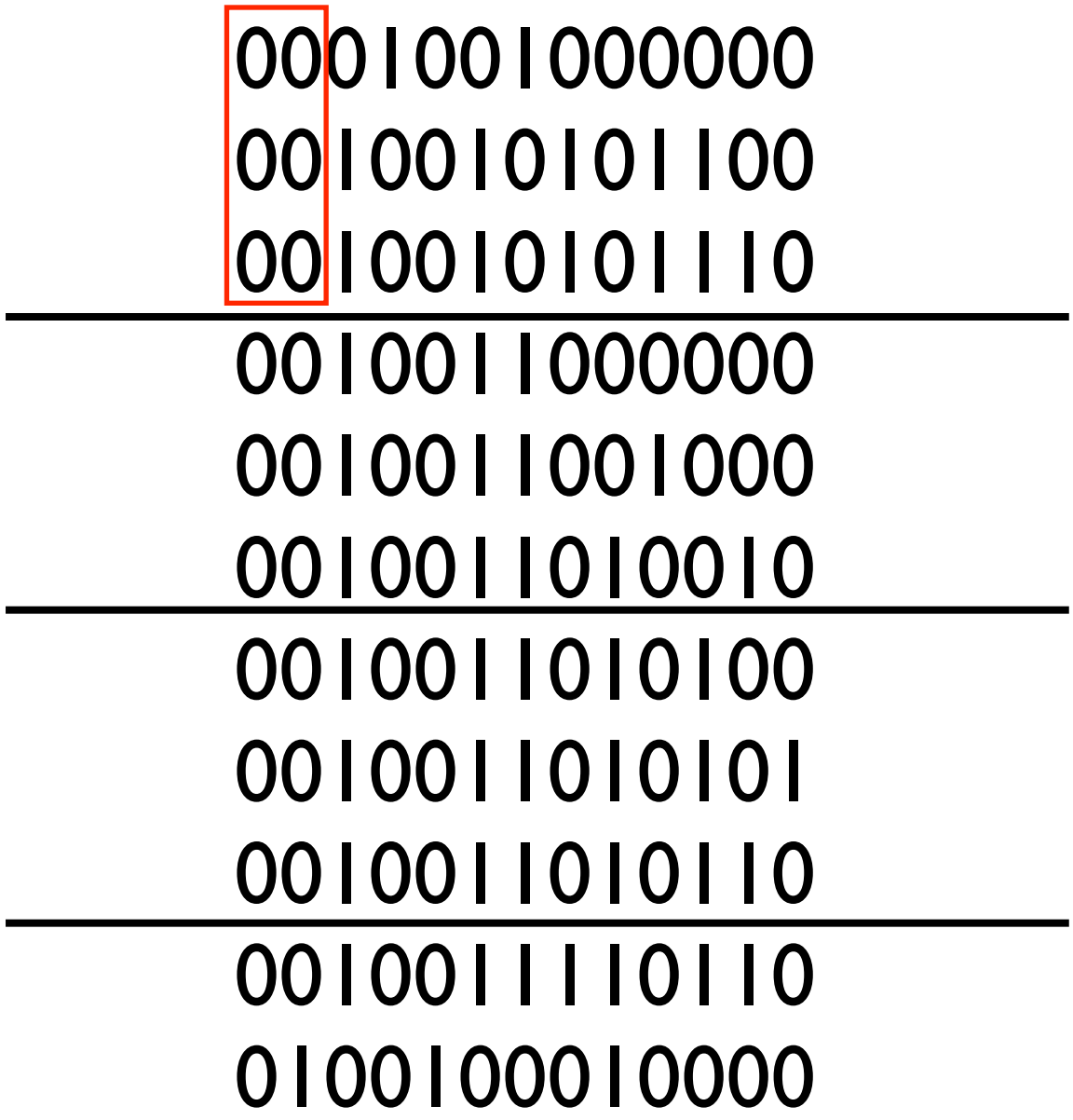
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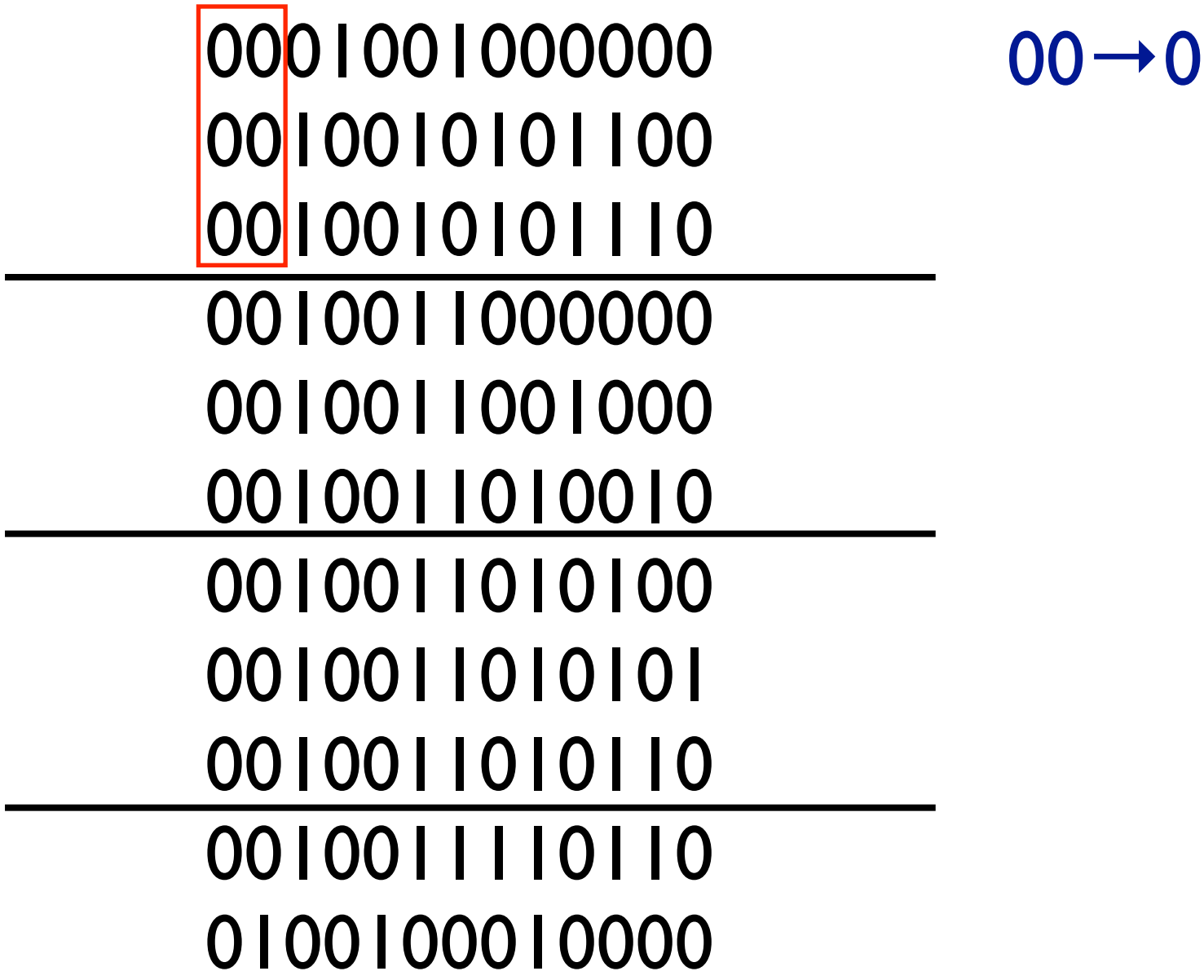
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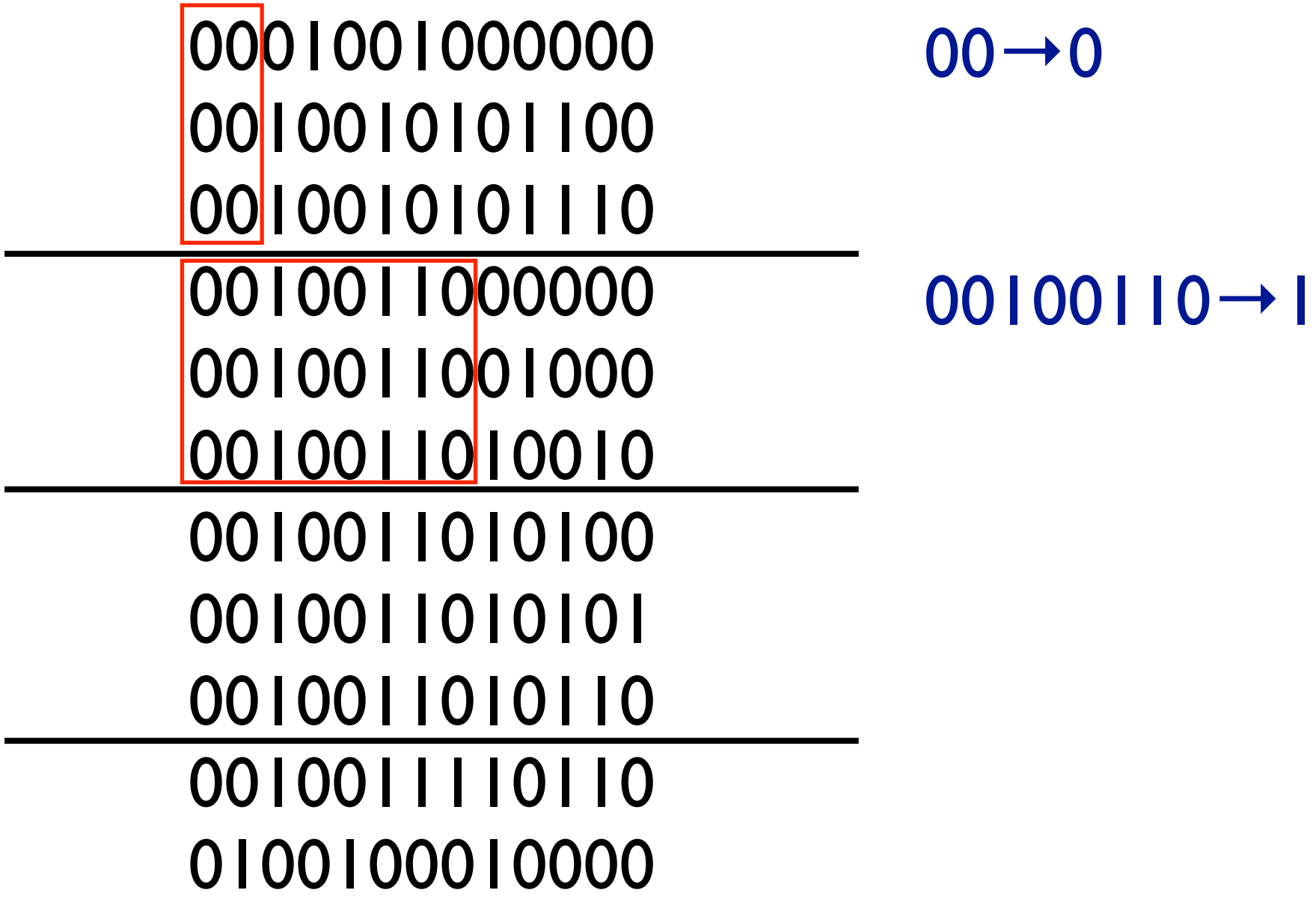
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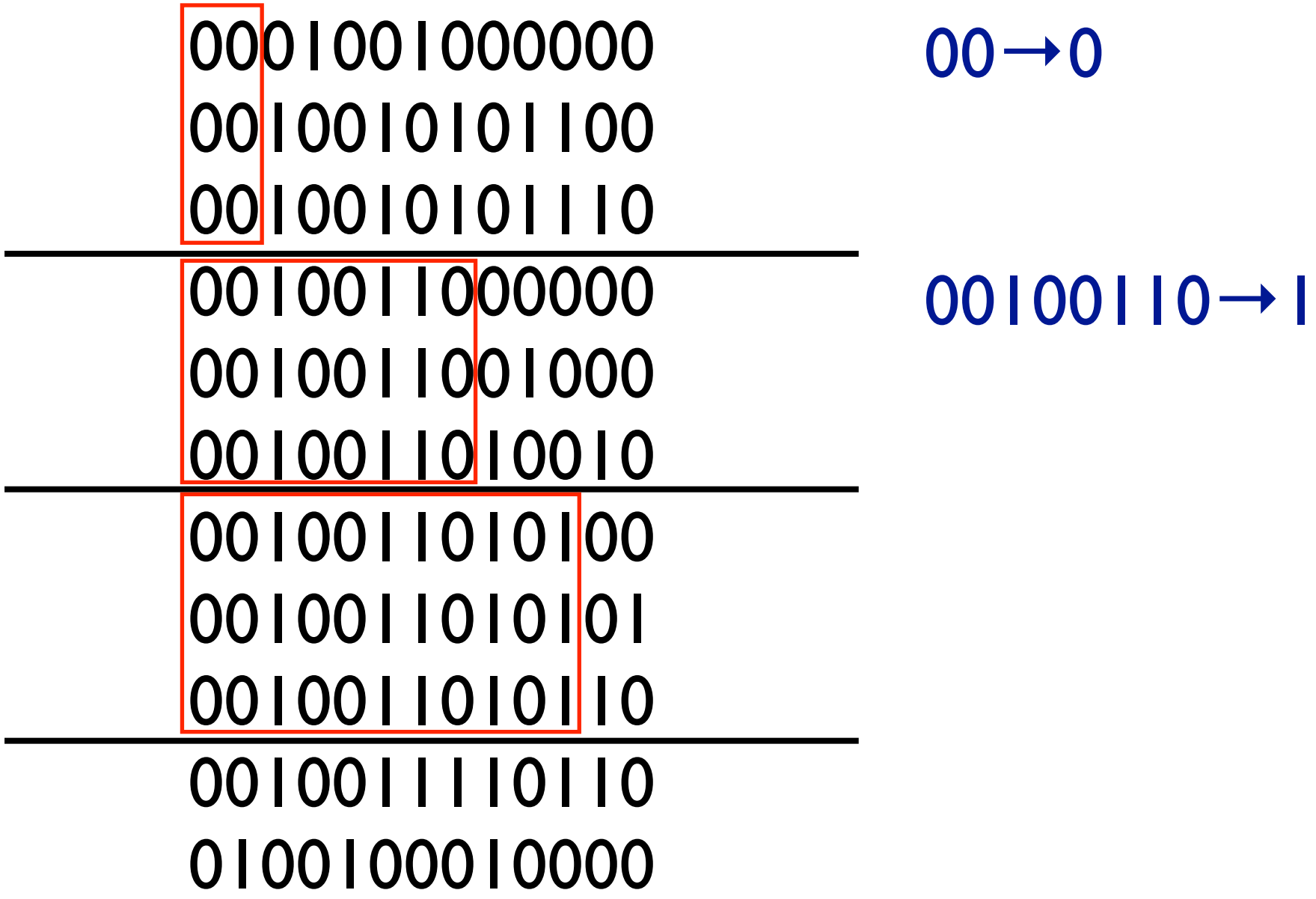
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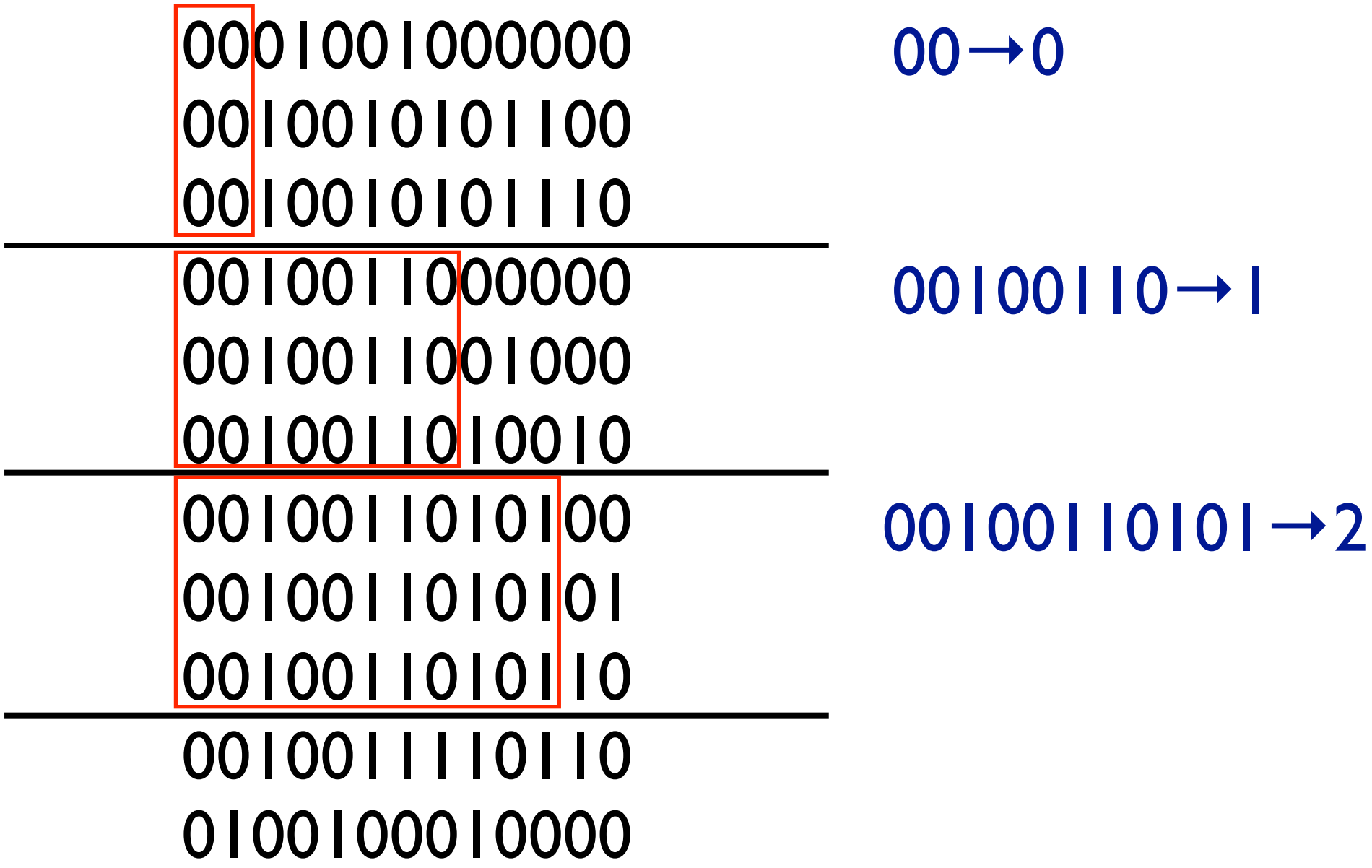
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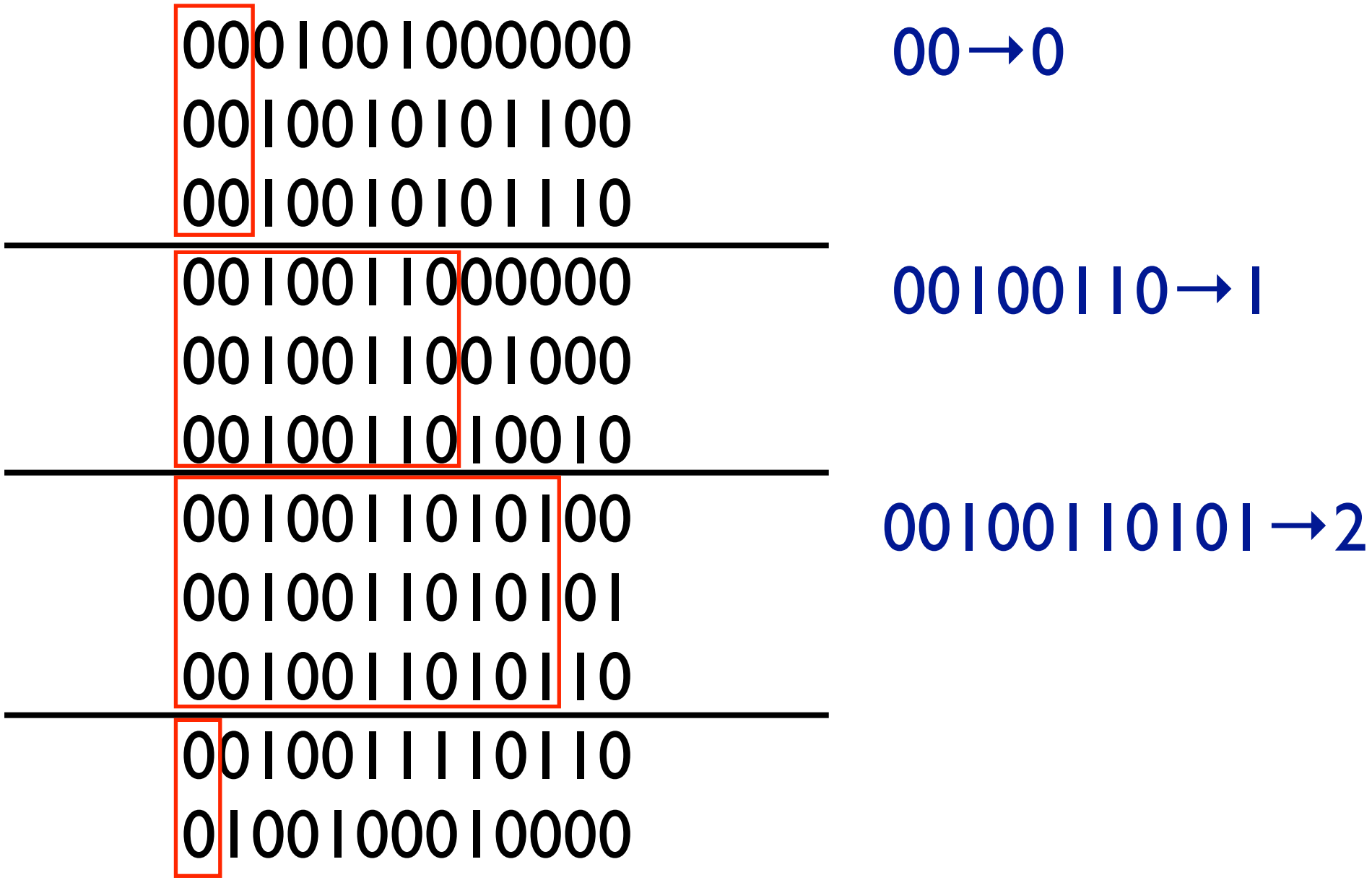
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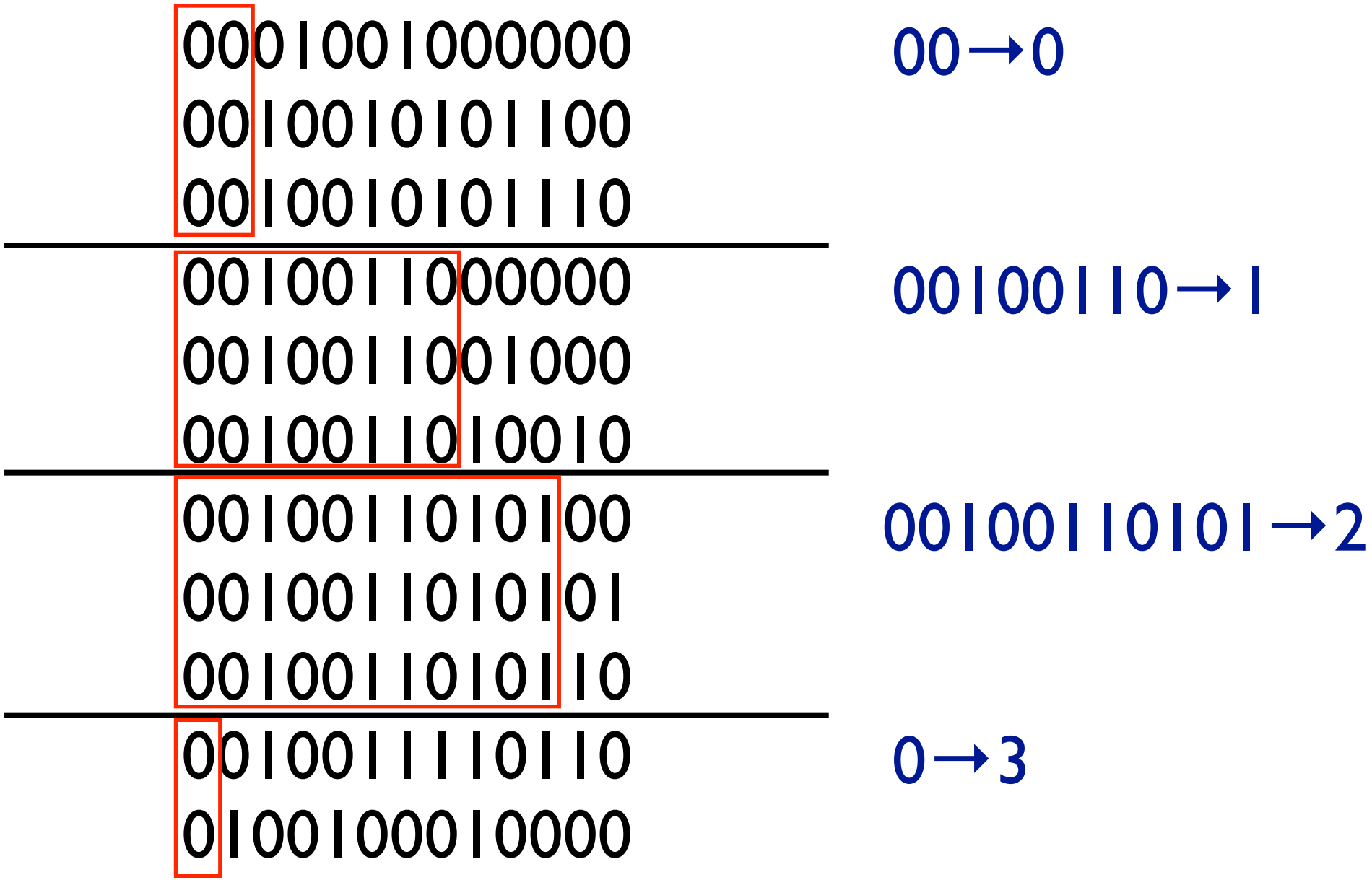
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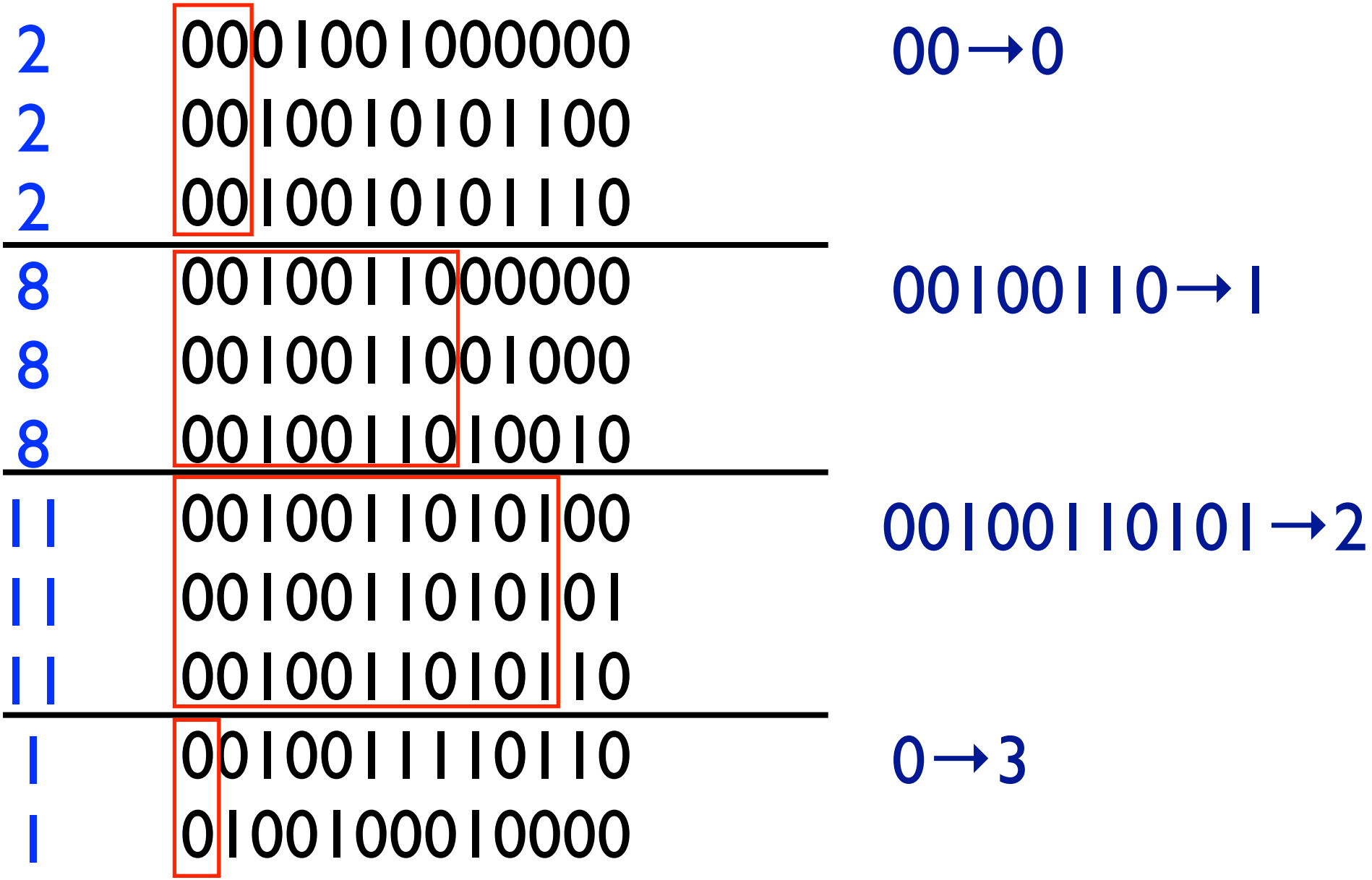
00|00|1|0 → 1
00|00|1|0|0|1 → 2
0 → 3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2 0010010101110
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8 0010011001000
8 0010011001010
0010011001→1
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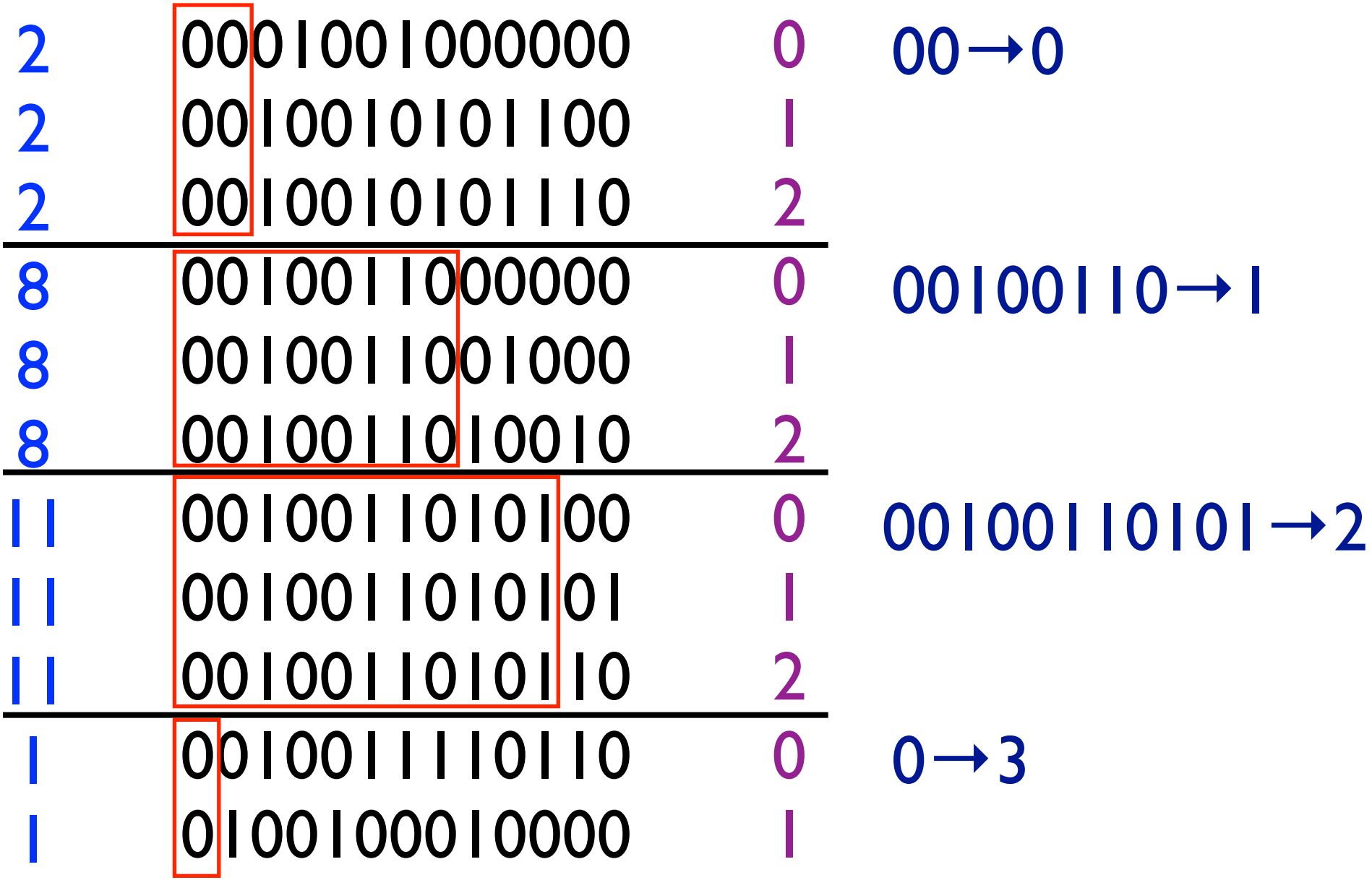
11 0010011010101
11 0010011010110
0011→2
1 0010011110110
1 0100100010000
0→3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0010011→1
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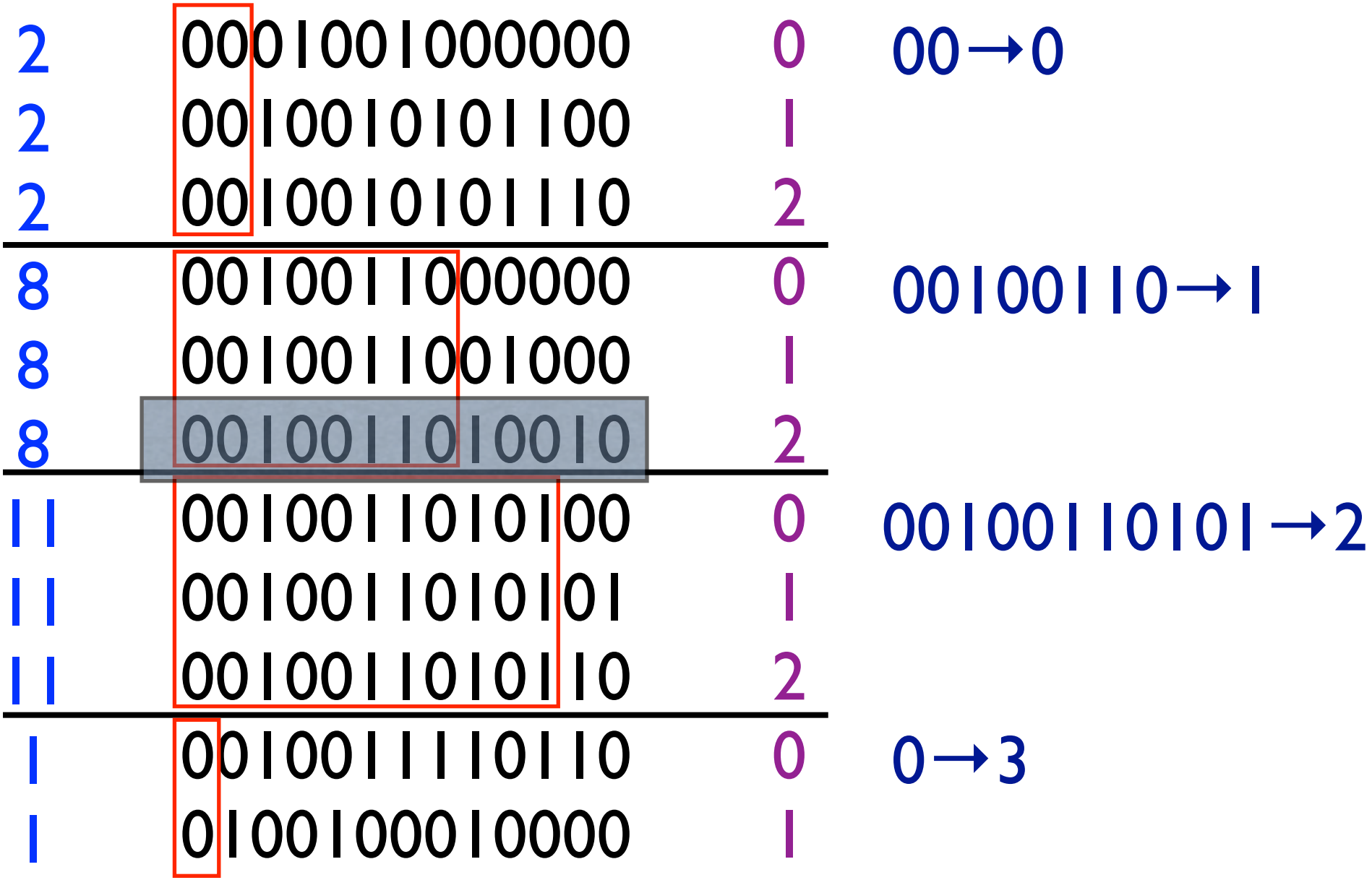
0→3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0010011→1
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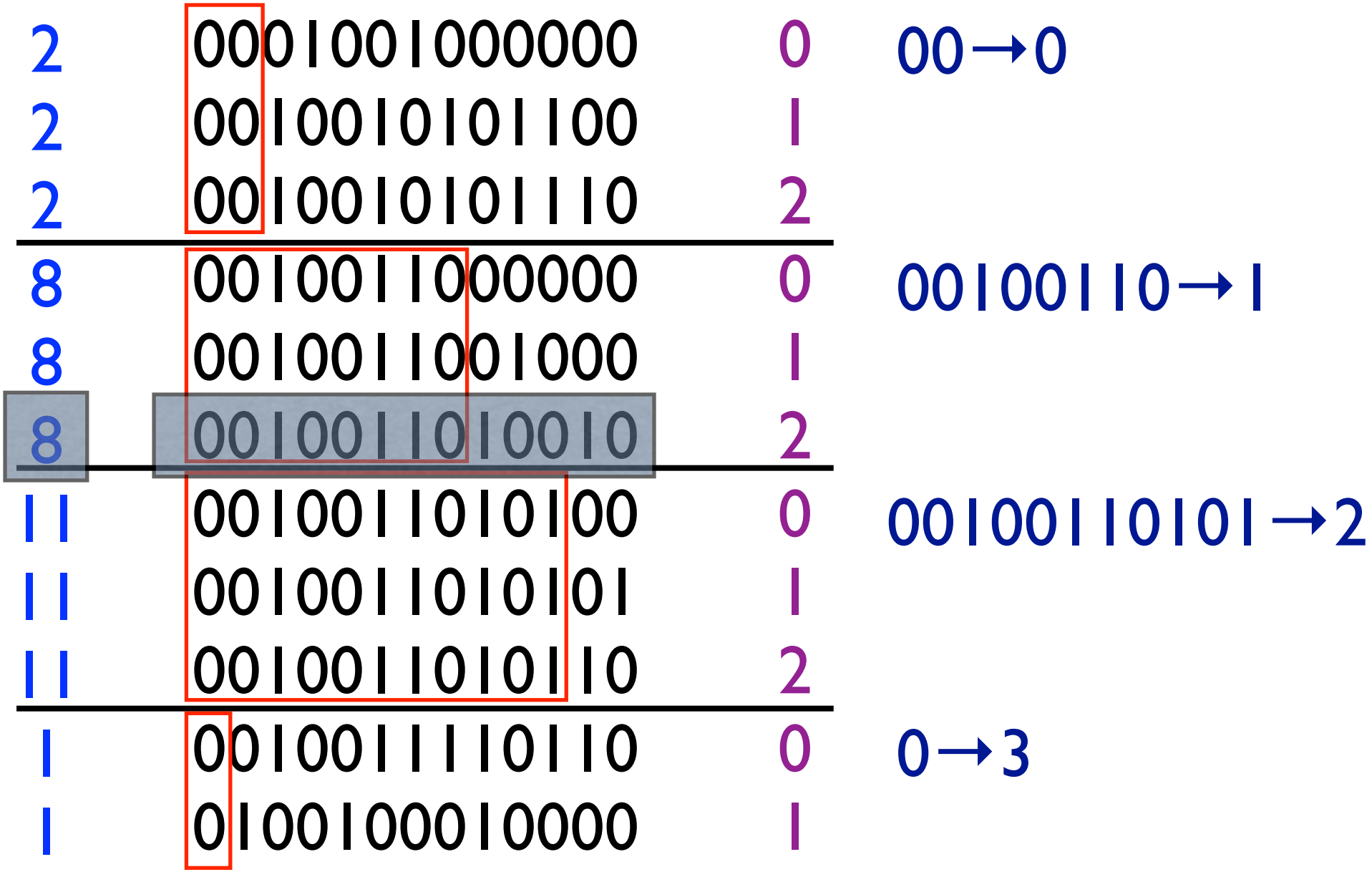
0→3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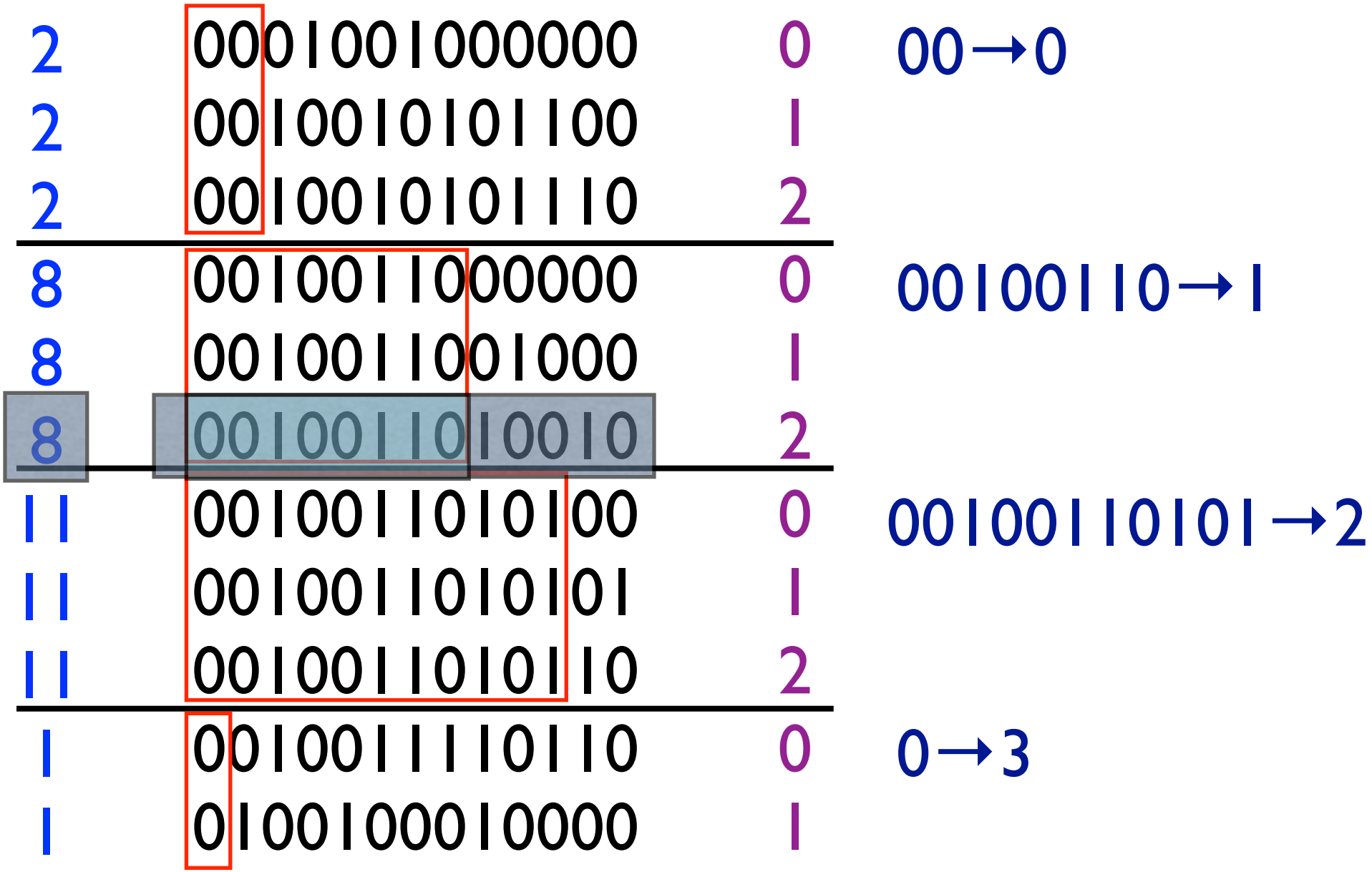
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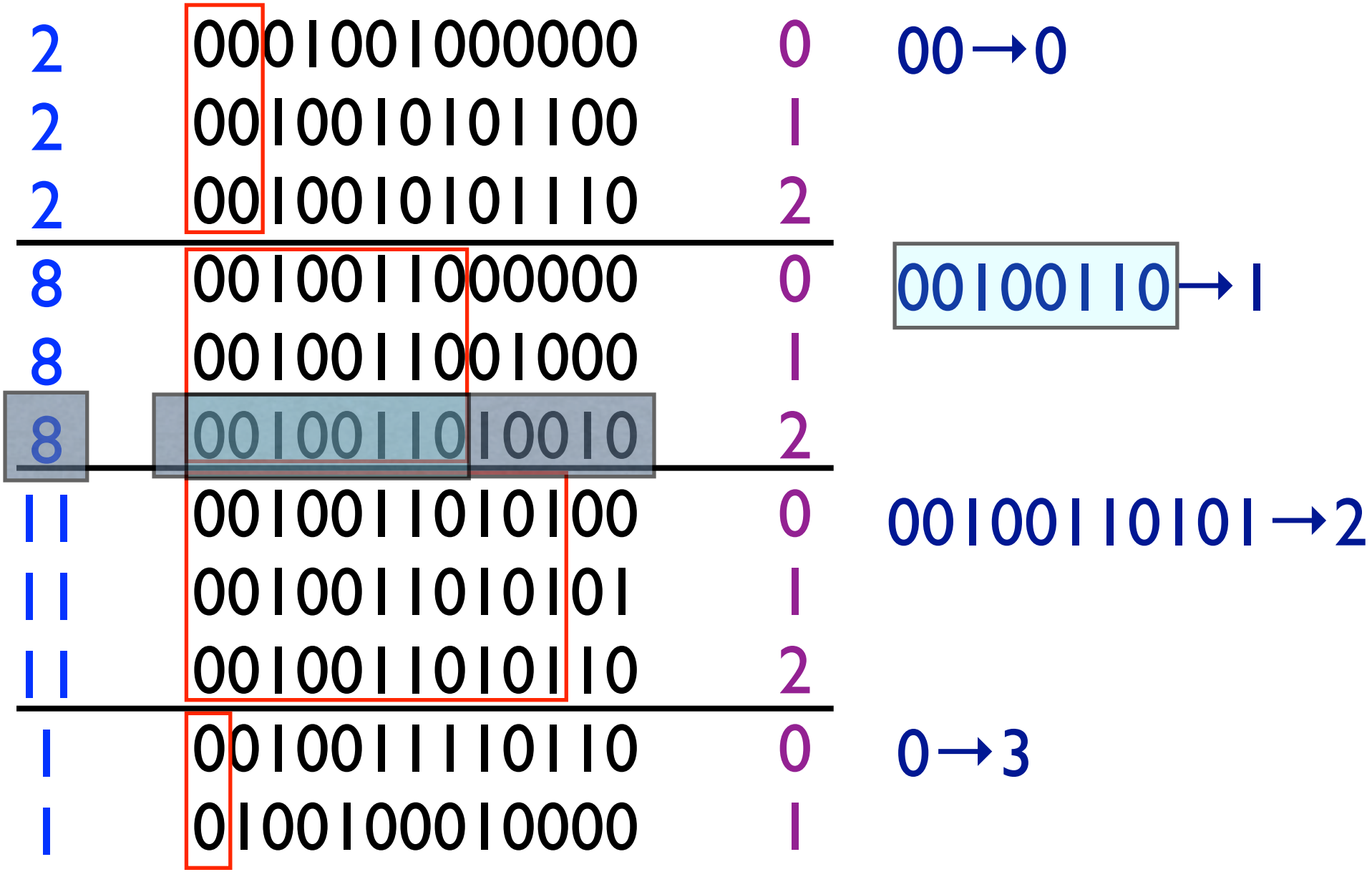
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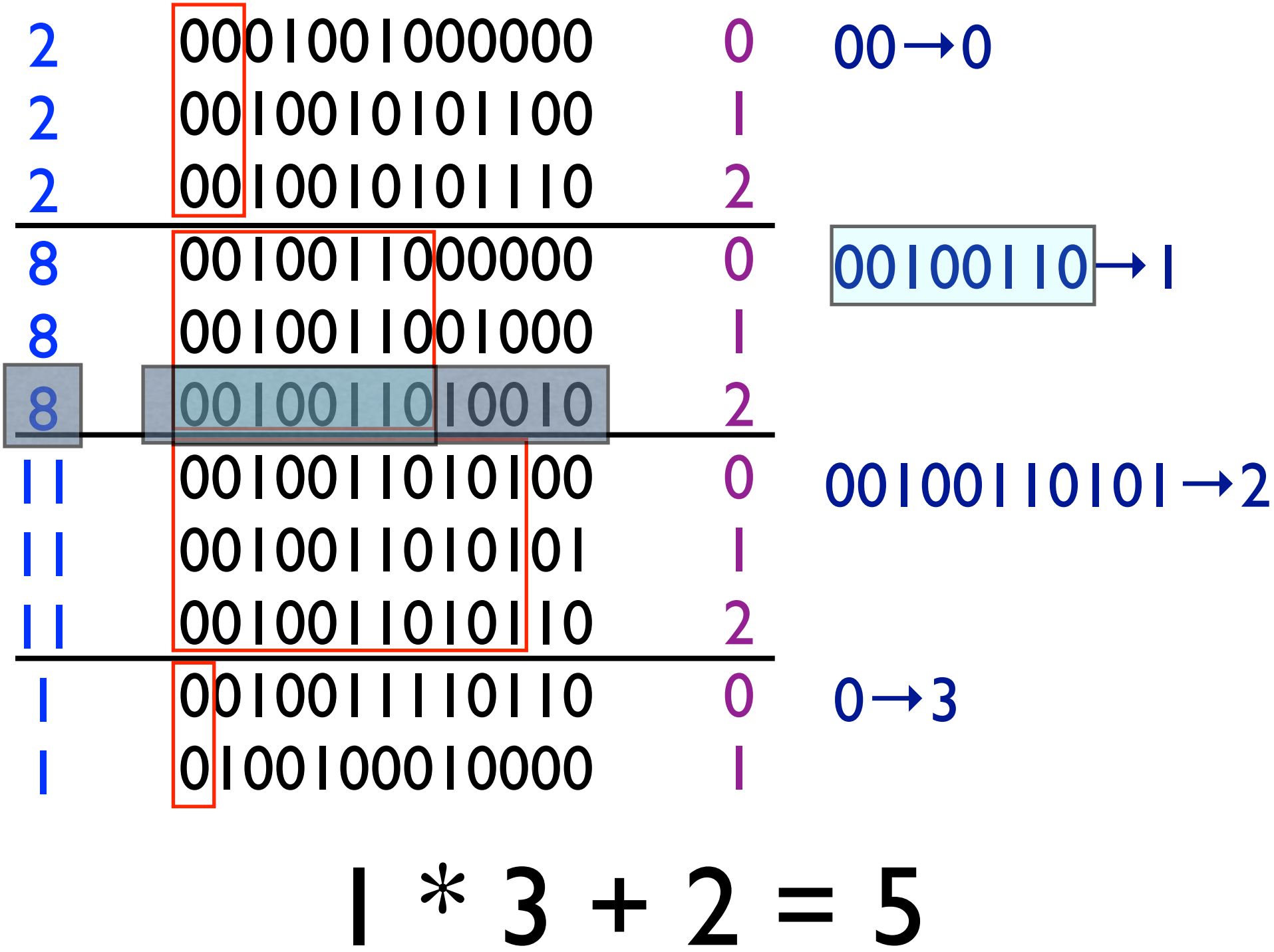
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11 0010011010101 1
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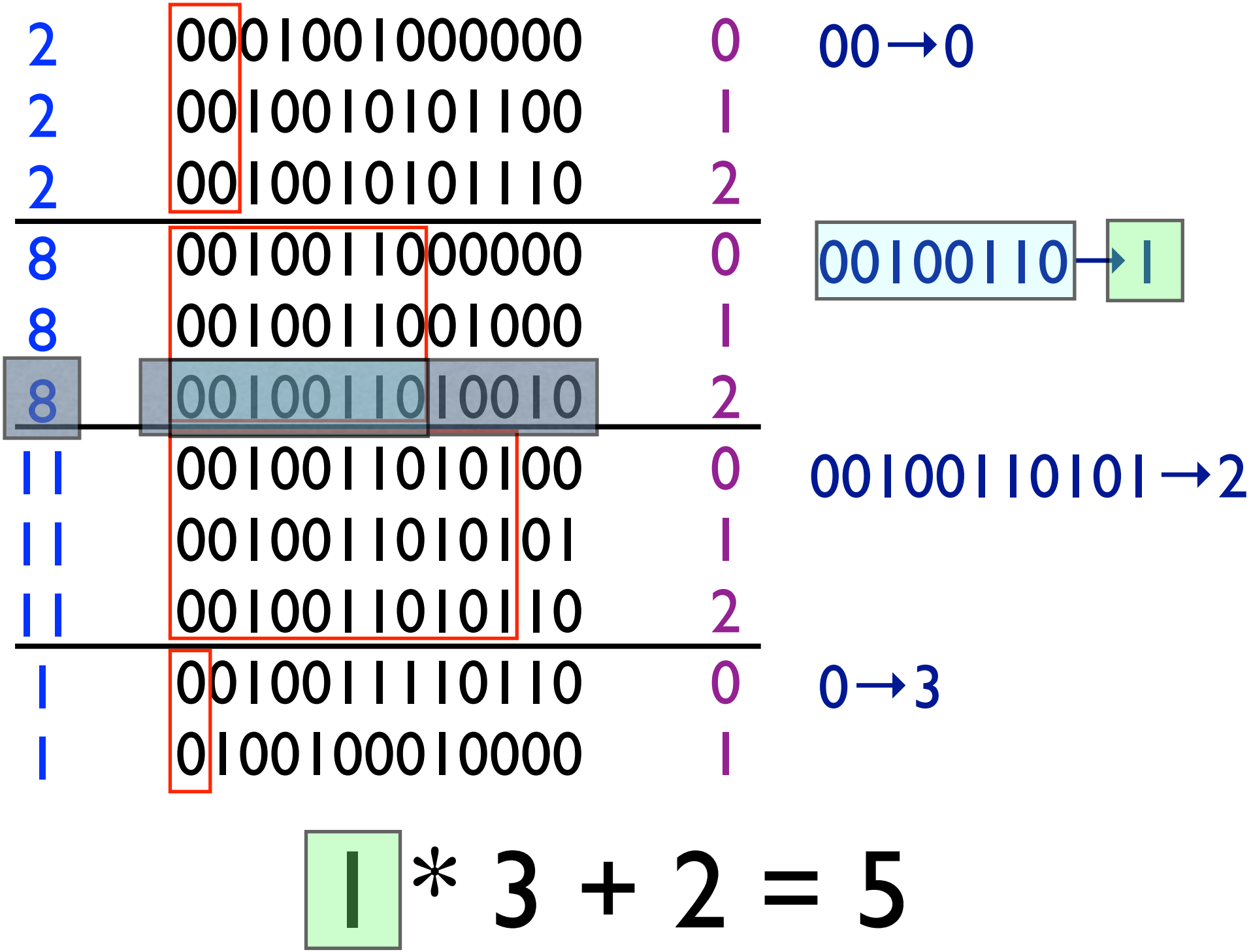
00100110→1
001001101101→2
0→3
1 * 3 + 2 = 5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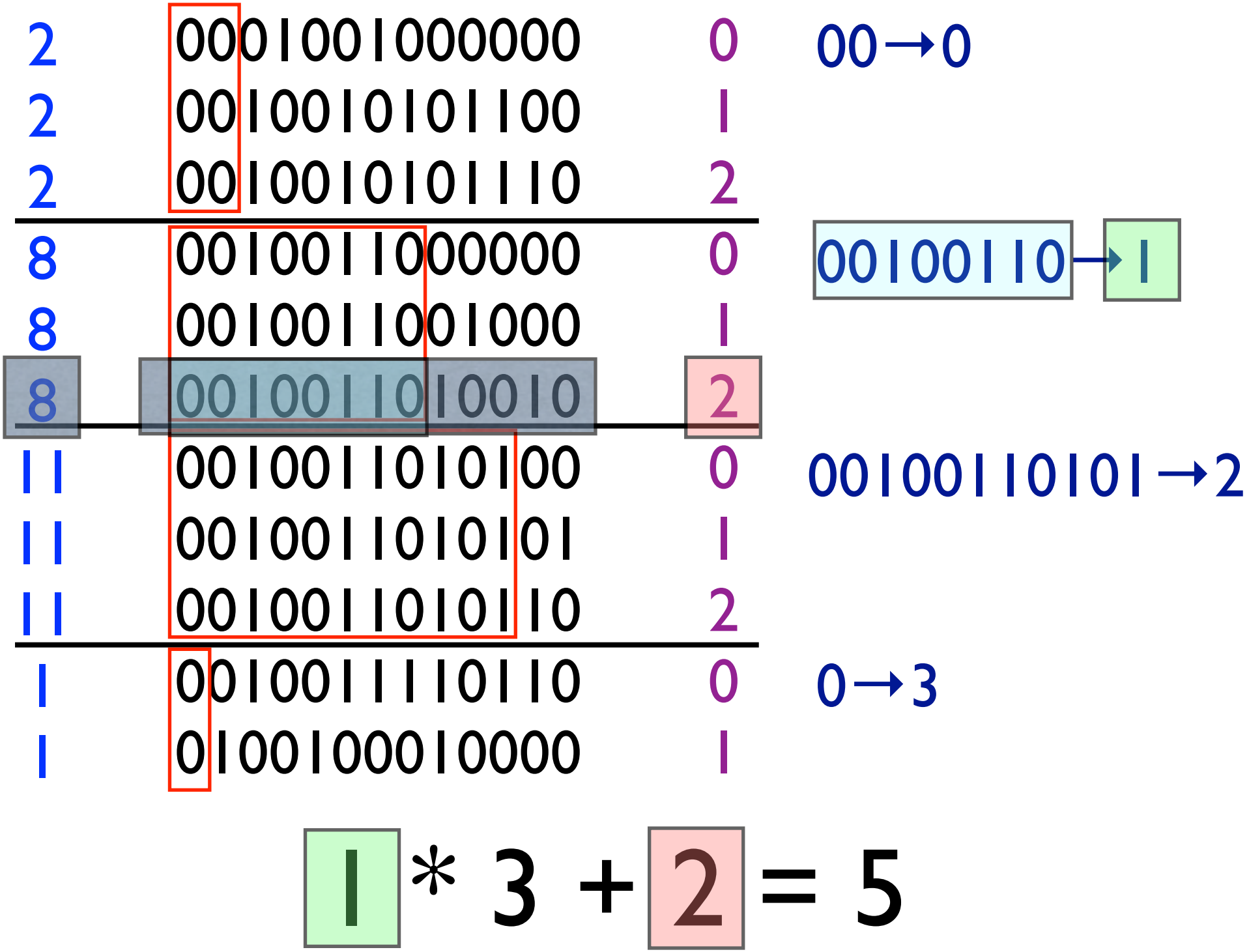
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11 0010011010101 1
11 0010011010110 2
1 0010011110110 0
1 0100100010000 1
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  - ...we get constant-time monotone minimal perfect hashing in $O(n \log w)$ bits

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- The structure is of independent interest; many other applications are coming...

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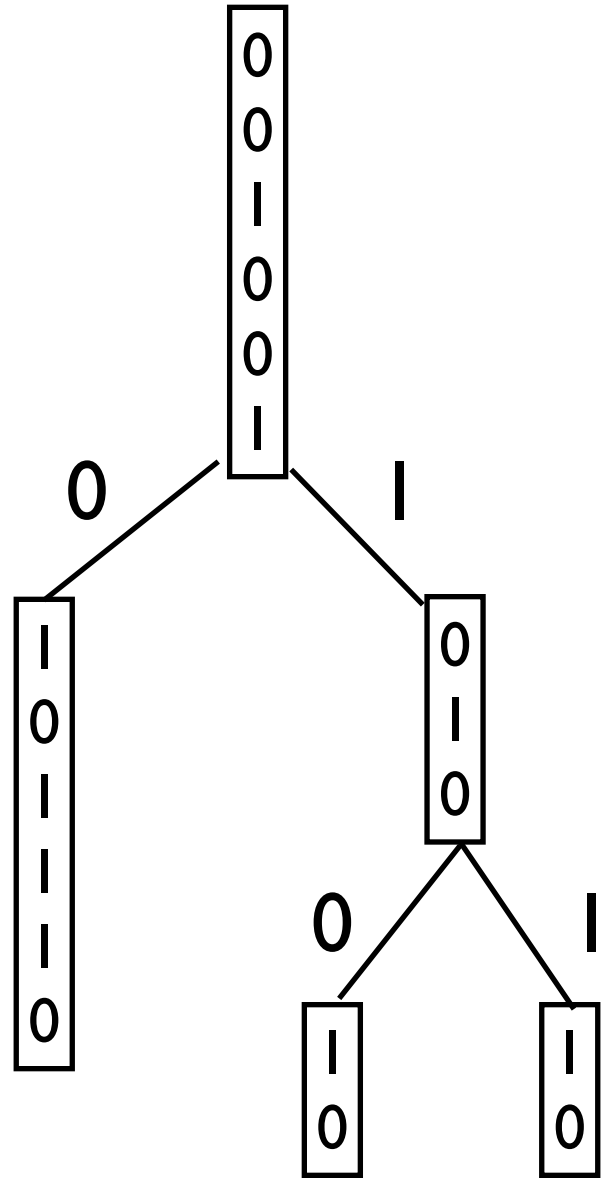
- Consider the *compacted trie* on S. Call:
  - *name* of a node the path leading to the node
  - *extent* of a node the path *represented* by the node
  - *skip interval* of a node the interval $[l..r)$ of positions, where l is the length of the extent of the parent (ε for the root) and r the length of the extent of the node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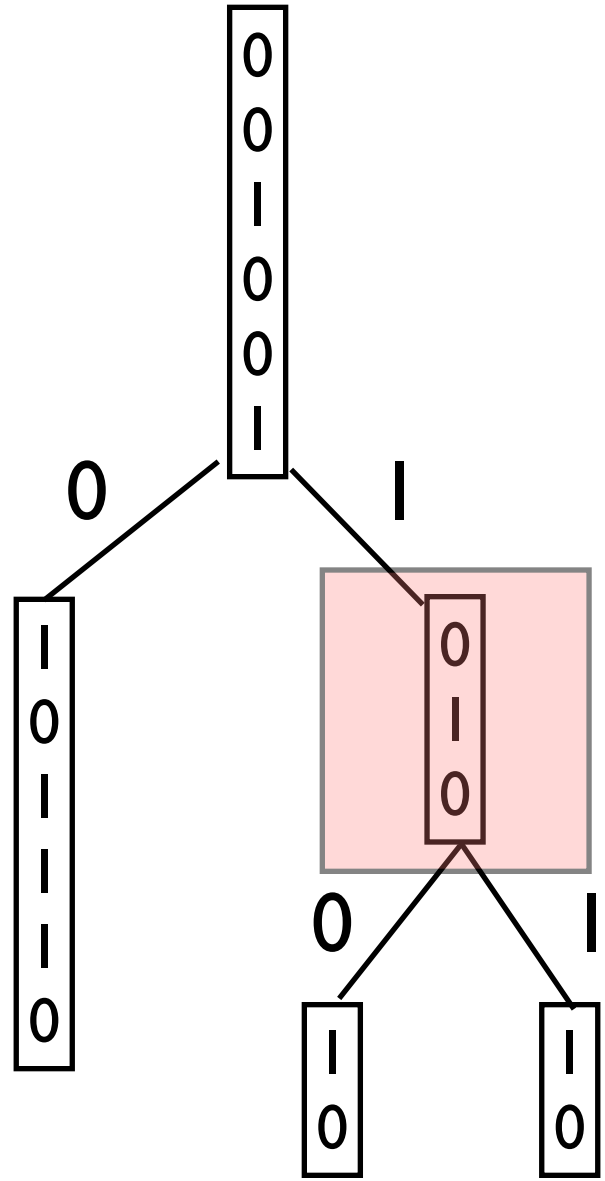
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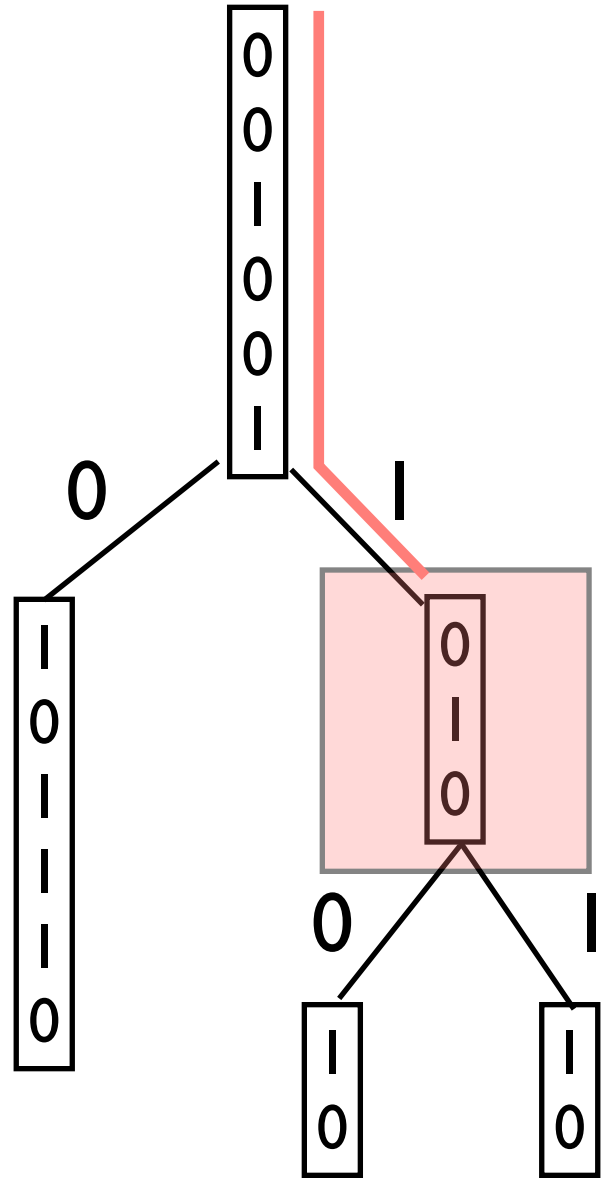
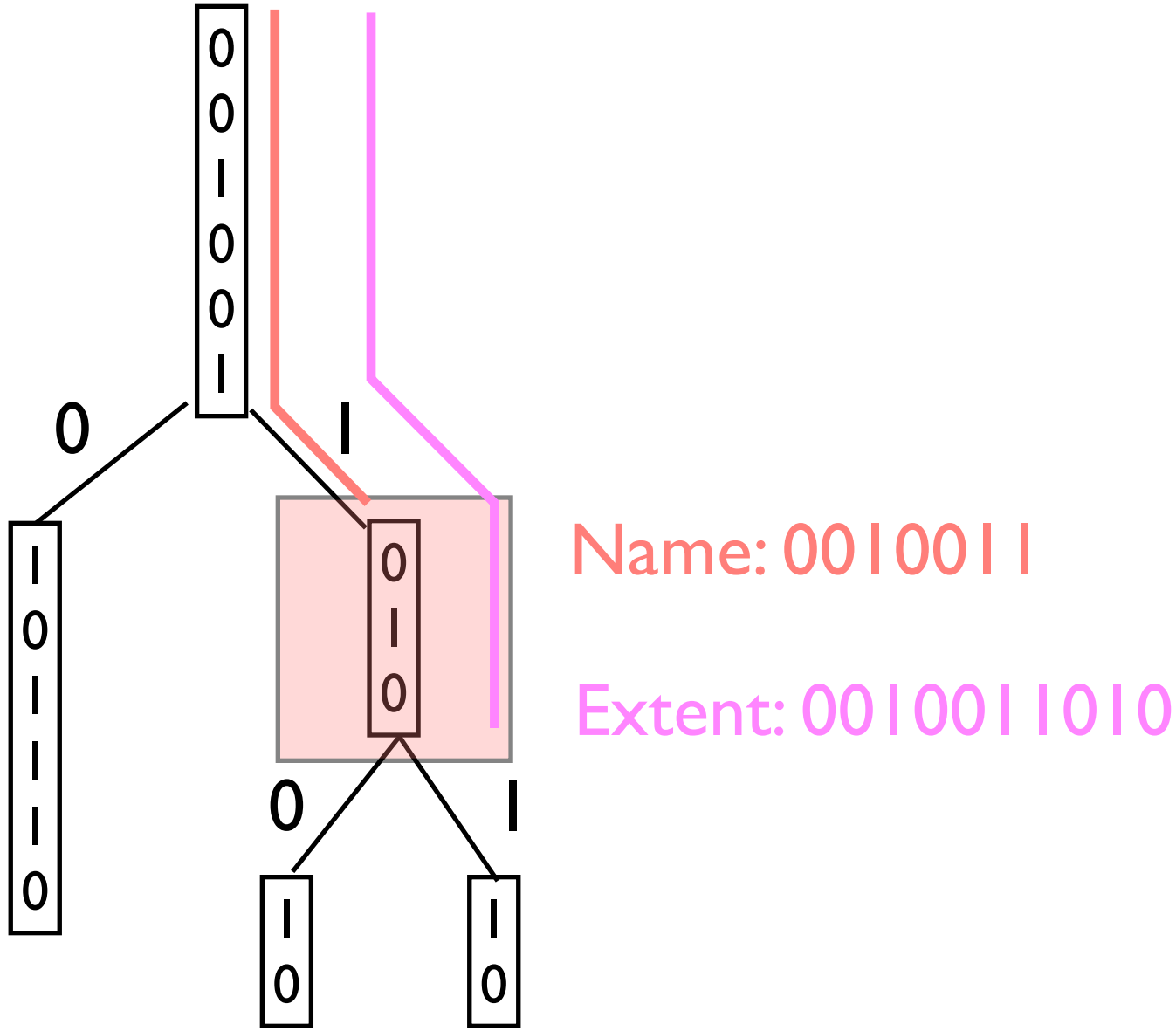


Name: 0010011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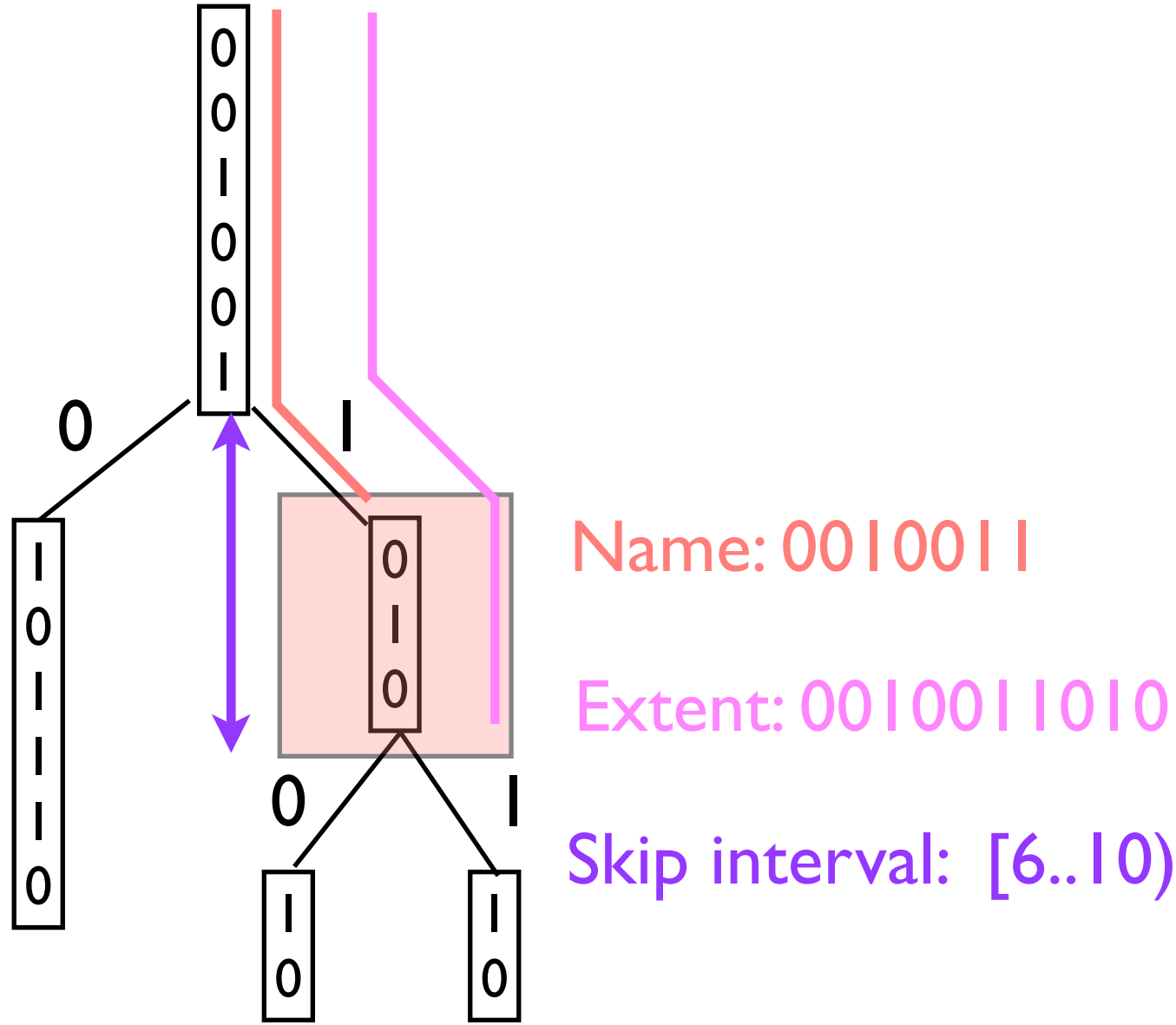
Extent: 00100111010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Name: 0010011
Extent: 00100111010
Skip interval: [6..10)

The Magic Trick

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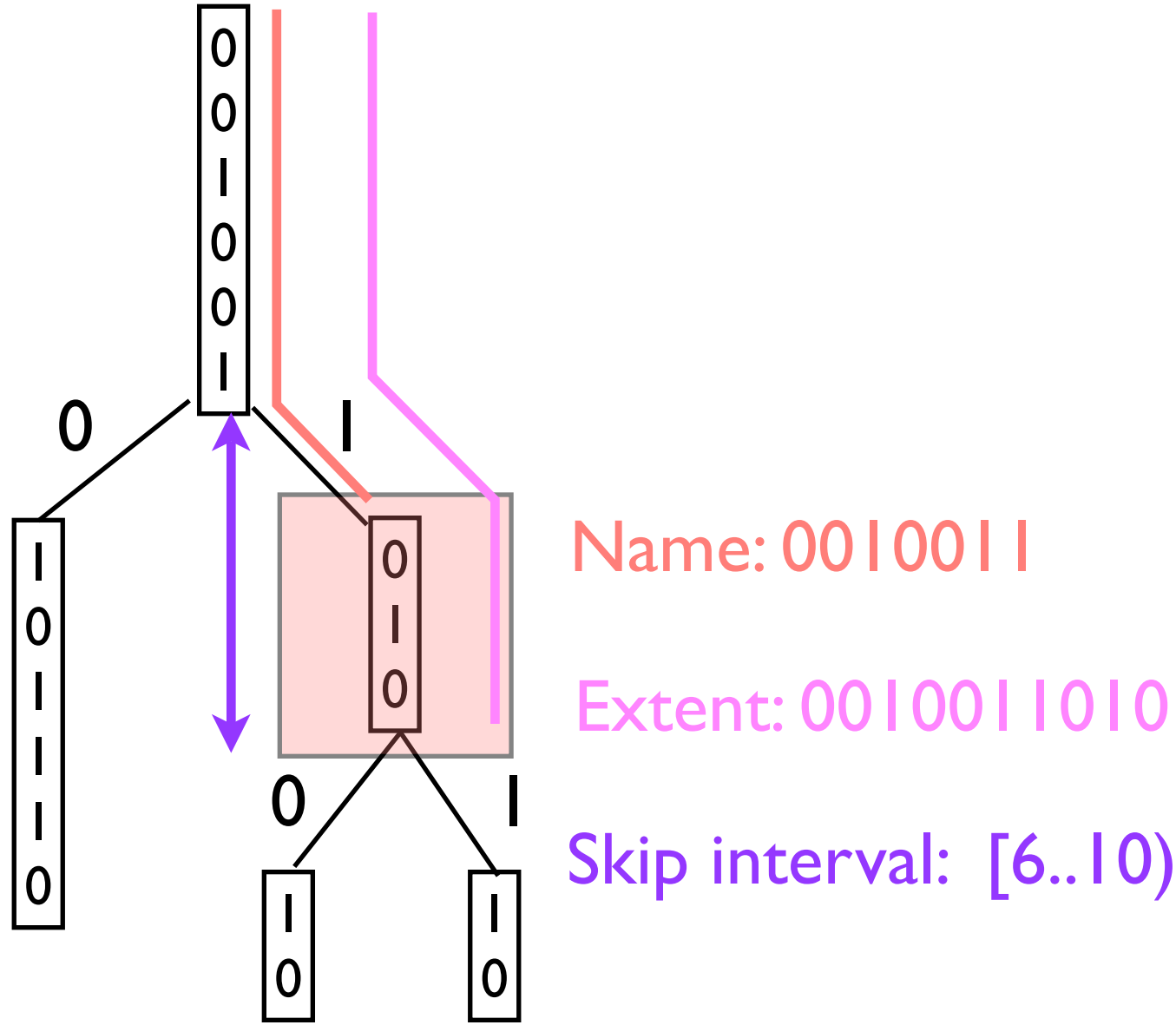
- This mapping requires linear (i.e., $O(nw)$) space and it is sufficient to navigate the trie!
- All in all, the z-fast trie is entirely specified by the mapping from handles to extent prefixes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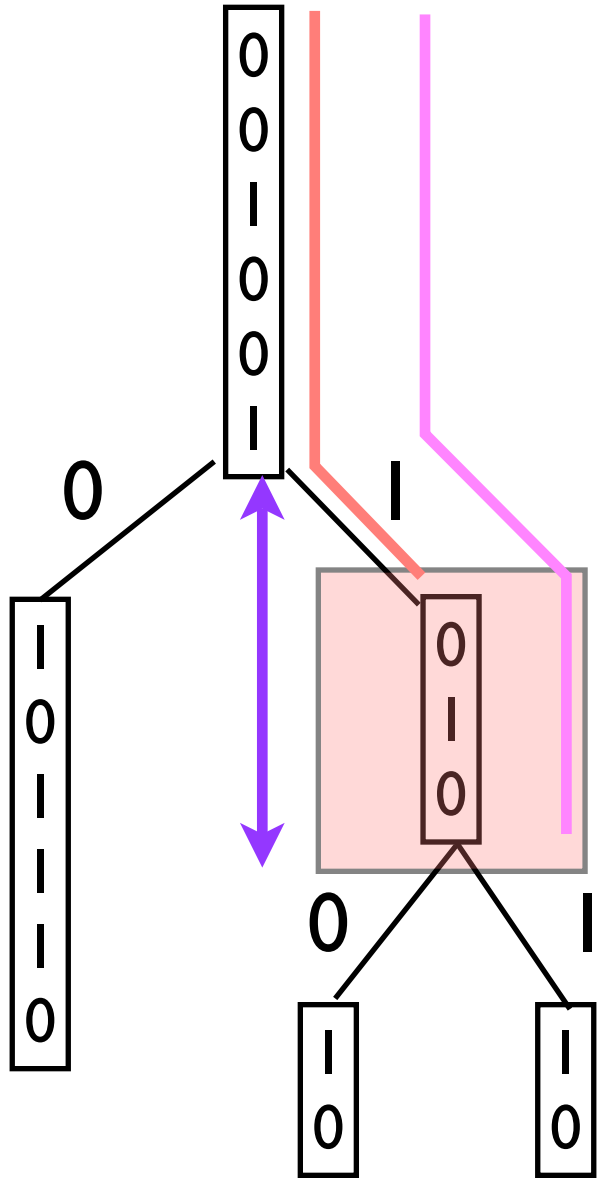
Name: 0010011
Extent: 00100111010
Skip interval: [6..10)

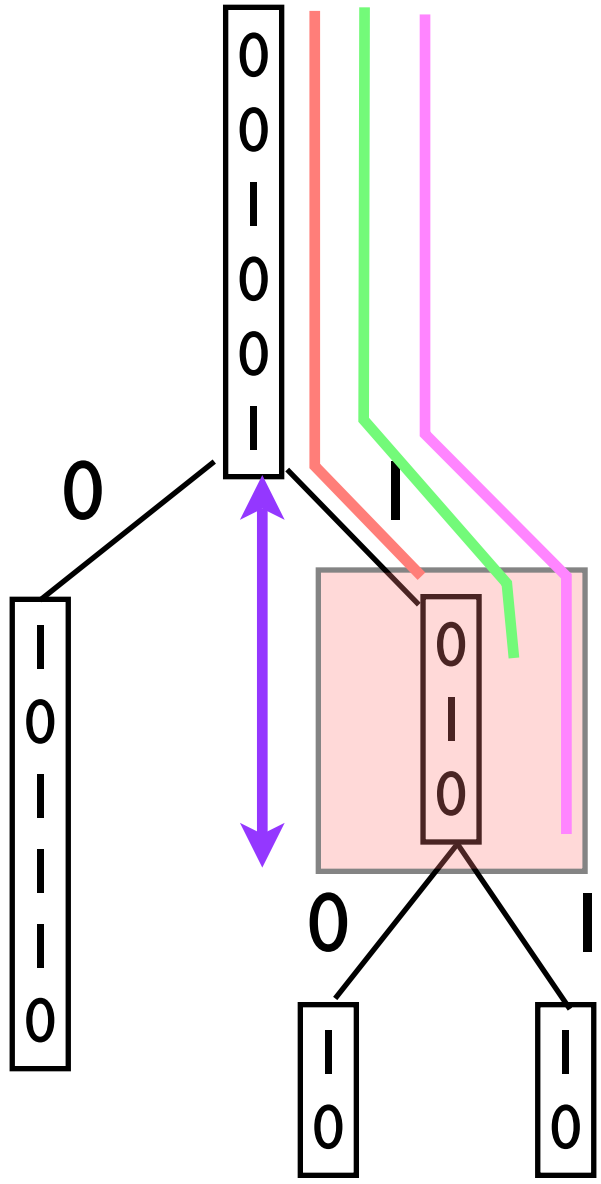


Name: 0010011

Extent: 00100111010

Skip interval: [6..10)

2-fattest is 8



Name: 0010011

Extent: 00100111010

Skip interval: [6..10)

2-fattest is 8

We store the mapping 00100110 → 00100111010

Fat Binary Search

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- A *fat binary search* is similar to a binary search, but probes always insist on the 2-fattest number in an interval, rather than on its middle point

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Notation: substring with indices in the given interval

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- In doing a fat binary search on x, our goal is to find *the longest extent e of the trie that is a prefix of x*
- Clearly, $x[0..|e|+1)$ is the name of the exit node (we are adding one bit from x to e)
- We will not know the exit direction (left/right)

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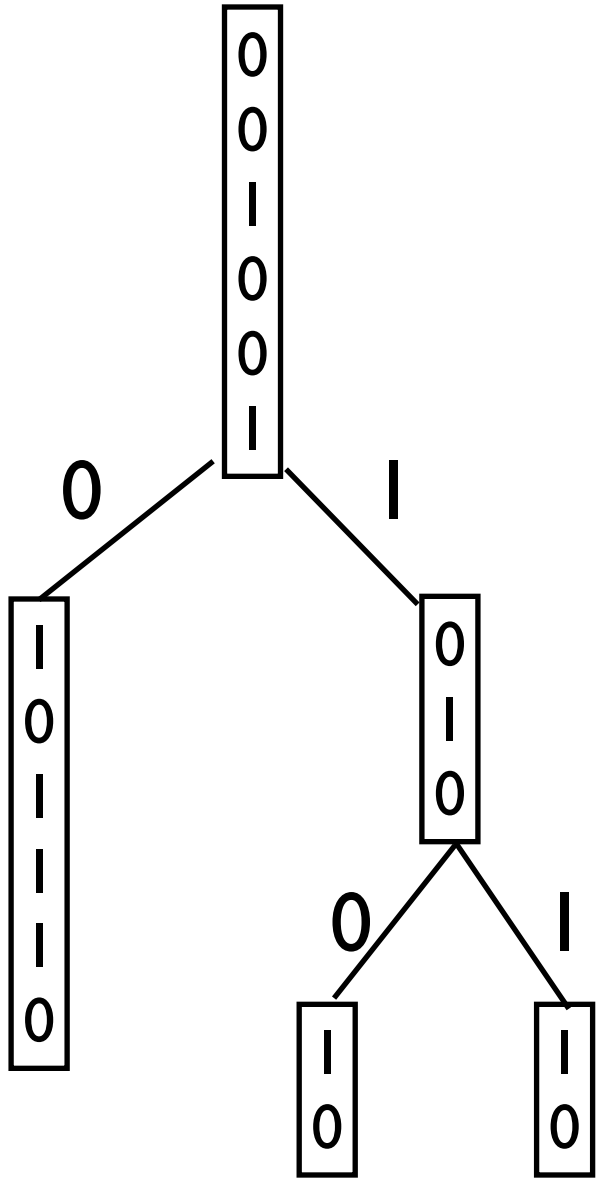
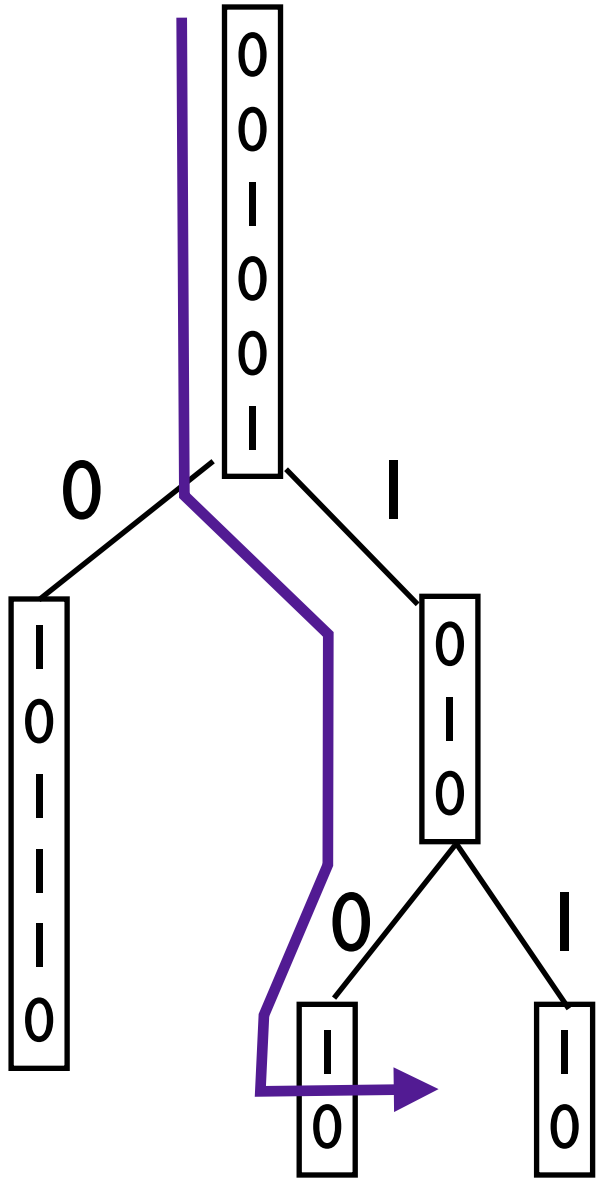
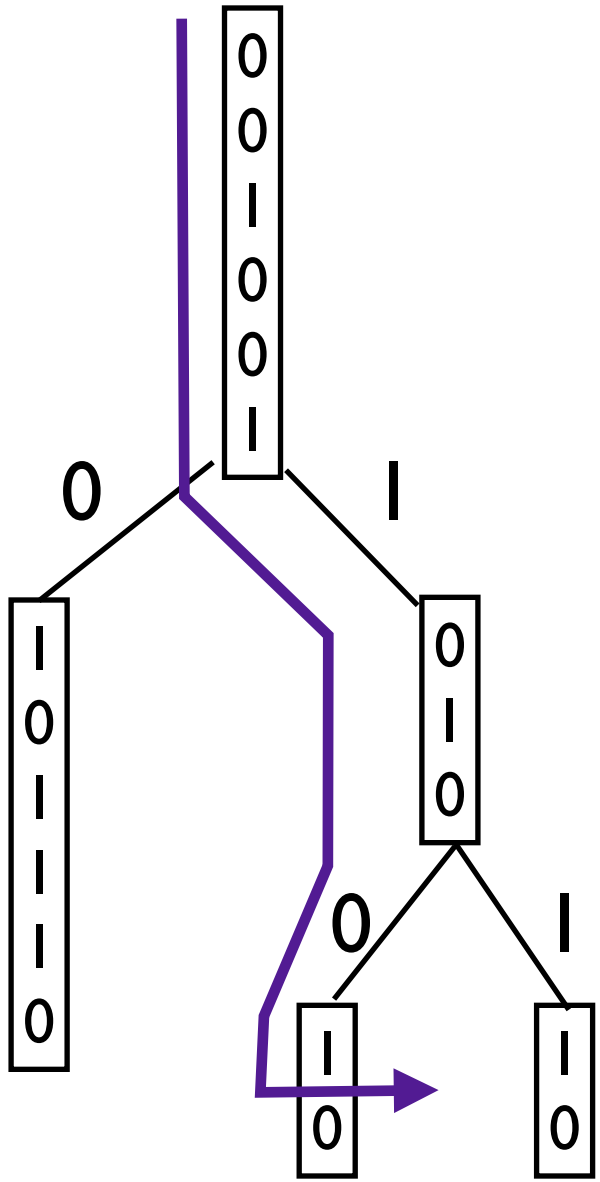
- At each step, we interrogate the trie with the prefix of x of length f—the 2-fattest number in $(l..r)$
- Key property: f is 2-fattest in every subinterval of $(l..r)$ that contains it
- Because of this fact, every time we query the z-fast trie with a prefix of x we will be using a handle, given that it is a prefix of some string in the trie
- If x is a prefix of the resulting string s *and* $s \in (l..r)$, we move to $(|s|..r)$; otherwise, we move to $(l..f)$



x = 0010011010011

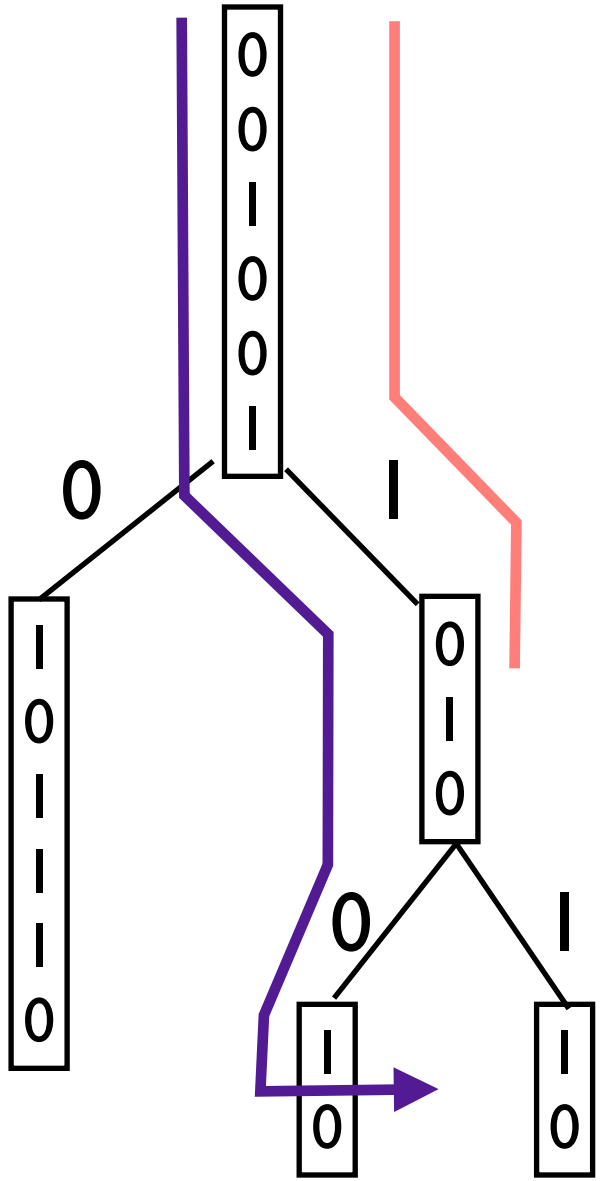


x = 0010011010011



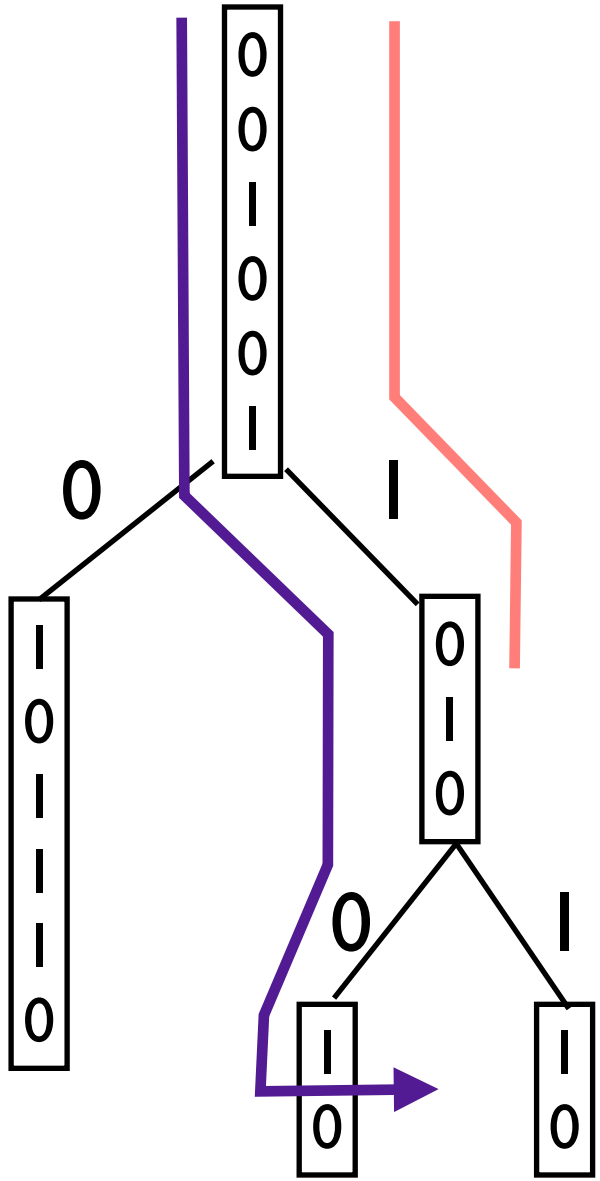
$(l..r) = (0..13) \Rightarrow f = 8$

x = 00100111010011



$(l..r) = (0..13) \Rightarrow f = 8$

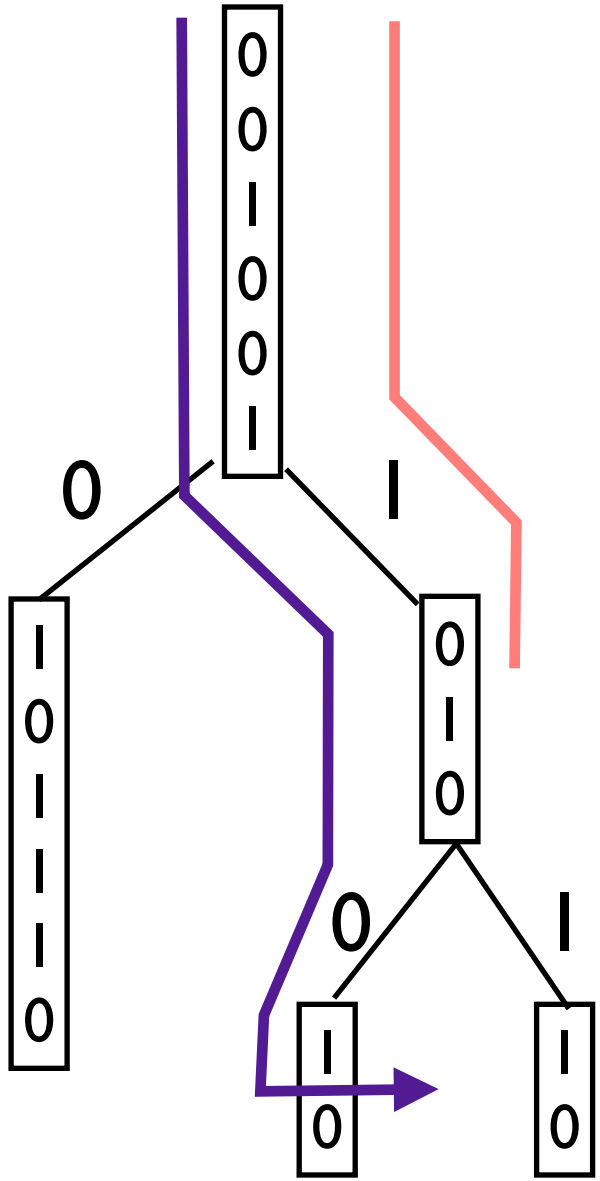
$x = 00100110100111$



$(l..r) = (0..13) \Rightarrow f = 8$

Function returns 00100111010 ≼ x

x = 00100111010011

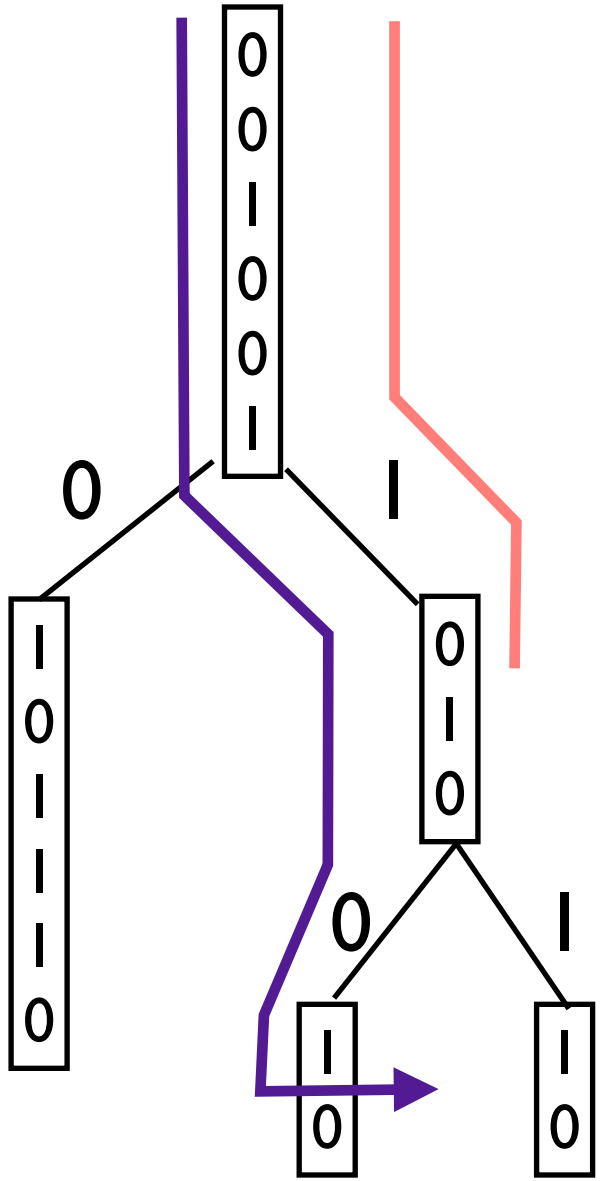


$(l..r) = (0..13) \Rightarrow f = 8$

Function returns 00100111010 $\preccurlyeq x$

$\Rightarrow (l..r) = (10..13) \Rightarrow f = 12$

x = 00100110 10011



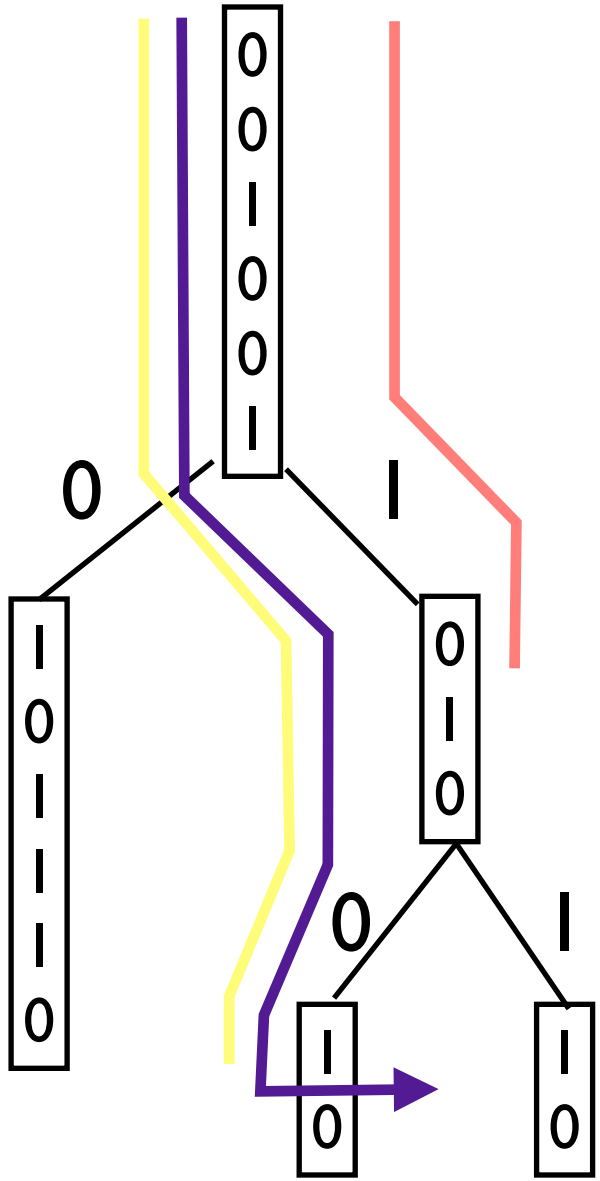
$(l..r) = (0..13) \Rightarrow f = 8$

Function returns $00100111010 \leqslant x$

$\Rightarrow (l..r) = (10..13) \Rightarrow f = 12$

Note: binary search would move to (8..13)!

$x = 00100110\,10011$



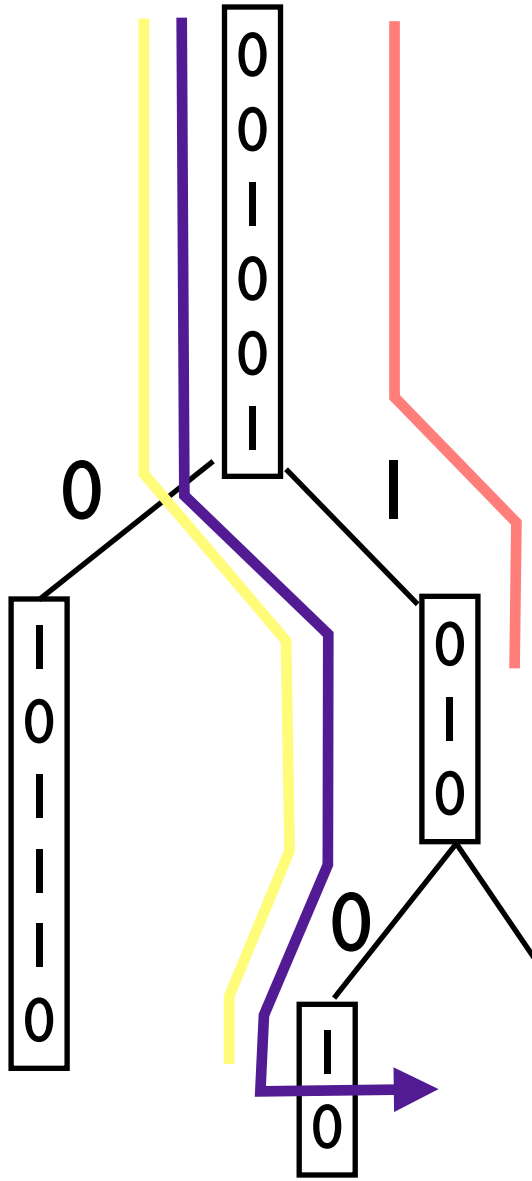
$(l..r) = (0..13) \Rightarrow f = 8$

Function returns $00100011010 \leqslant x$

$\Rightarrow (l..r) = (10..13) \Rightarrow f = 12$

Note: binary search would move to (8..13)!

$x = 00100011010011$



$(l..r) = (0..13) \Rightarrow f = 8$

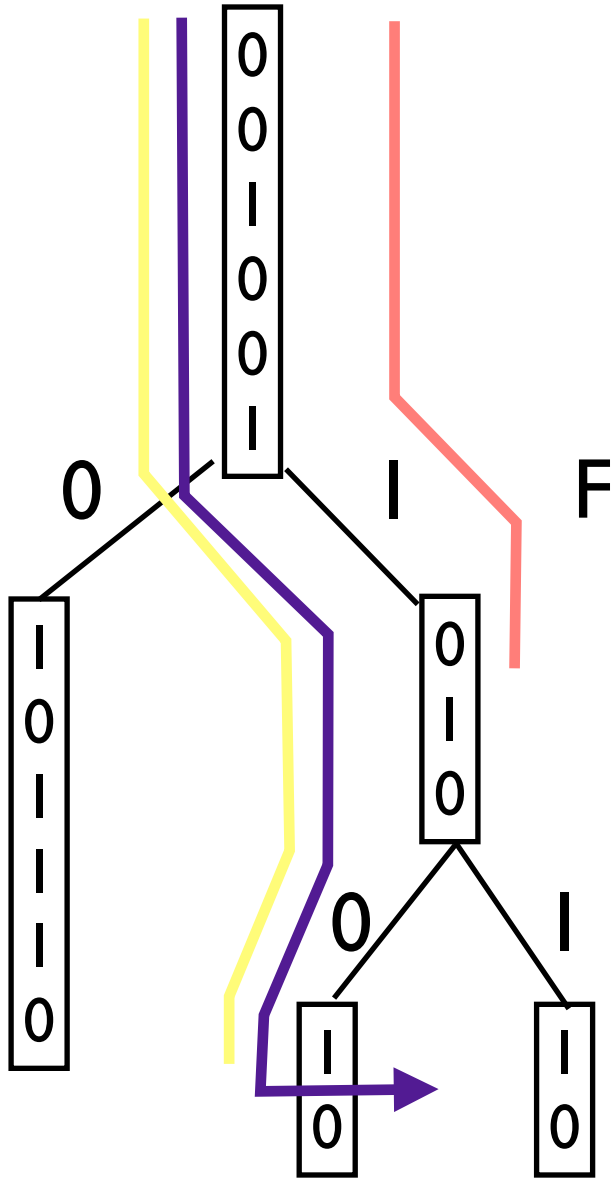
Function returns $0010011010 \leqslant x$

$\Rightarrow (l..r) = (10..13) \Rightarrow f = 12$

Note: binary search would move to (8..13)!

Function returns $00100110100010 \not\leqslant x$

$x = 00100110100111$



$(l..r) = (0..13) \Rightarrow f = 8$

Function returns $0010011010 \preccurlyeq x$

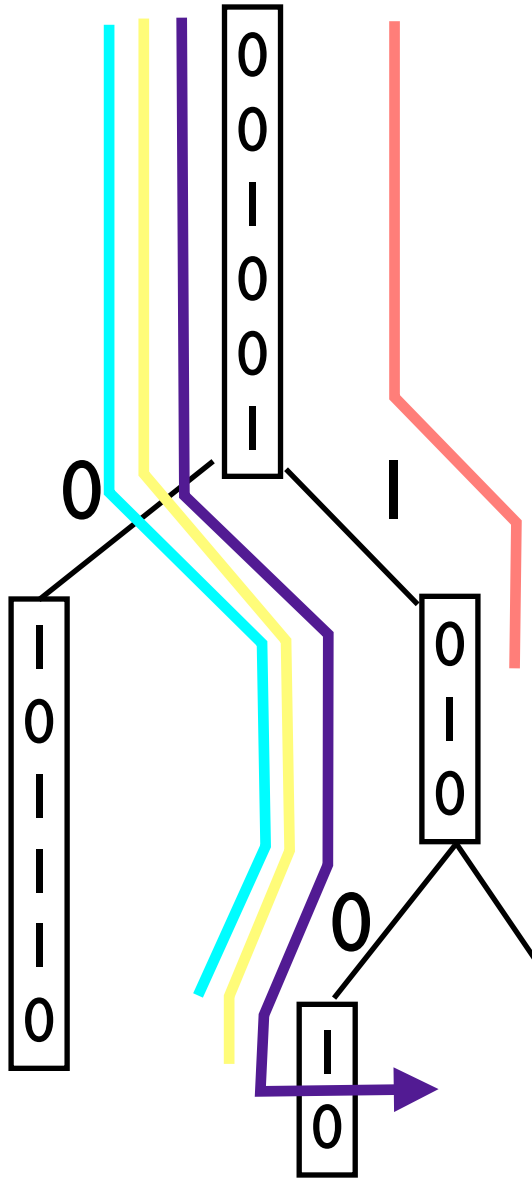
$\Rightarrow (l..r) = (10..13) \Rightarrow f = 12$

Note: binary search would move to (8..13)!

Function returns $00100110010010 \not\preccurlyeq x$

$\Rightarrow (l..r) = (10..12) \Rightarrow f = 11$

x = 00100110100111



$(l..r) = (0..13) \Rightarrow f = 8$

Function returns $0010011010 \preccurlyeq x$

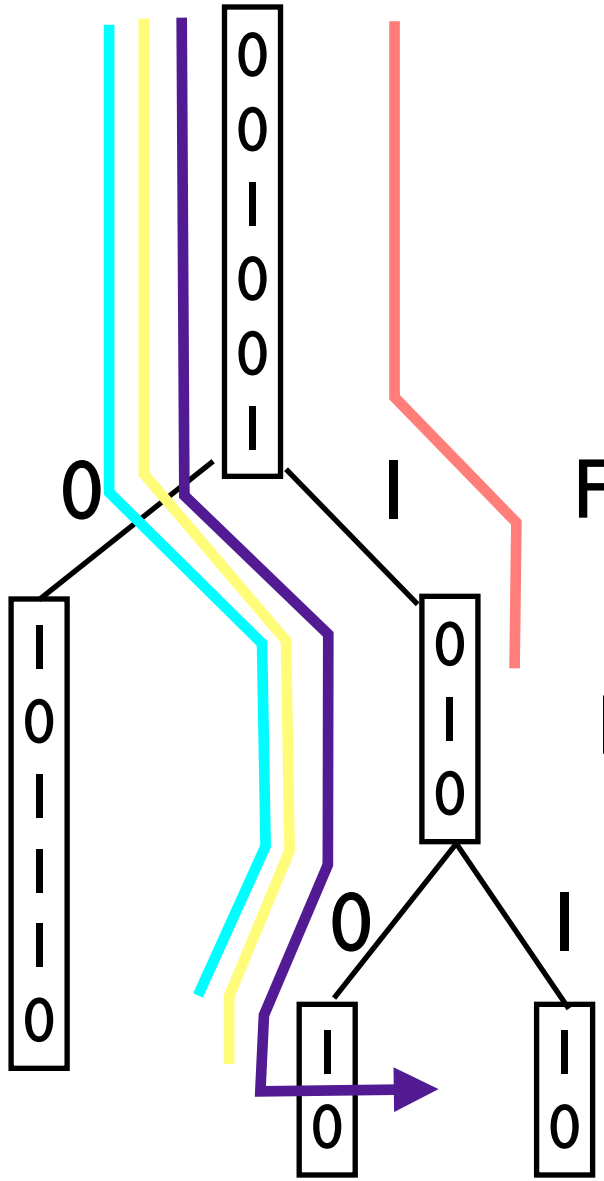
$\Rightarrow (l..r) = (10..13) \Rightarrow f = 12$

Note: binary search would move to (8..13)!

Function returns $00100110100 10 \not\preccurlyeq x$

$\Rightarrow (l..r) = (10..12) \Rightarrow f = 11$

x = 00100110100 11



$(l..r) = (0..13) \Rightarrow f = 8$

Function returns $0010011010 \preccurlyeq x$

$\Rightarrow (l..r) = (10..13) \Rightarrow f = 12$

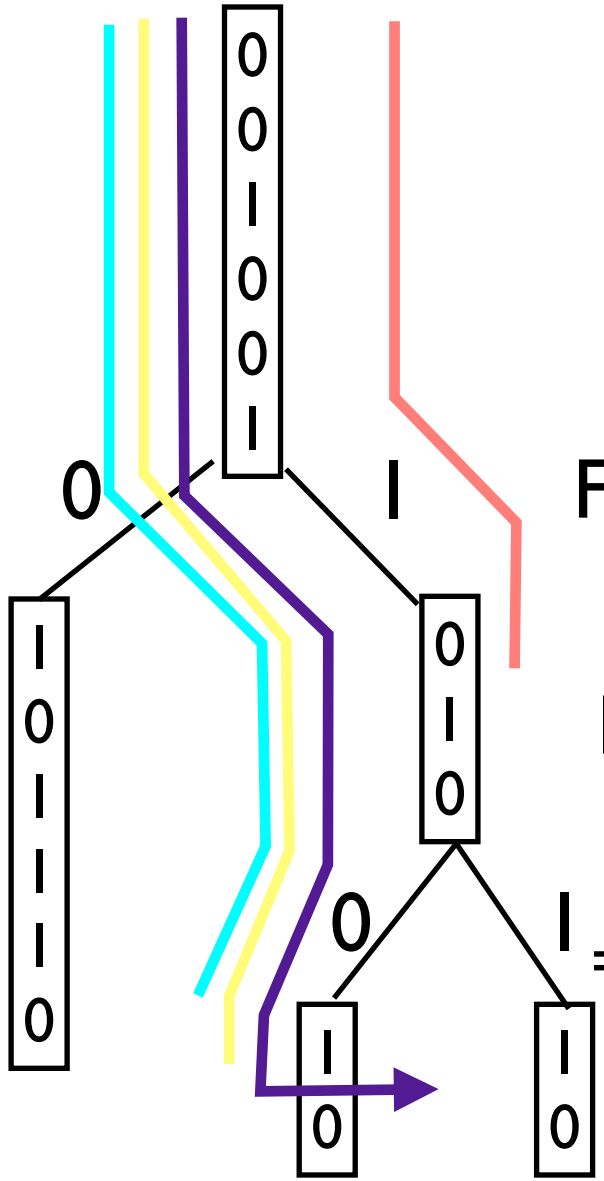
Note: binary search would move to (8..13)!

Function returns $00100110100 10 \not\preccurlyeq x$

$\Rightarrow (l..r) = (10..12) \Rightarrow f = 11$

Function returns a random string s and either $s \not\preccurlyeq x$ or $|s| \leq 10$

$x =$ 00100110100 11



$(l..r) = (0..13) \Rightarrow f = 8$

Function returns $0010011010 \preccurlyeq x$

Note: binary search would move to (8..13)!

$\Rightarrow (l..r) = (10..13) \Rightarrow f = 12$

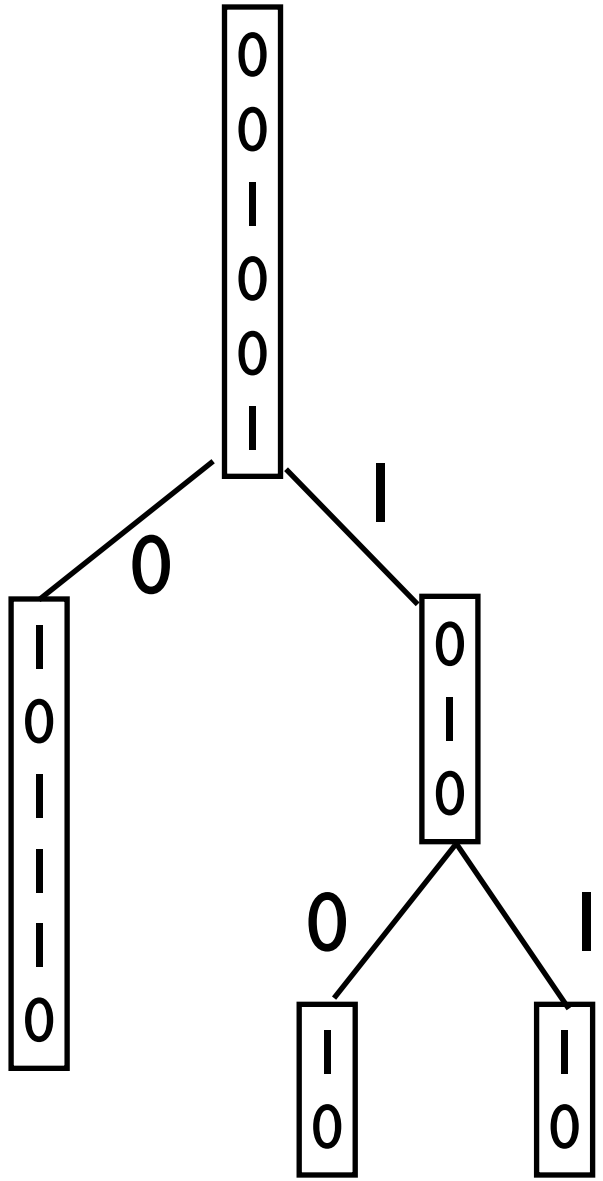
Function returns $00100110100 10 \not\preccurlyeq x$

$\Rightarrow (l..r) = (10..12) \Rightarrow f = 11$

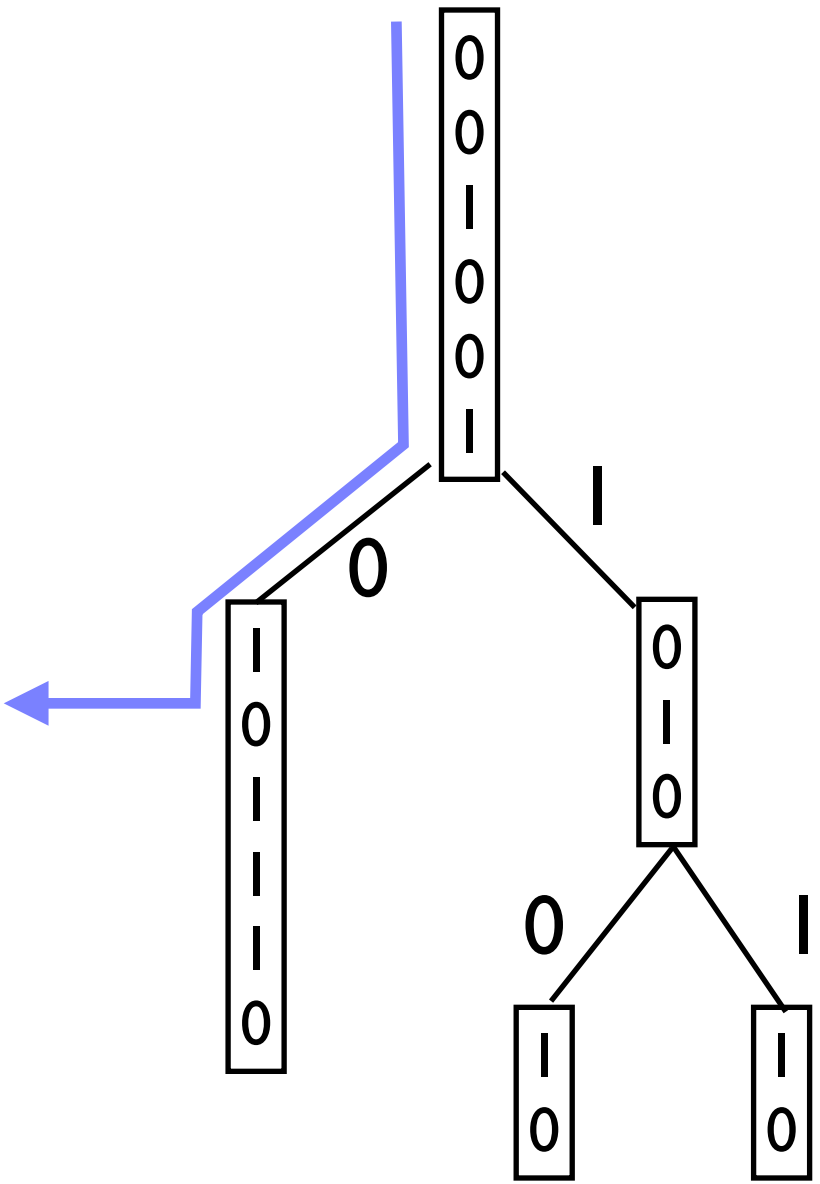
Function returns a random string s and either $s \not\preccurlyeq x$ or $|s| \leq 10$

$\Rightarrow x$ exits at node with name x[0..11)

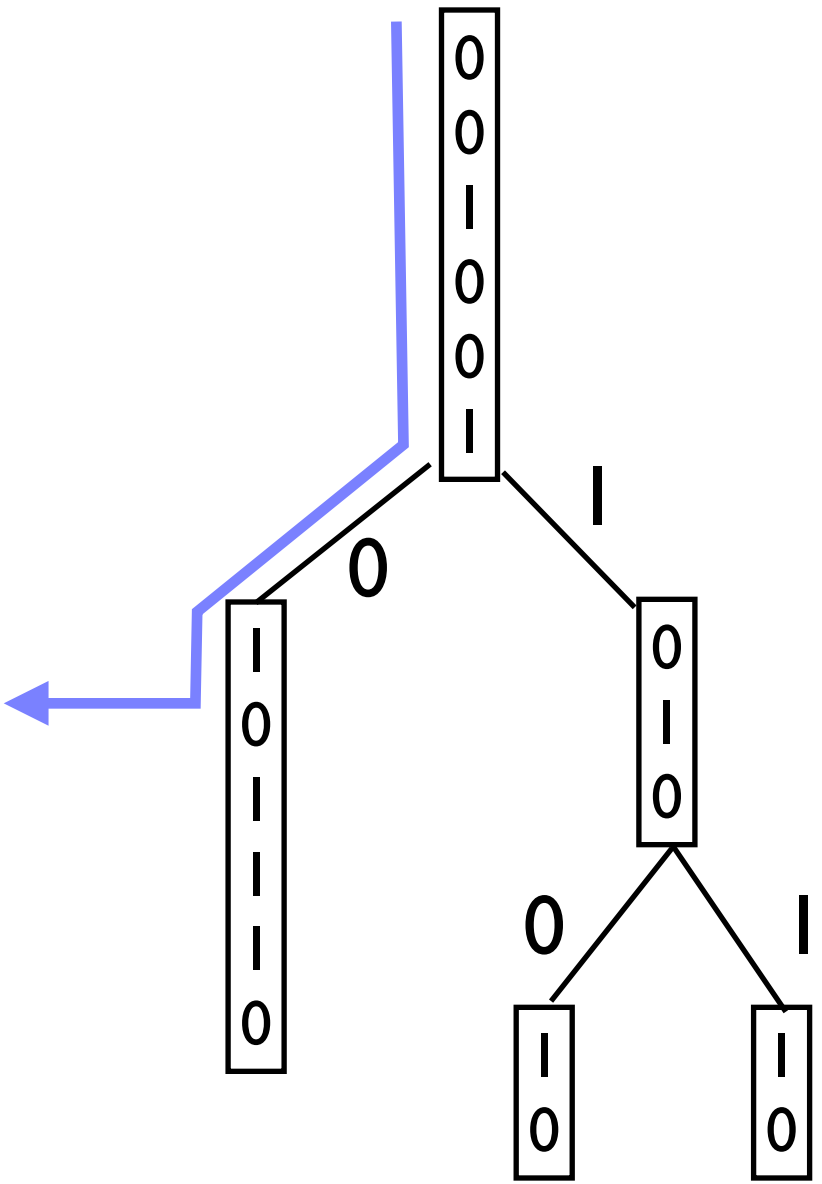
x = 00100110100 11



x = 001001010011 0

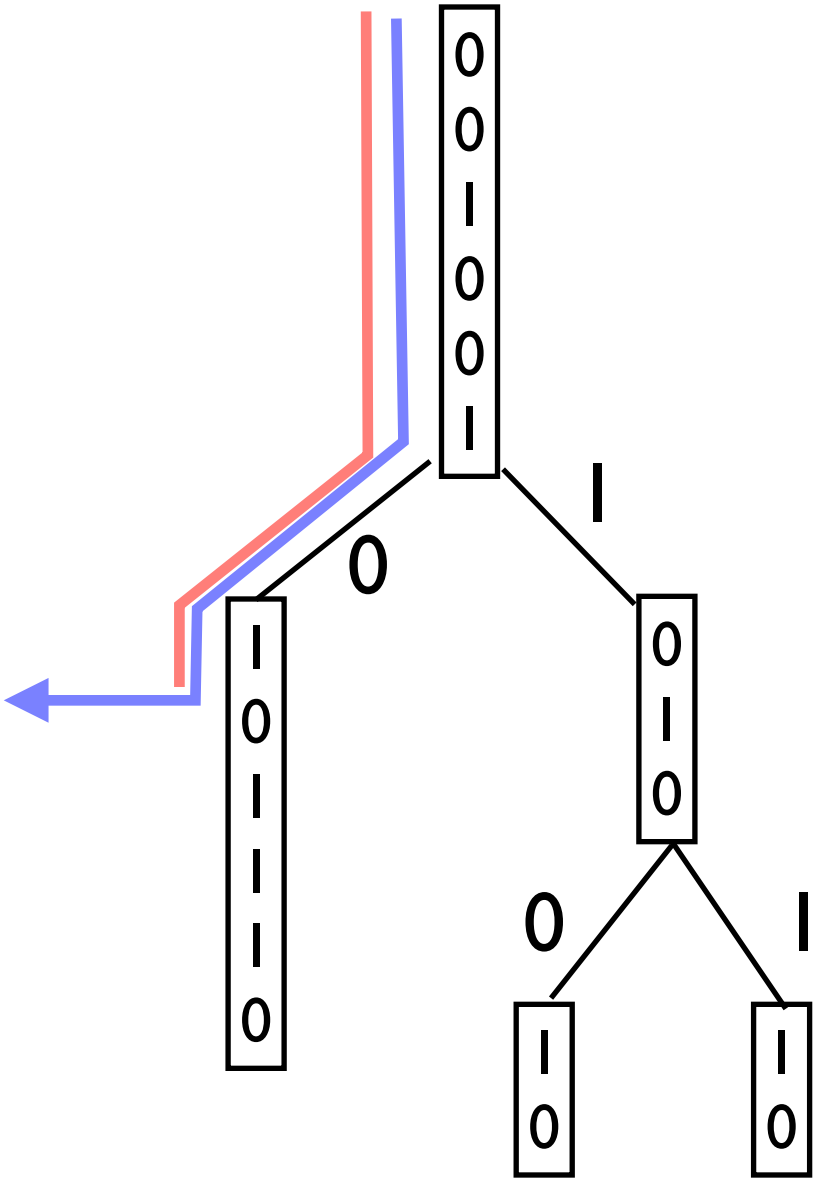


x = 001001010011 0



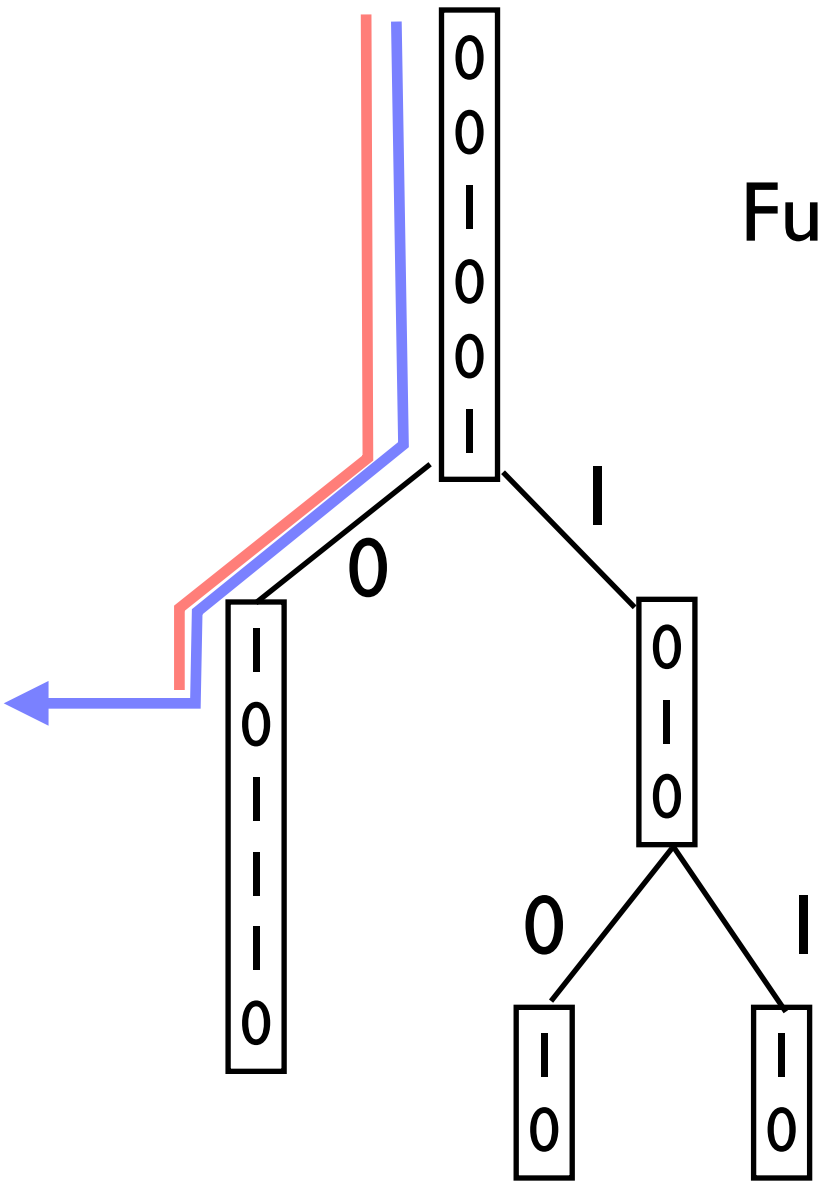
$(l..r) = (0..13) \Rightarrow f = 8$

x = 00100101001110



$(l..r) = (0..13) \Rightarrow f = 8$

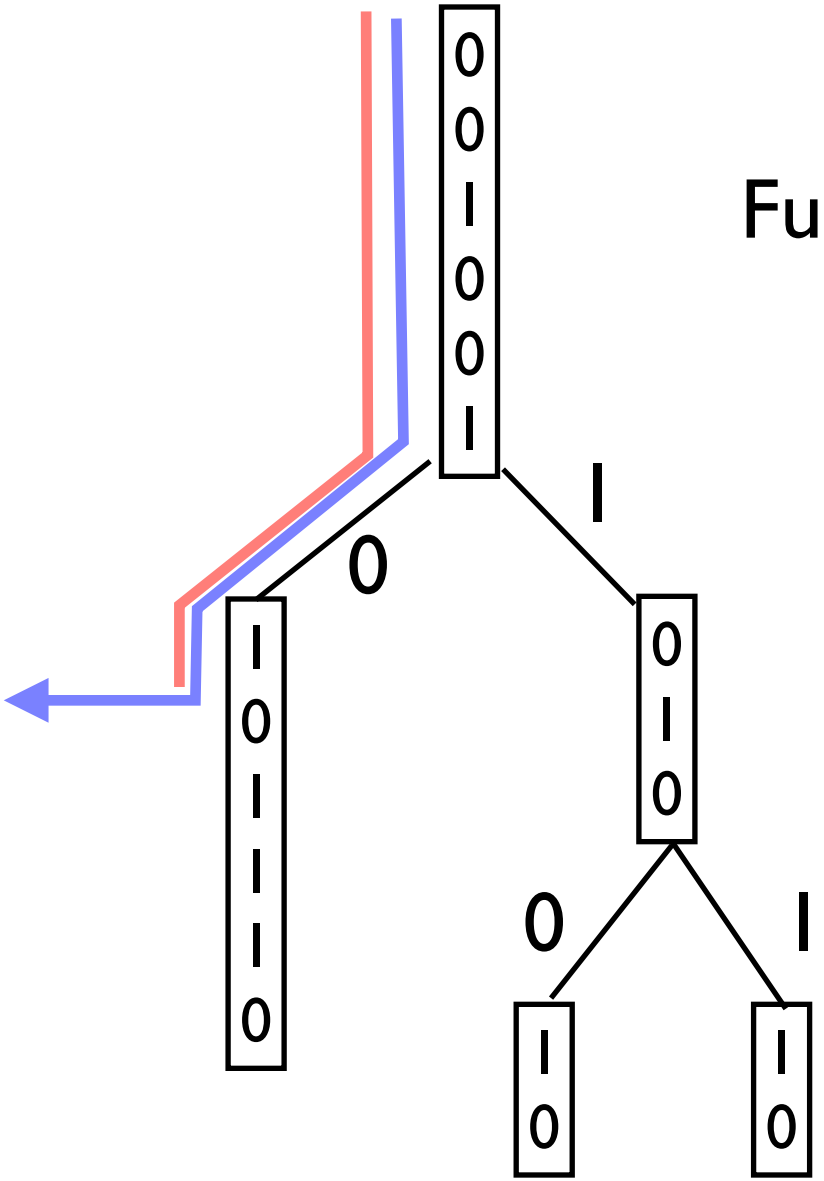
$x = 00100101\,00110$



$(l..r) = (0..13) \Rightarrow f = 8$

Function returns $00100101011110 \nleq x$

$x = 00100101\,00110$

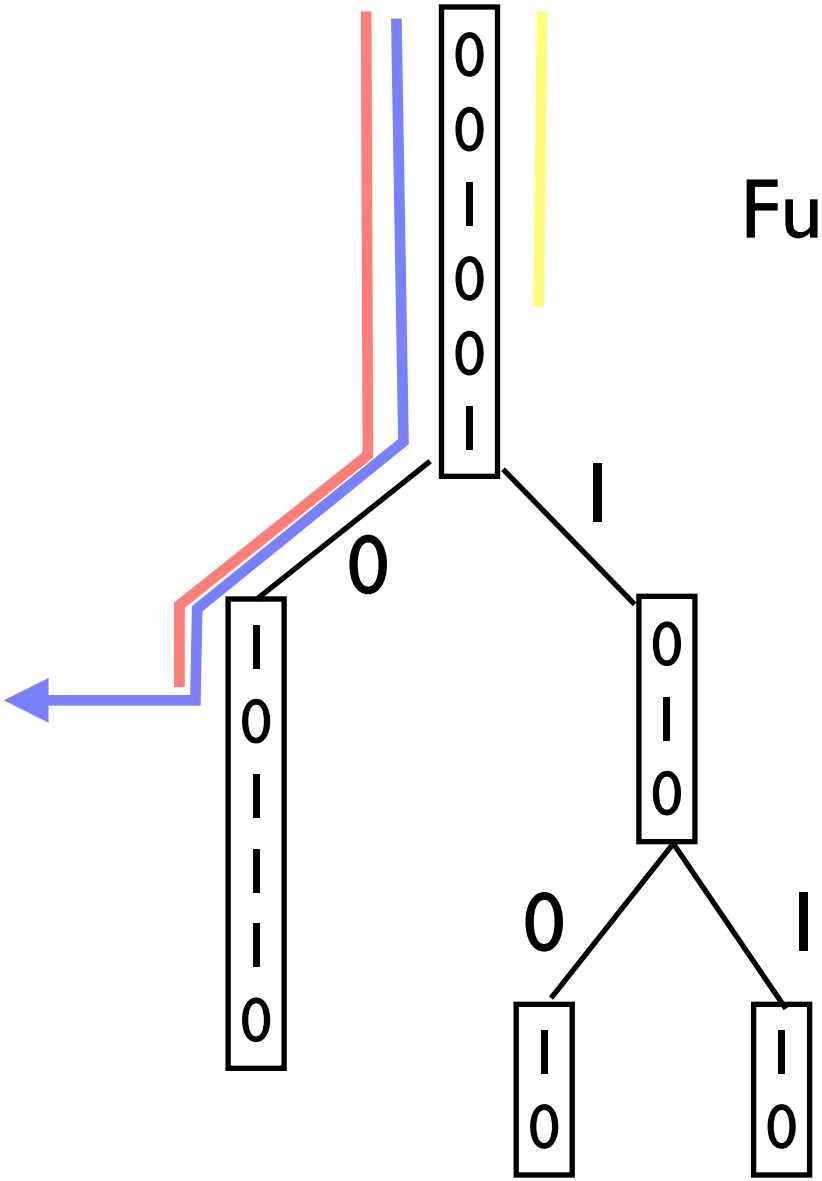


$(l..r) = (0..13) \Rightarrow f = 8$

Function returns 00100101011110 $\not< x$

$\Rightarrow (l..r) = (0..8) \Rightarrow f = 4$

x = 00100101 00110

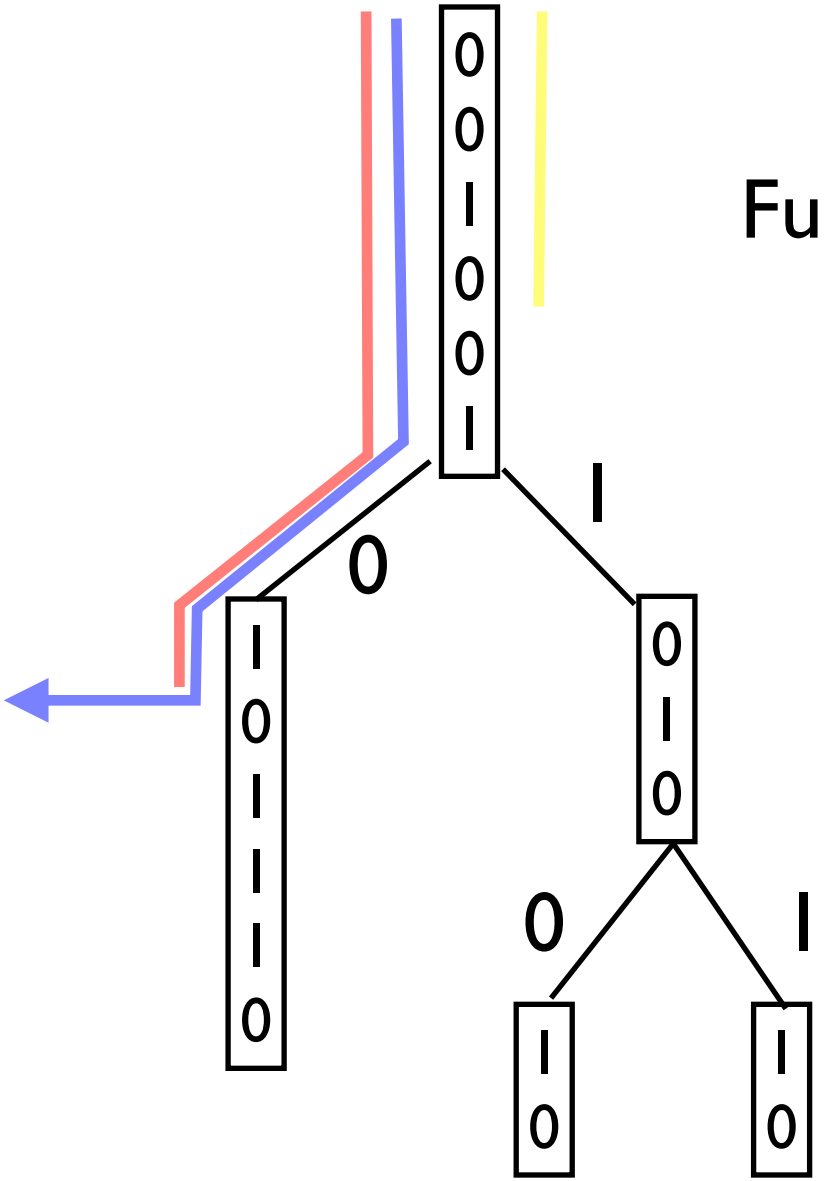


$(l..r) = (0..13) \Rightarrow f = 8$

Function returns 00100101011110 $\nleq x$

$\Rightarrow (l..r) = (0..8) \Rightarrow f = 4$

x = 00100101 00110



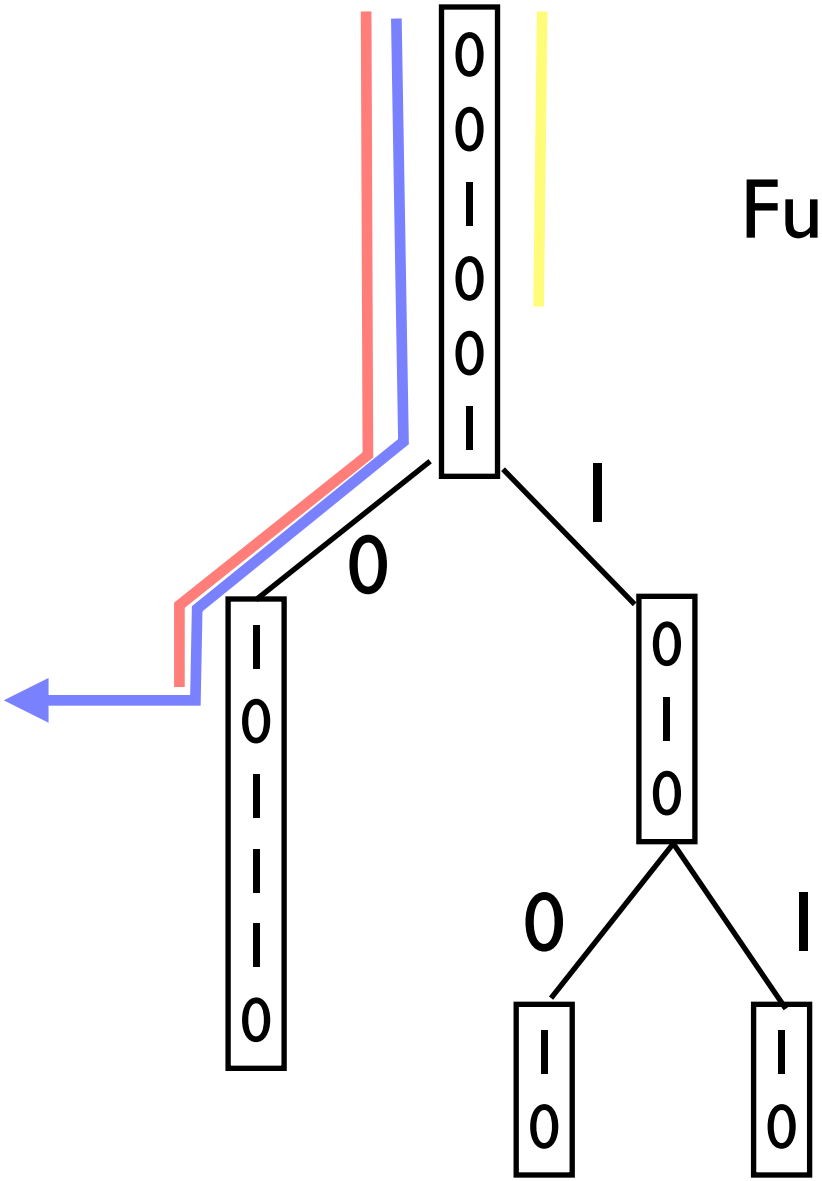
$(l..r) = (0..13) \Rightarrow f = 8$

Function returns 00100101011110 $\not\preccurlyeq x$

$\Rightarrow (l..r) = (0..8) \Rightarrow f = 4$

Function returns 001001 $\preccurlyeq x$

x = 00100101 00110



$(l..r) = (0..13) \Rightarrow f = 8$

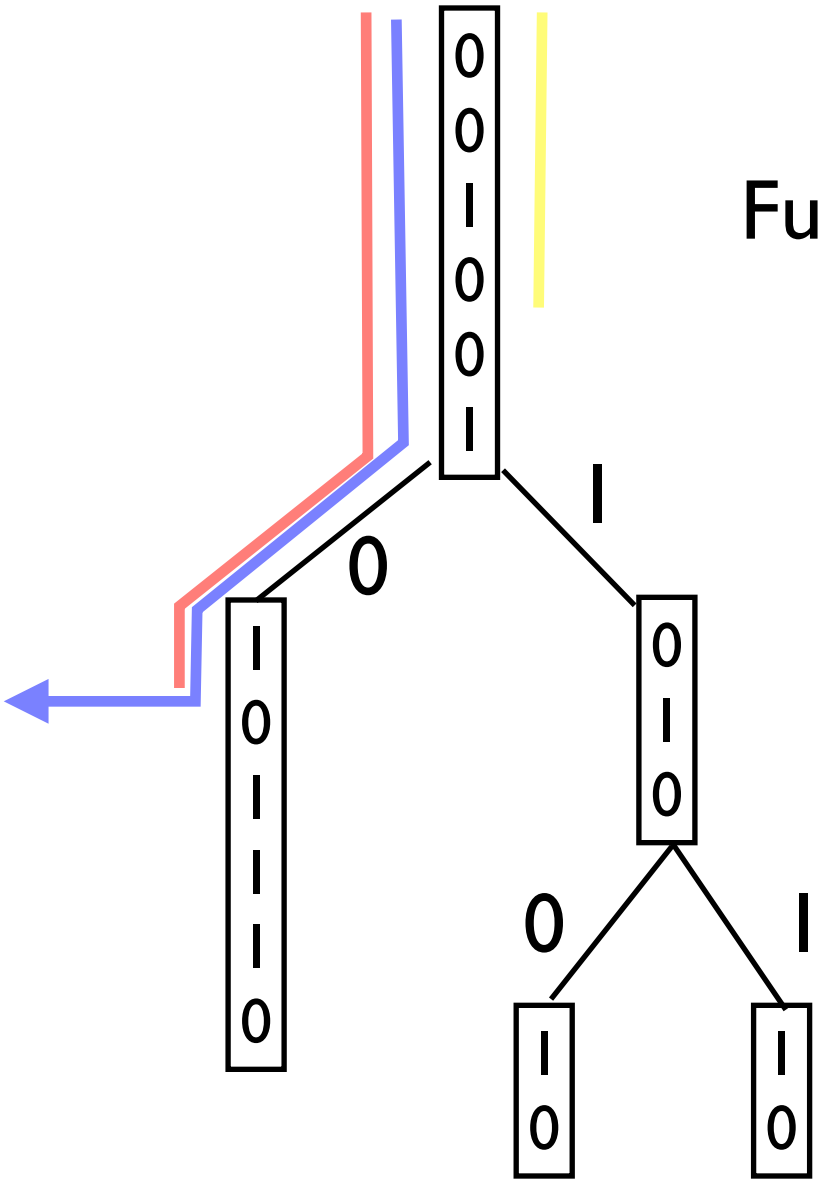
Function returns 00100101011110 $\nleq x$

$\Rightarrow (l..r) = (0..8) \Rightarrow f = 4$

Function returns 001001 $\leqslant x$

$\Rightarrow (l..r) = (6..8) \Rightarrow f = 7$

x = 00100101 00110



$(l..r) = (0..13) \Rightarrow f = 8$

Function returns 00100101011110 $\nleqslant x$

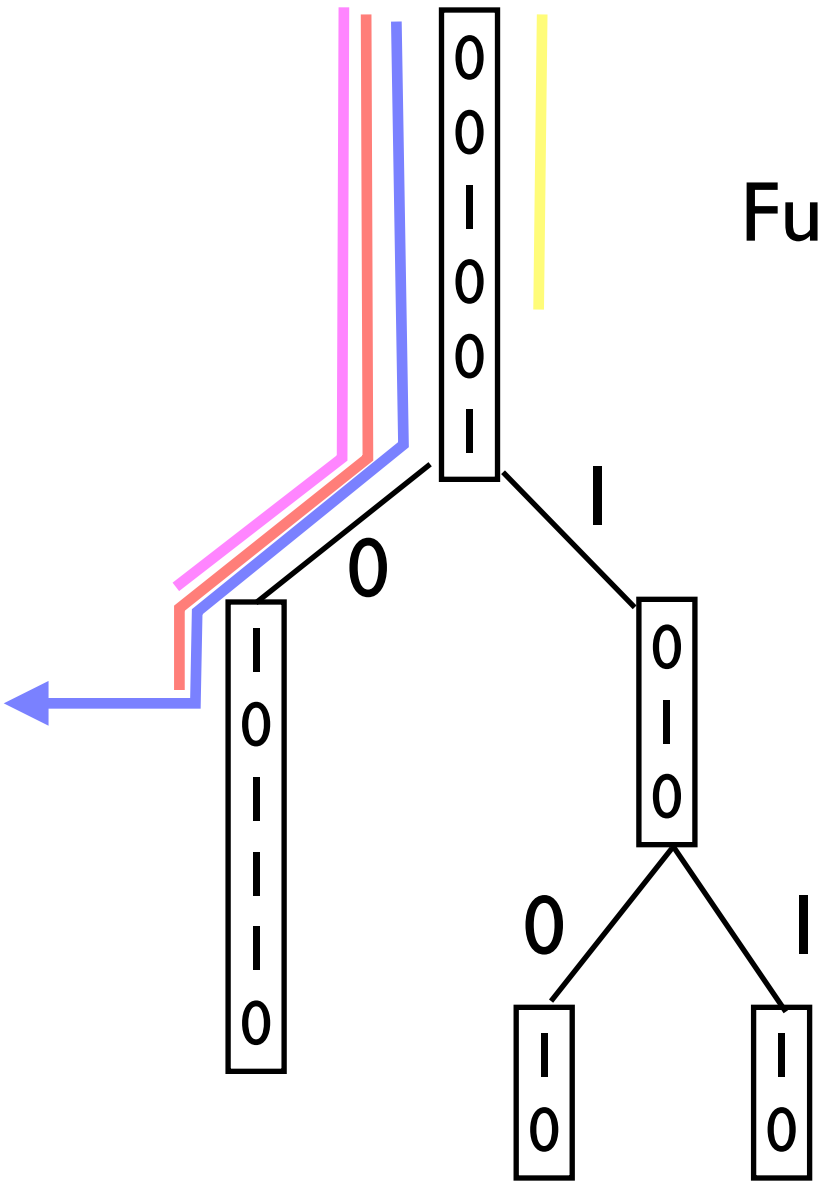
$\Rightarrow (l..r) = (0..8) \Rightarrow f = 4$

Function returns 001001 $\leqslant x$

$\Rightarrow (l..r) = (6..8) \Rightarrow f = 7$

Note: binary search would choose 6!

x = 00100101 00110



$(l..r) = (0..13) \Rightarrow f = 8$

Function returns 00100101011110 $\nleq x$

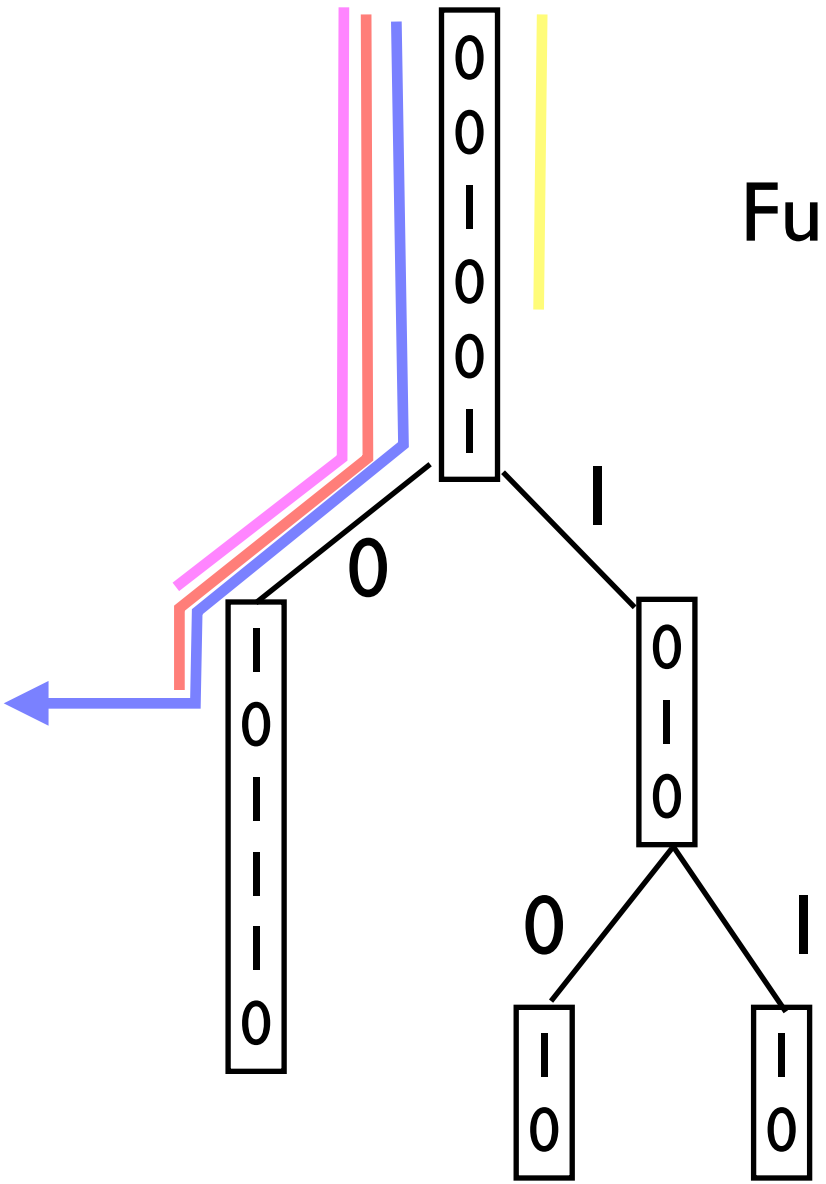
$\Rightarrow (l..r) = (0..8) \Rightarrow f = 4$

Function returns 001001 $\leqslant x$

$\Rightarrow (l..r) = (6..8) \Rightarrow f = 7$

Note: binary search would choose 6!

x = 00100101100110



$(l..r) = (0..13) \Rightarrow f = 8$

Function returns $00100101011110 \not\preccurlyeq x$

$\Rightarrow (l..r) = (0..8) \Rightarrow f = 4$

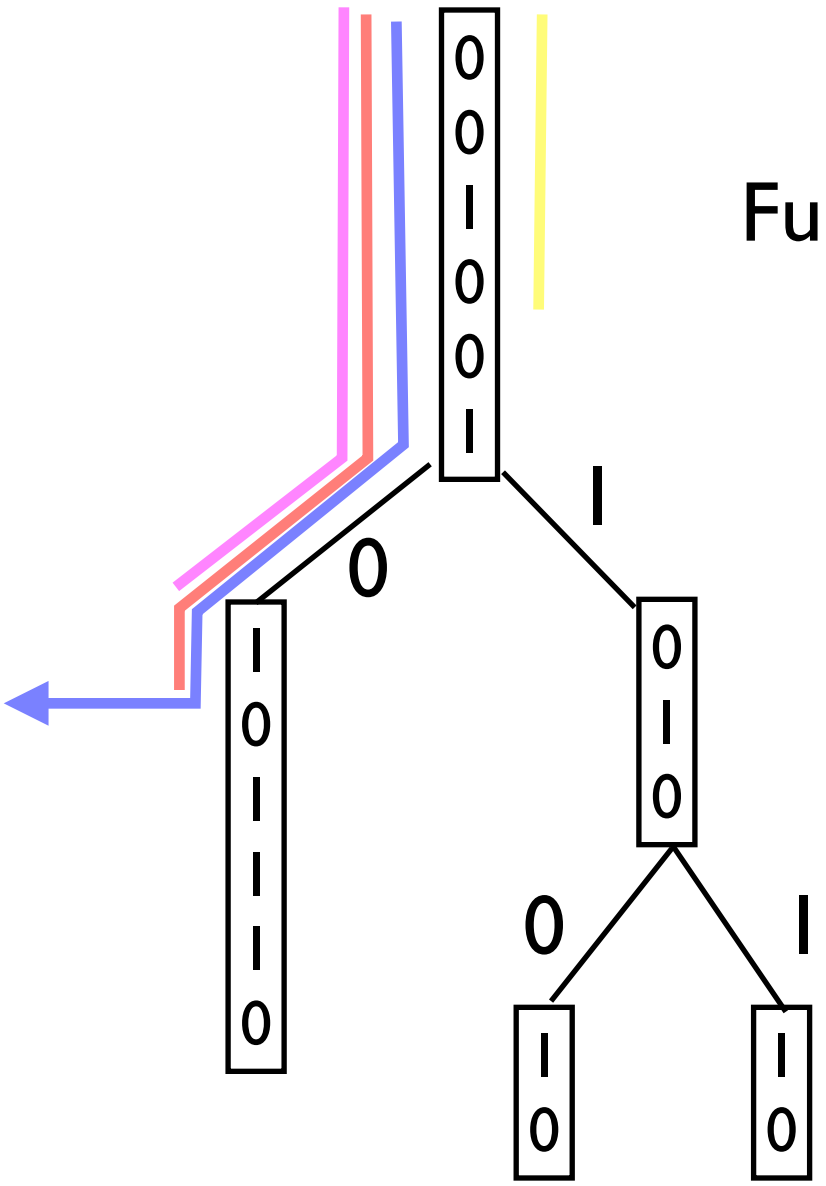
Function returns $001001 \preccurlyeq x$

$\Rightarrow (l..r) = (6..8) \Rightarrow f = 7$

Function returns a random string

Note: binary search would choose 6!

$x = 00100101100110$



$(l..r) = (0..13) \Rightarrow f = 8$

Function returns $00100101011110 \nleq x$

$\Rightarrow (l..r) = (0..8) \Rightarrow f = 4$

Function returns $001001 \leqslant x$

$\Rightarrow (l..r) = (6..8) \Rightarrow f = 7$

Function returns a random string

Note: binary search would choose 6!

$\Rightarrow x$ exits at node x[0..7)

x = 00100101100110

We Don’t Keep The Paths!

We Don’t Keep The Paths!

- The main point of all this is that we will *not* map handles to extents, but handles to log *w* bits *signatures* of extents

We Don’t Keep The Paths!

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- So how can we use the z-fast trie built on the delimiter set *D* as a distributor?

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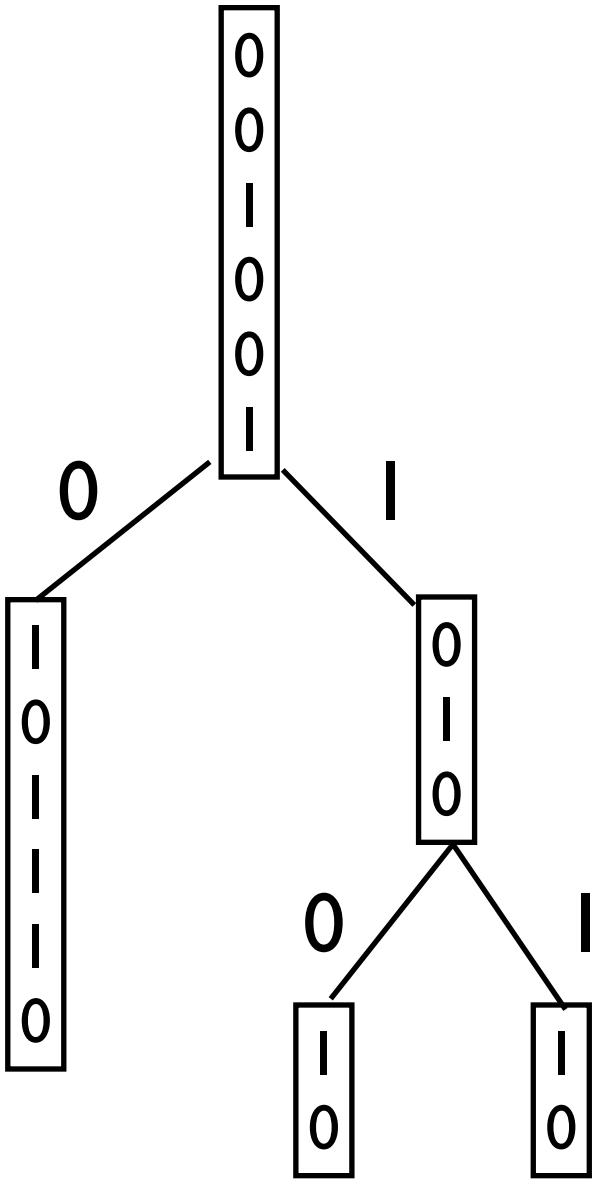
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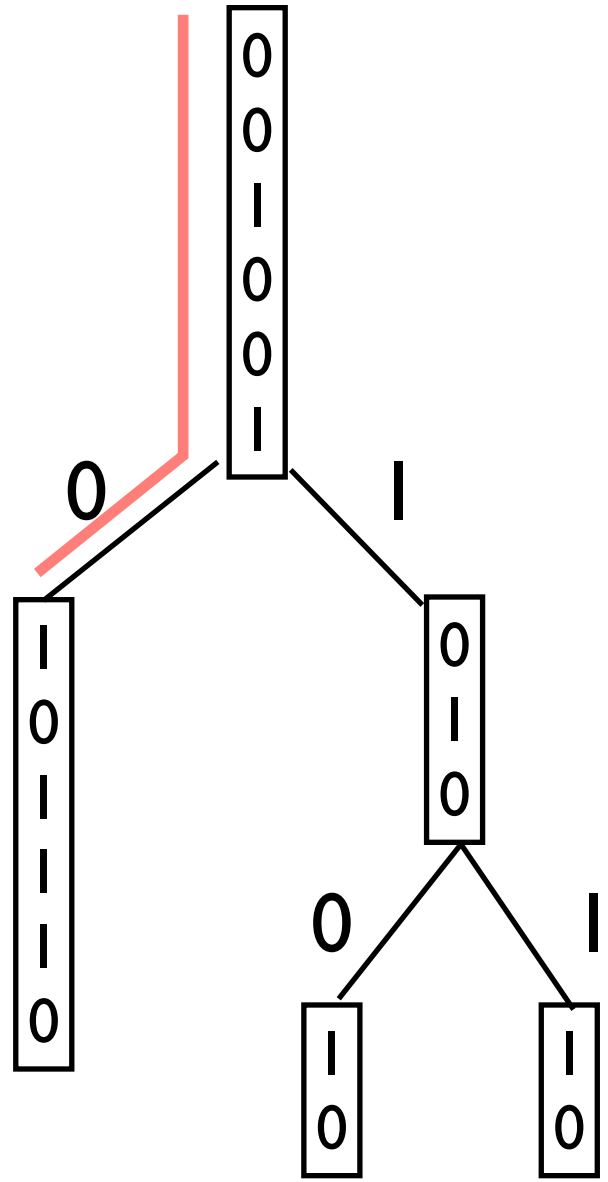
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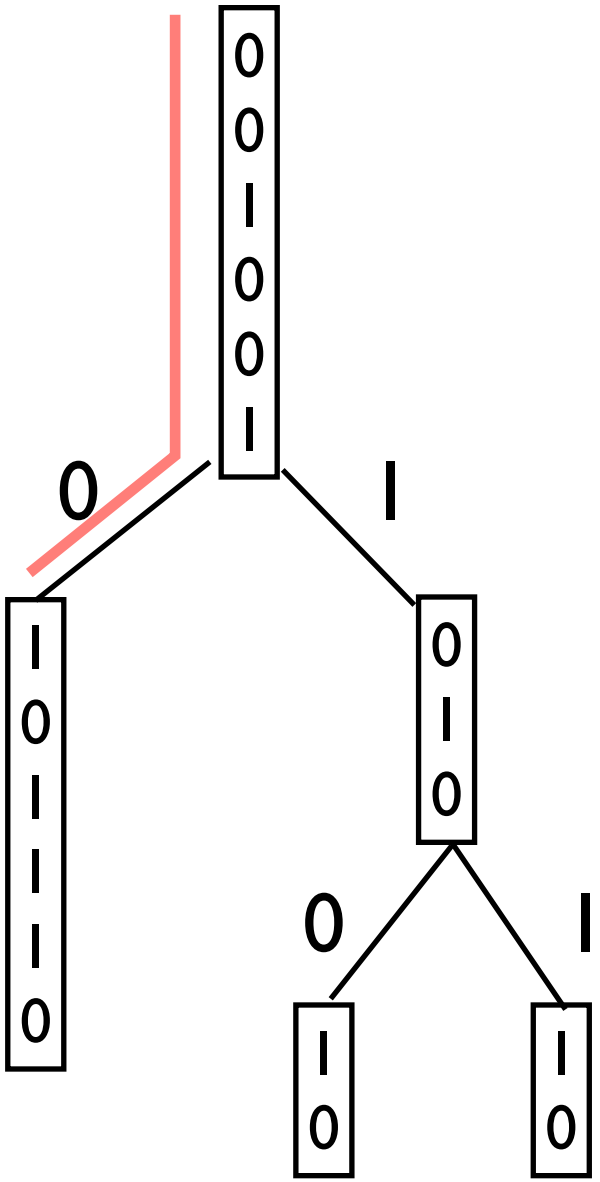

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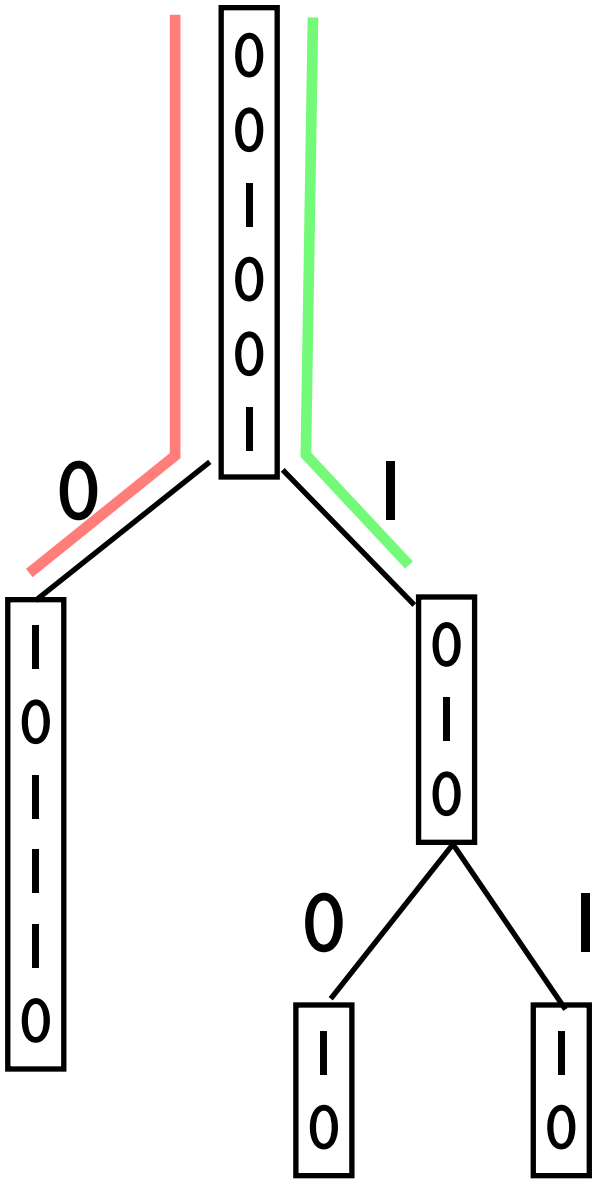

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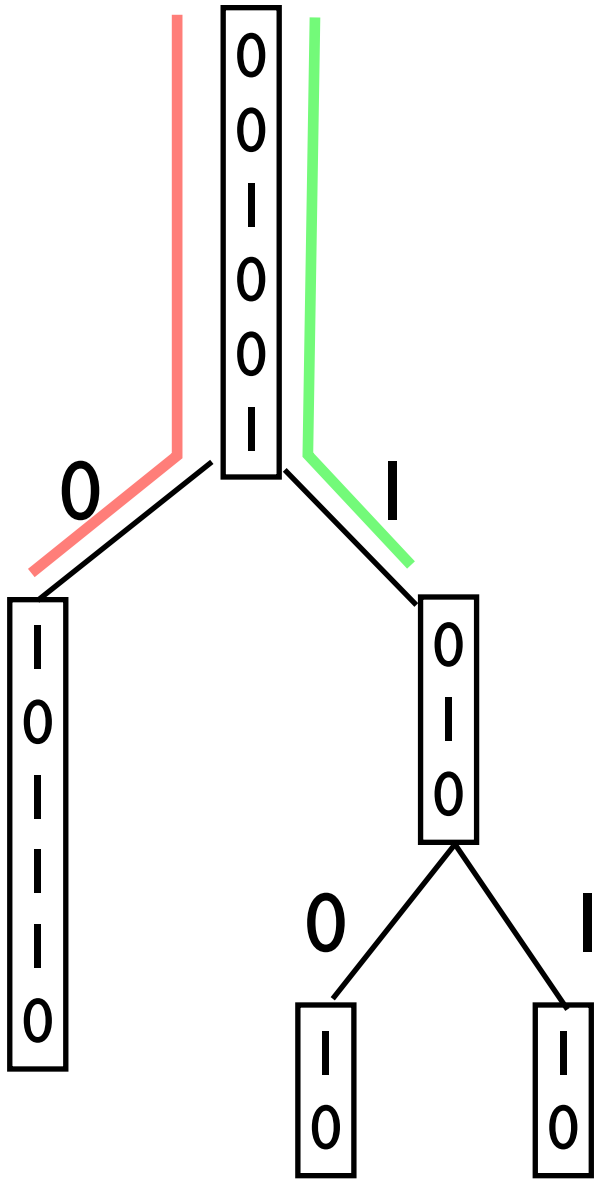

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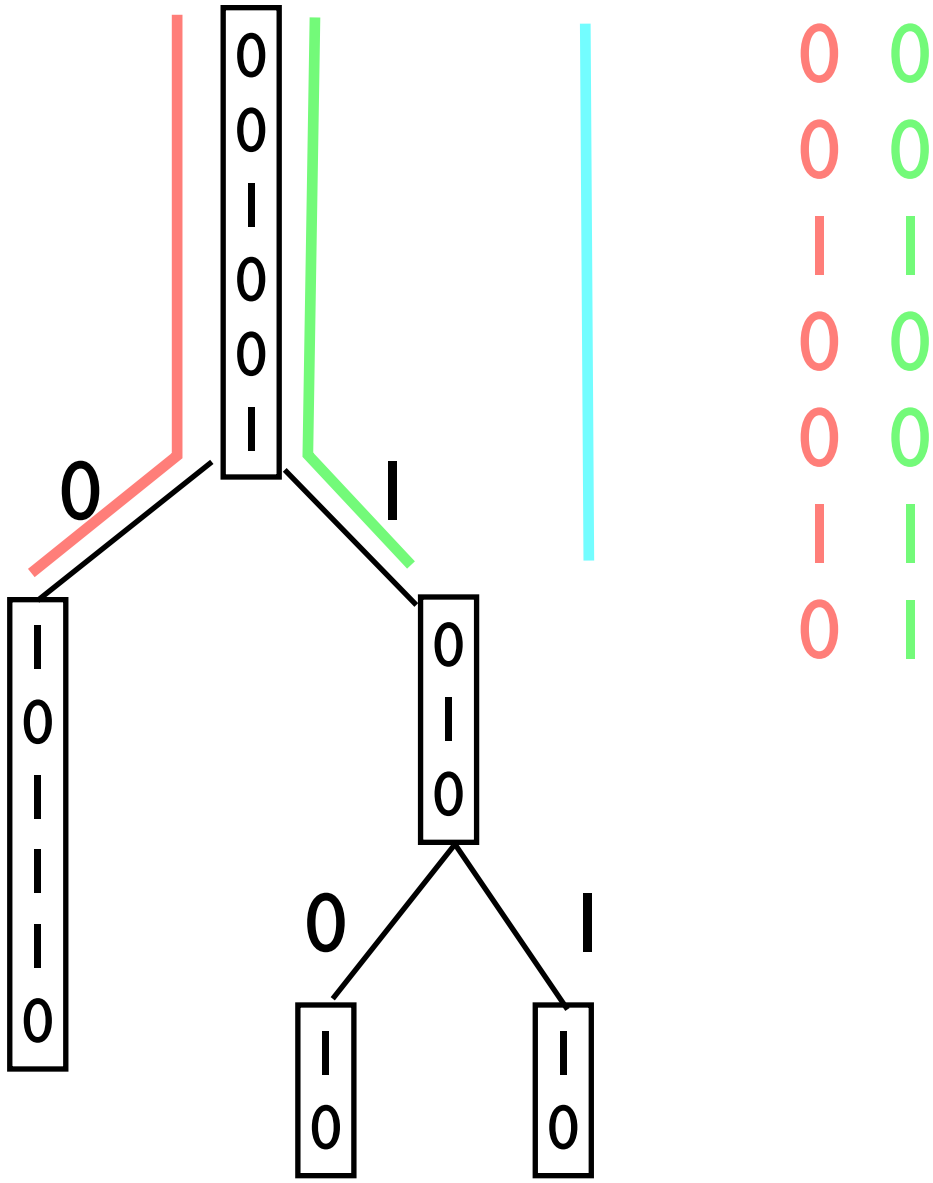
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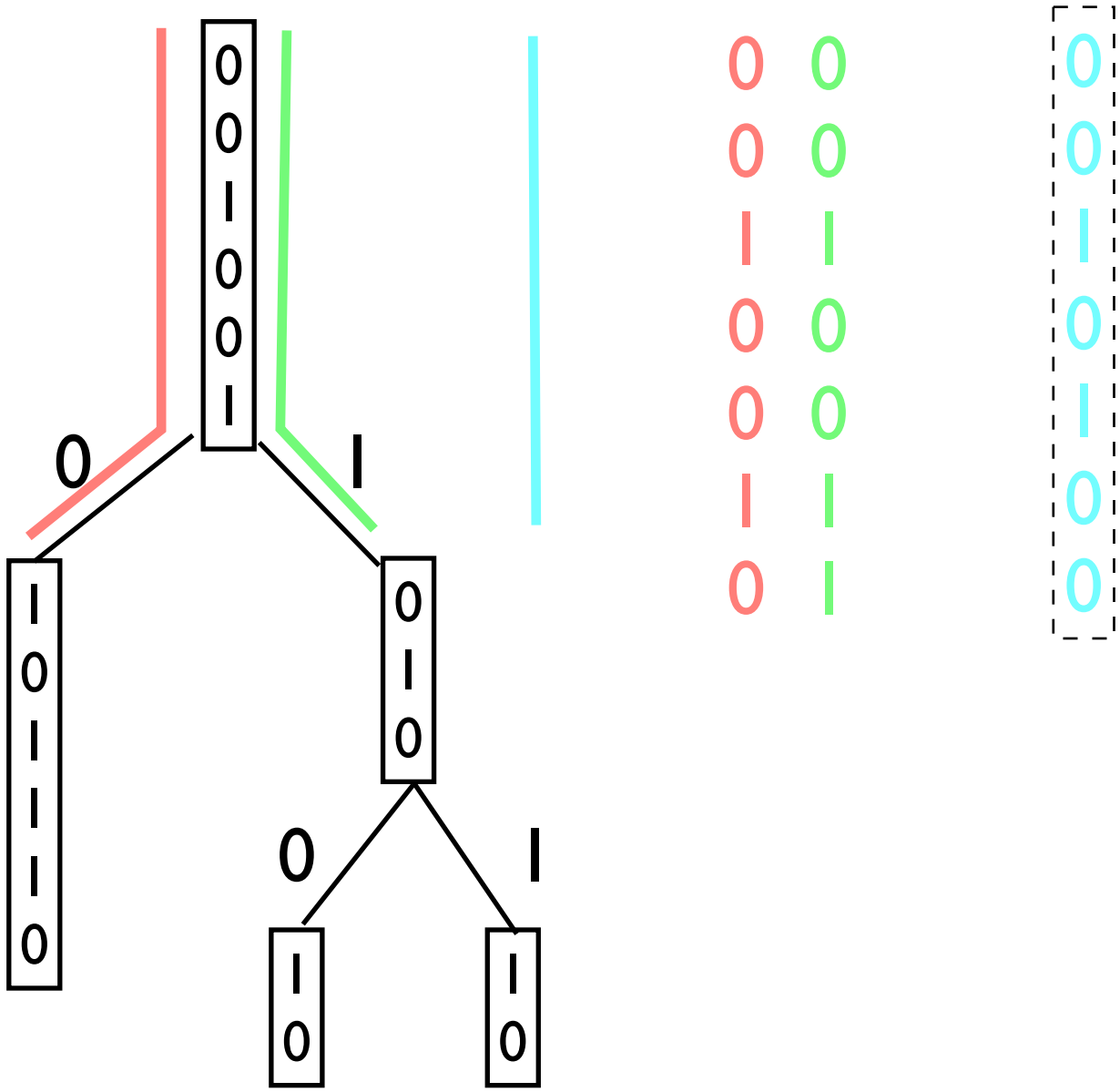
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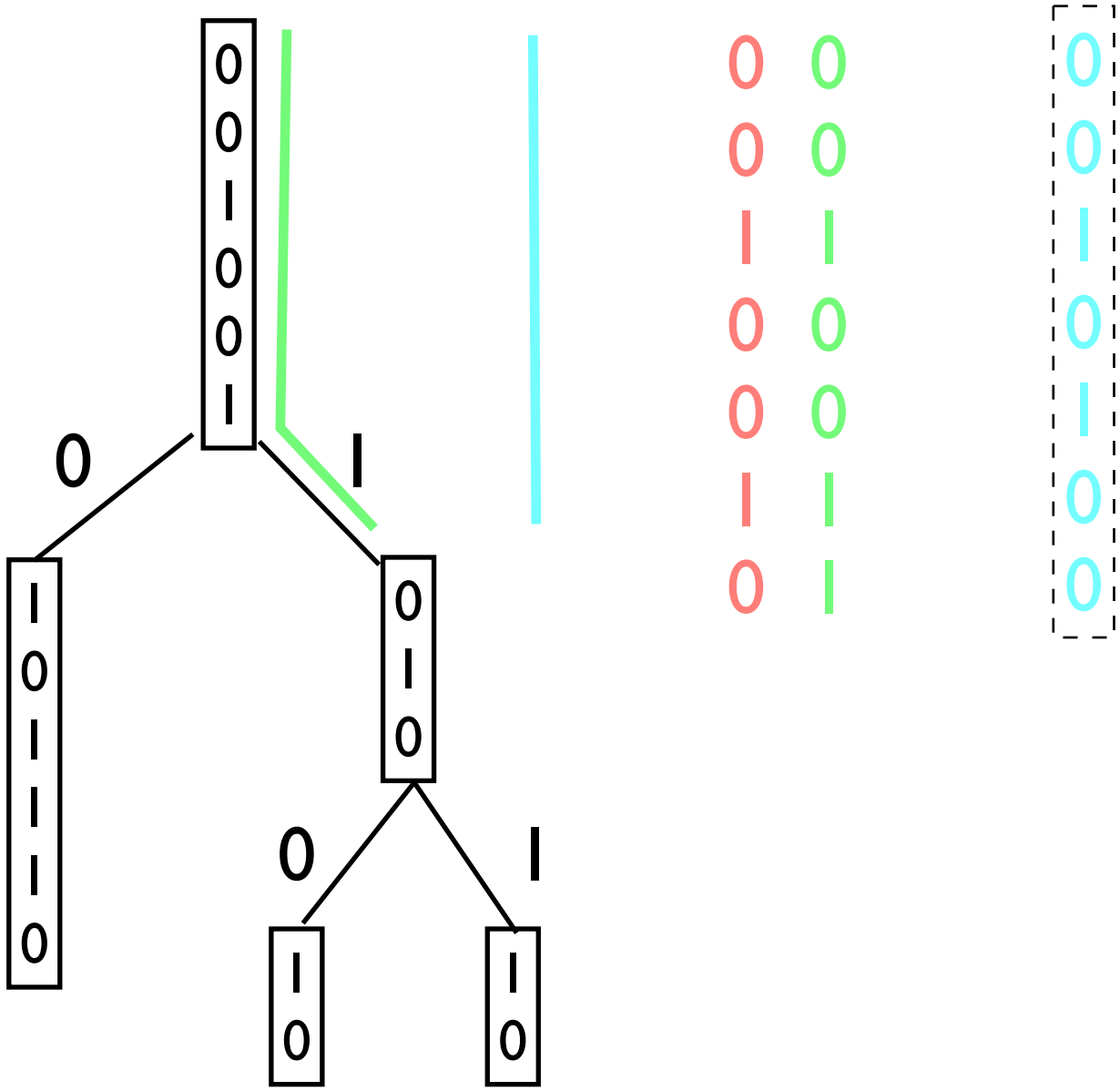
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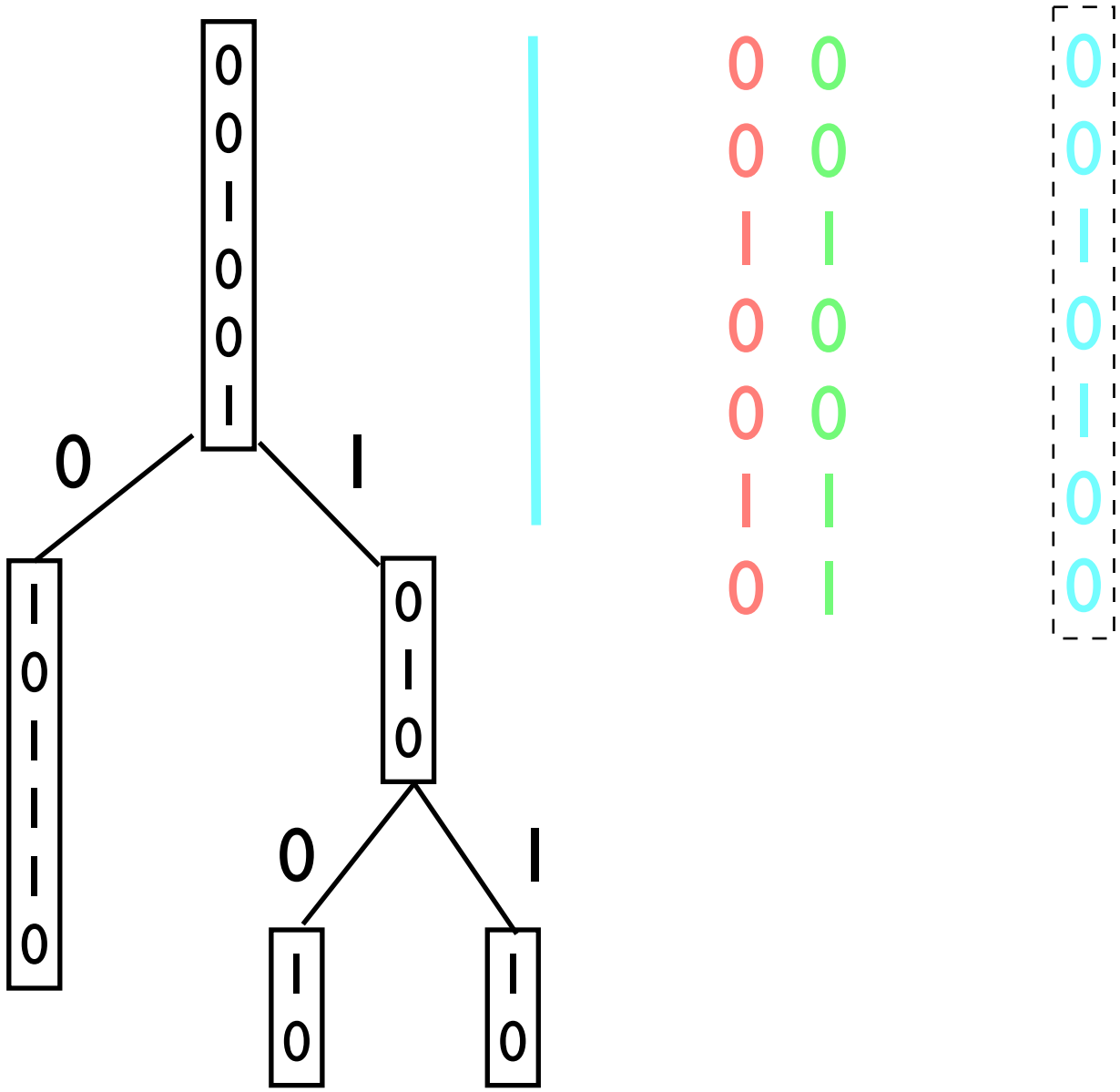
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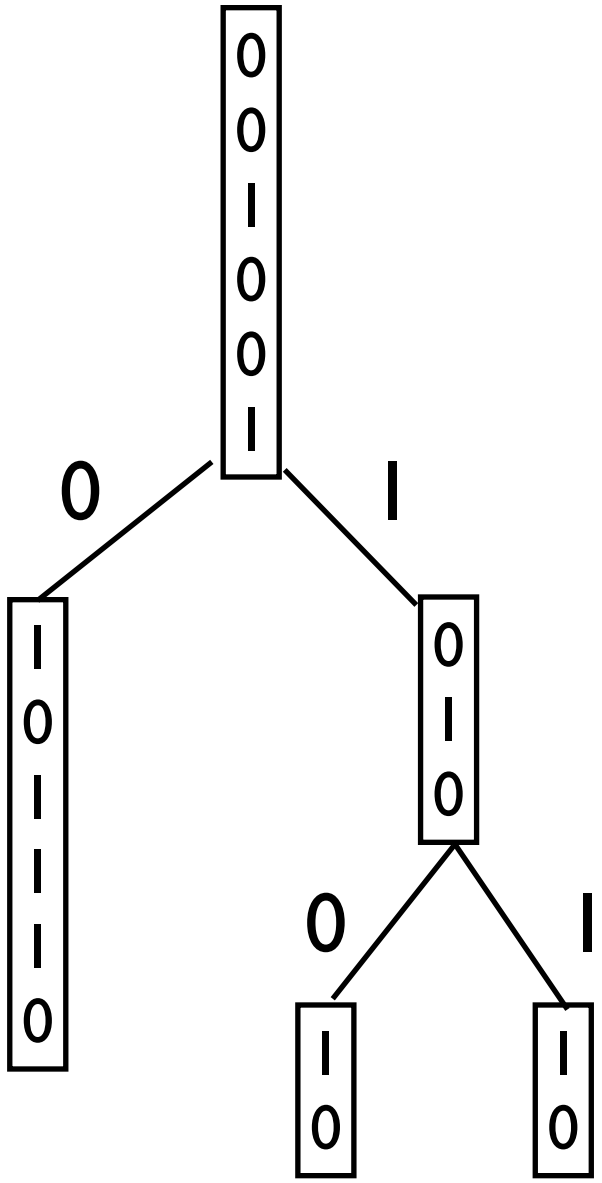
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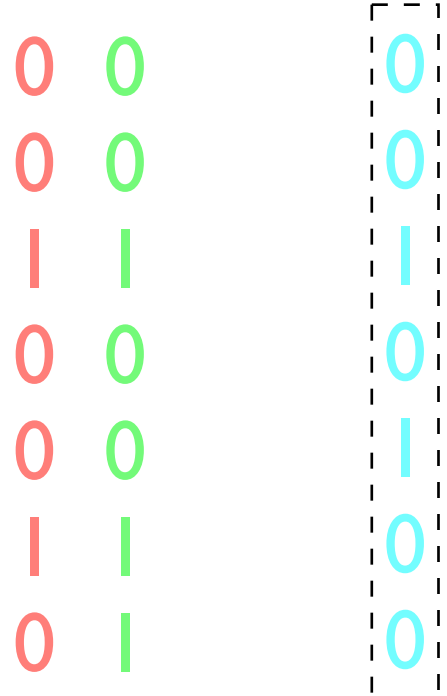
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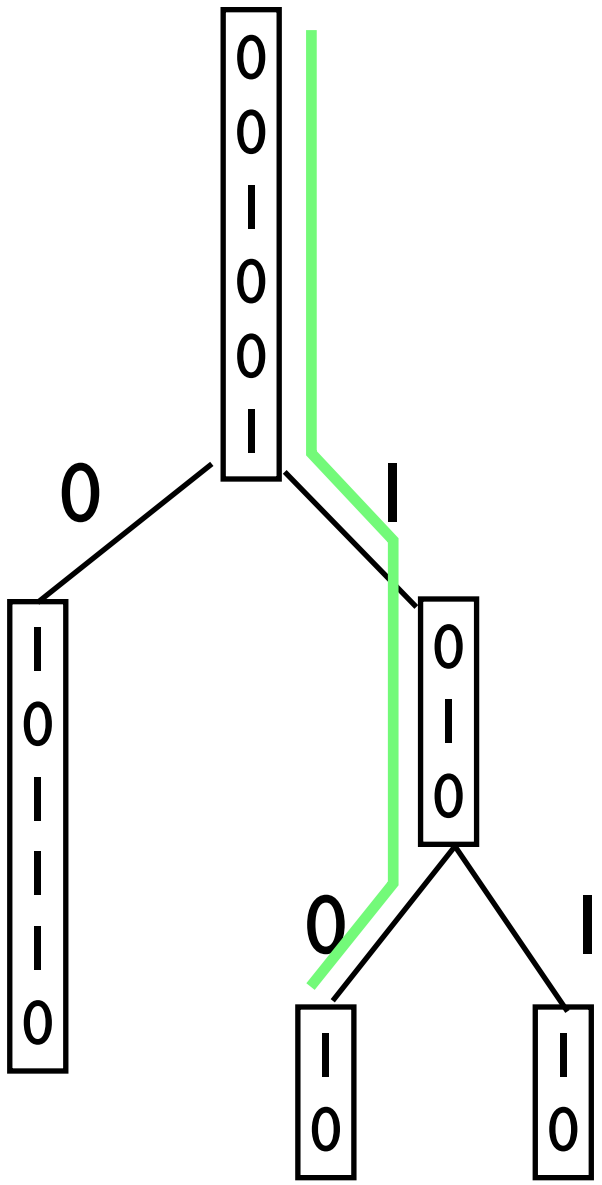
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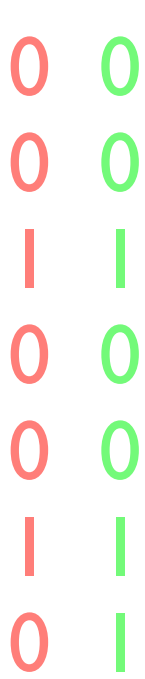
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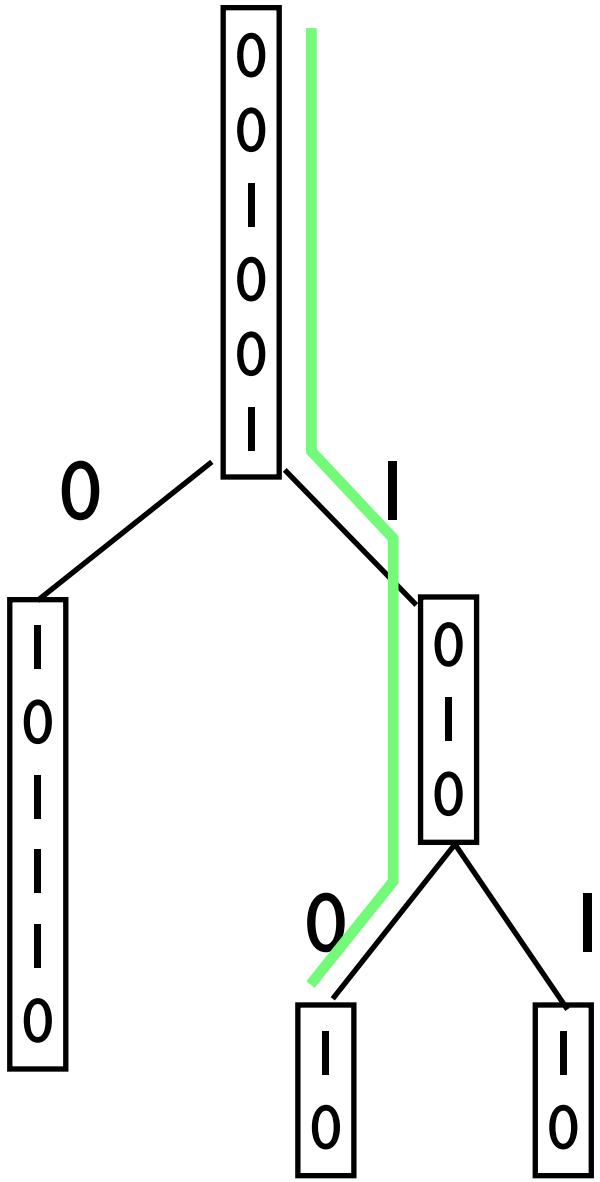
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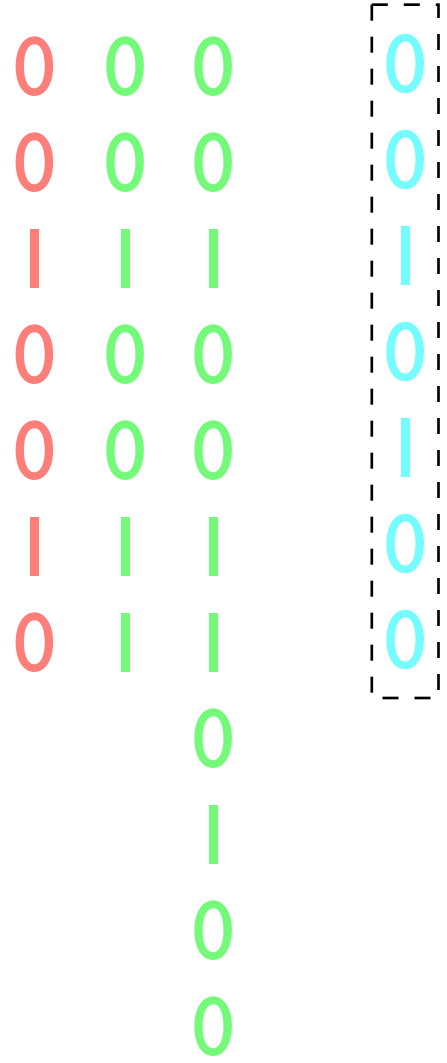
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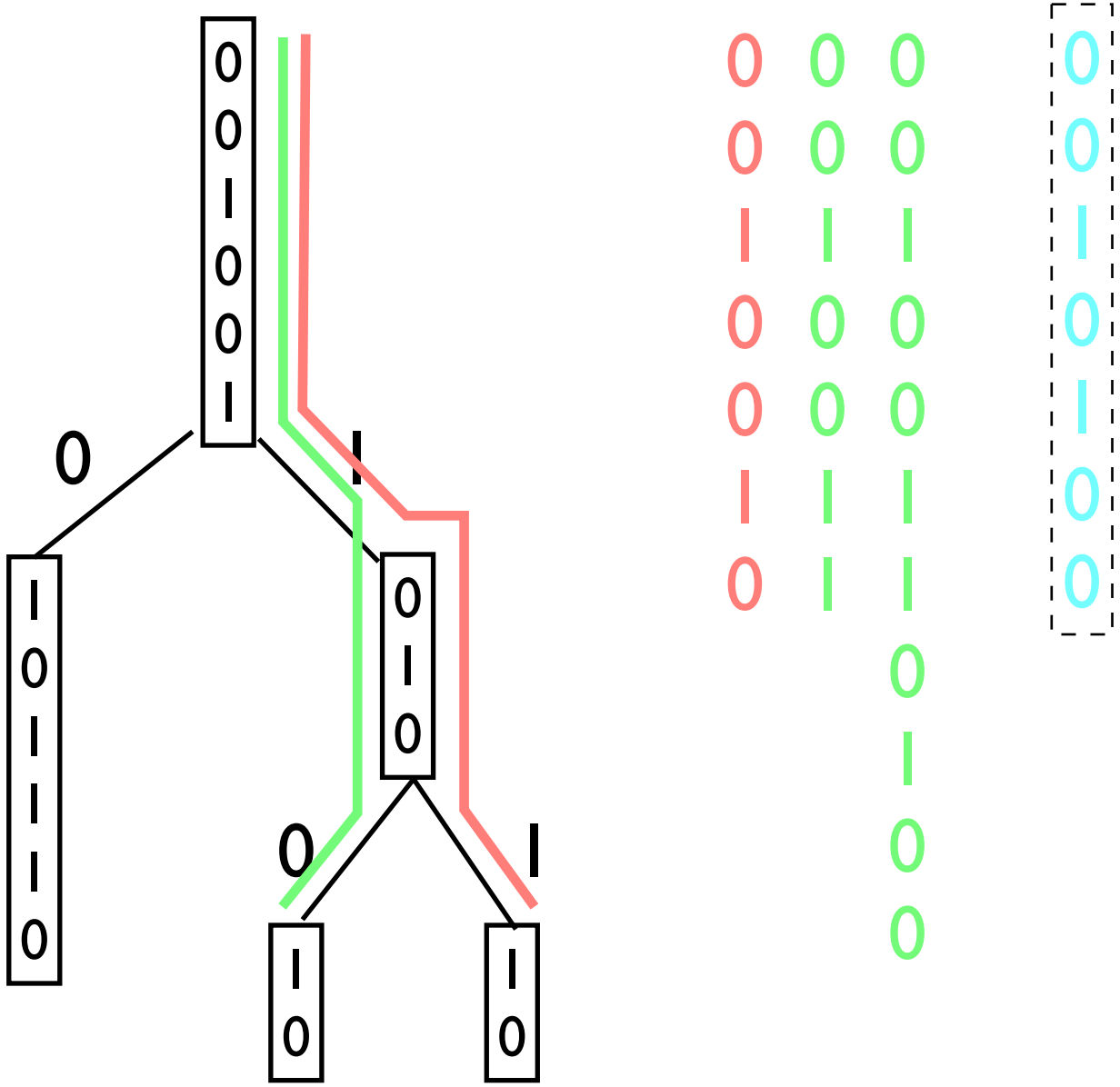
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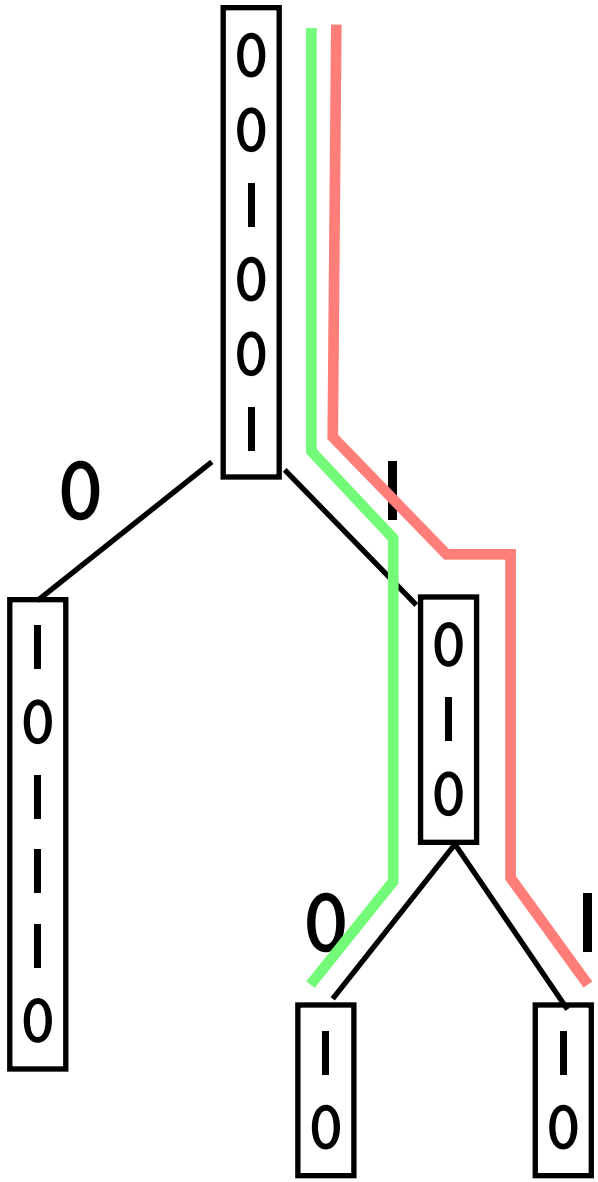
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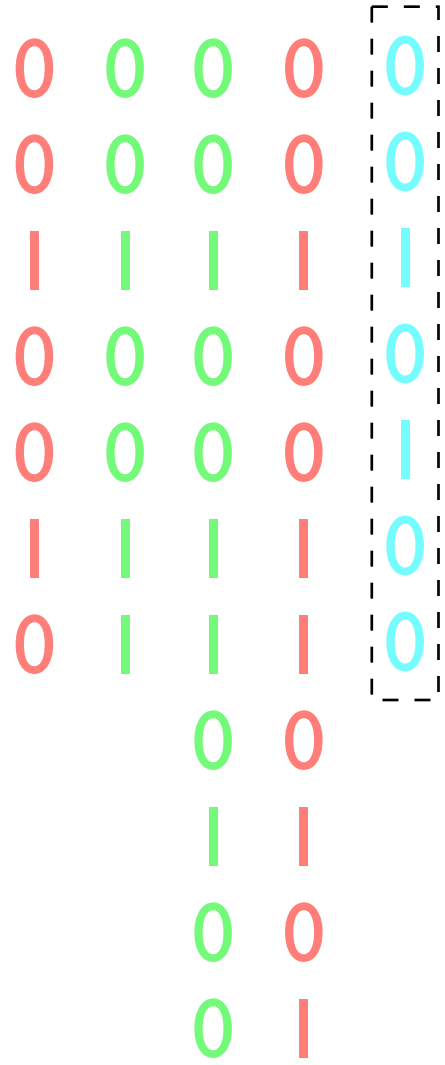

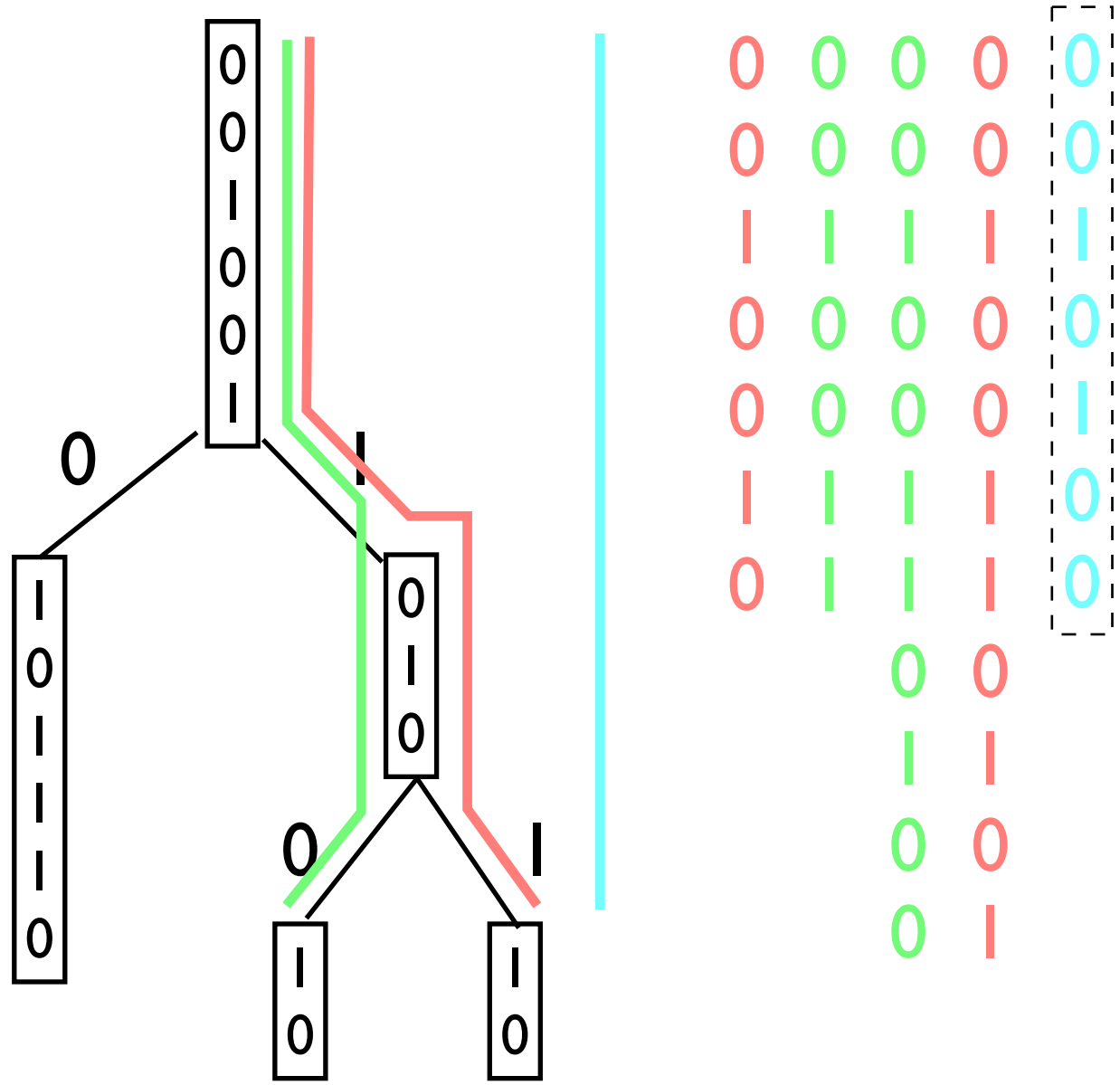
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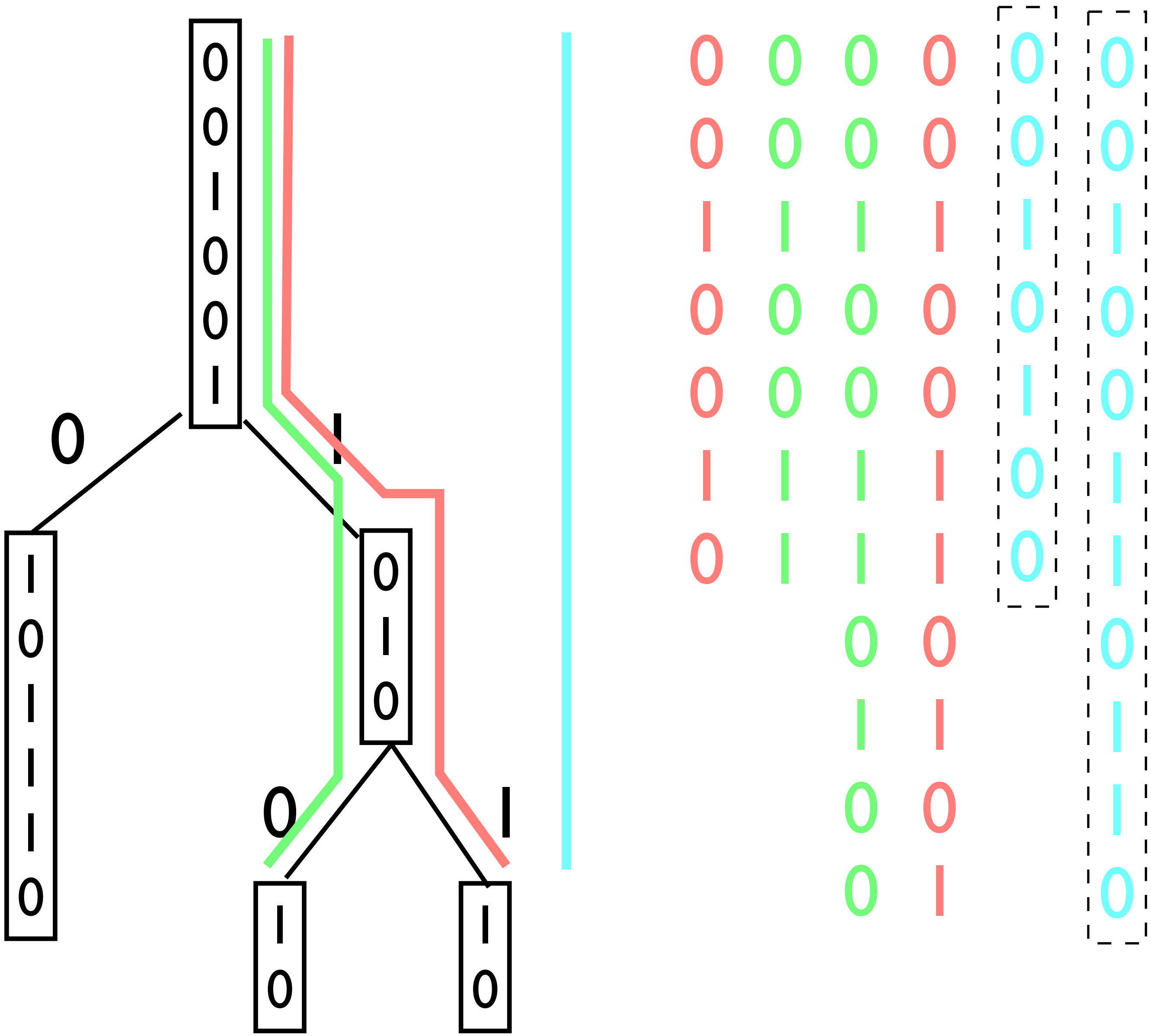

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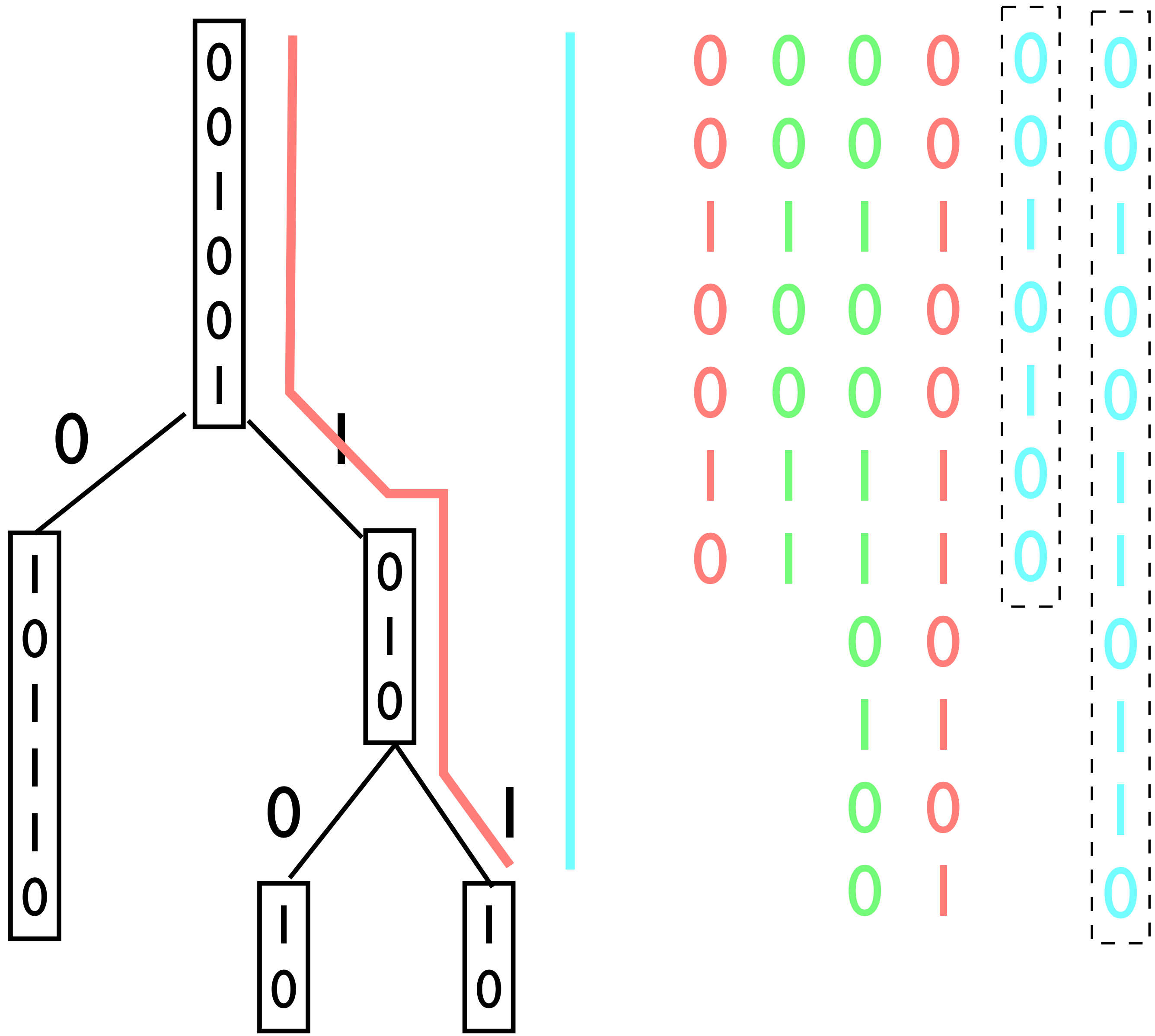
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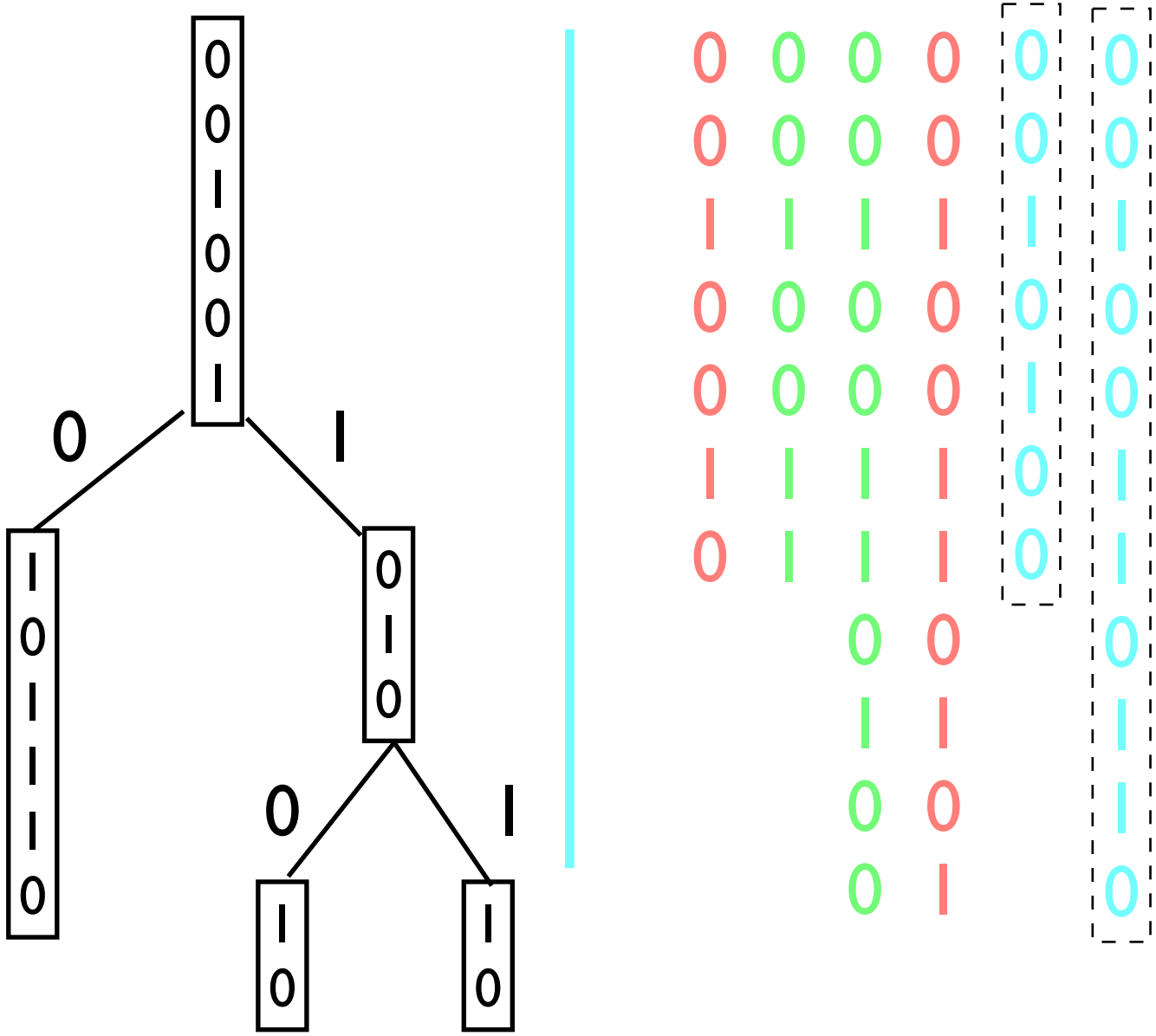
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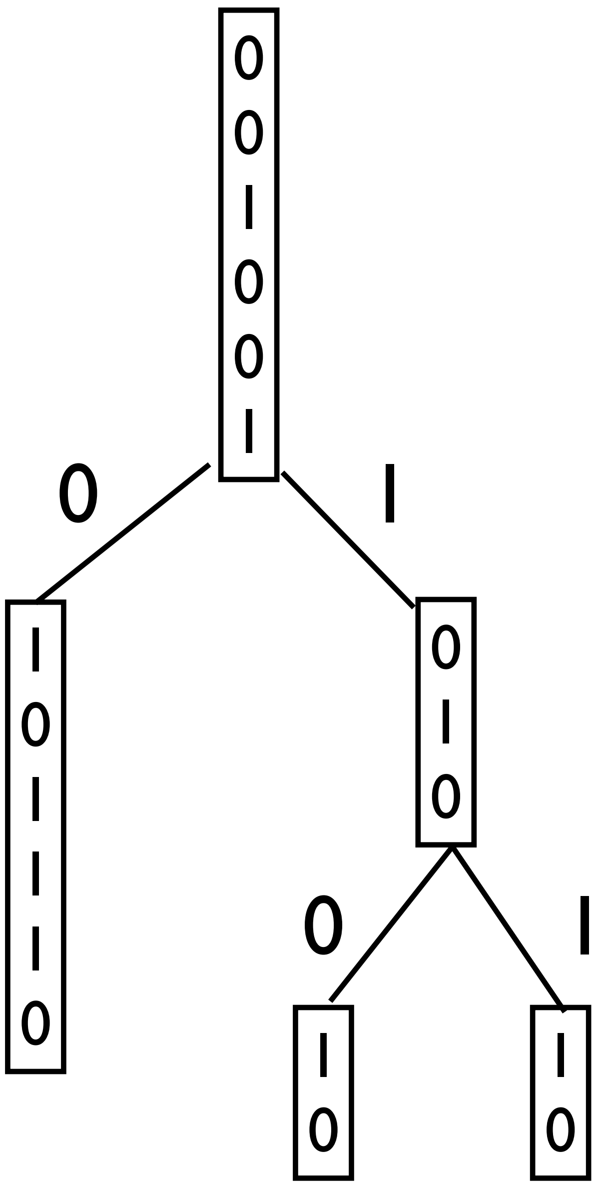

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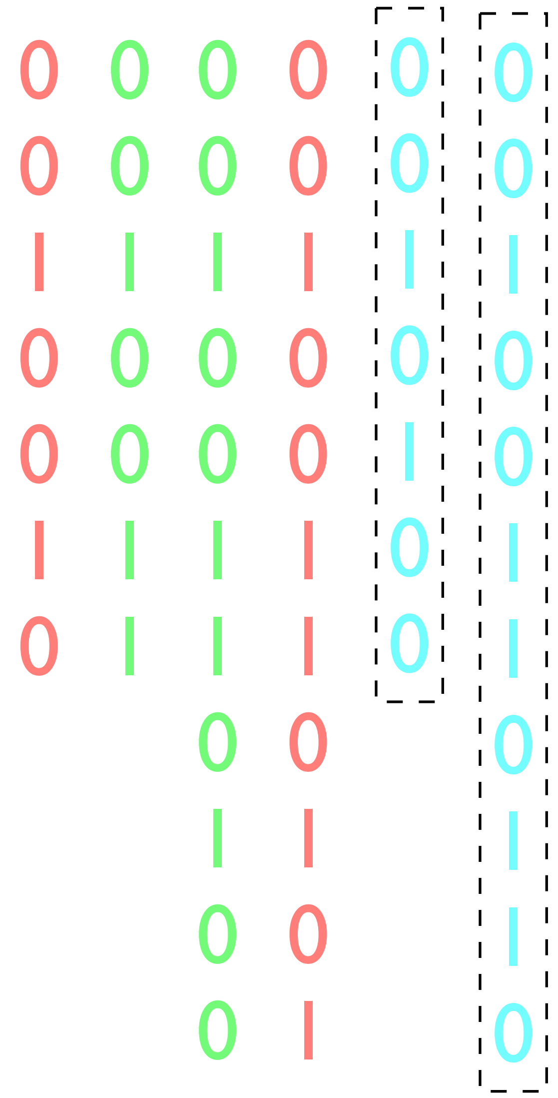

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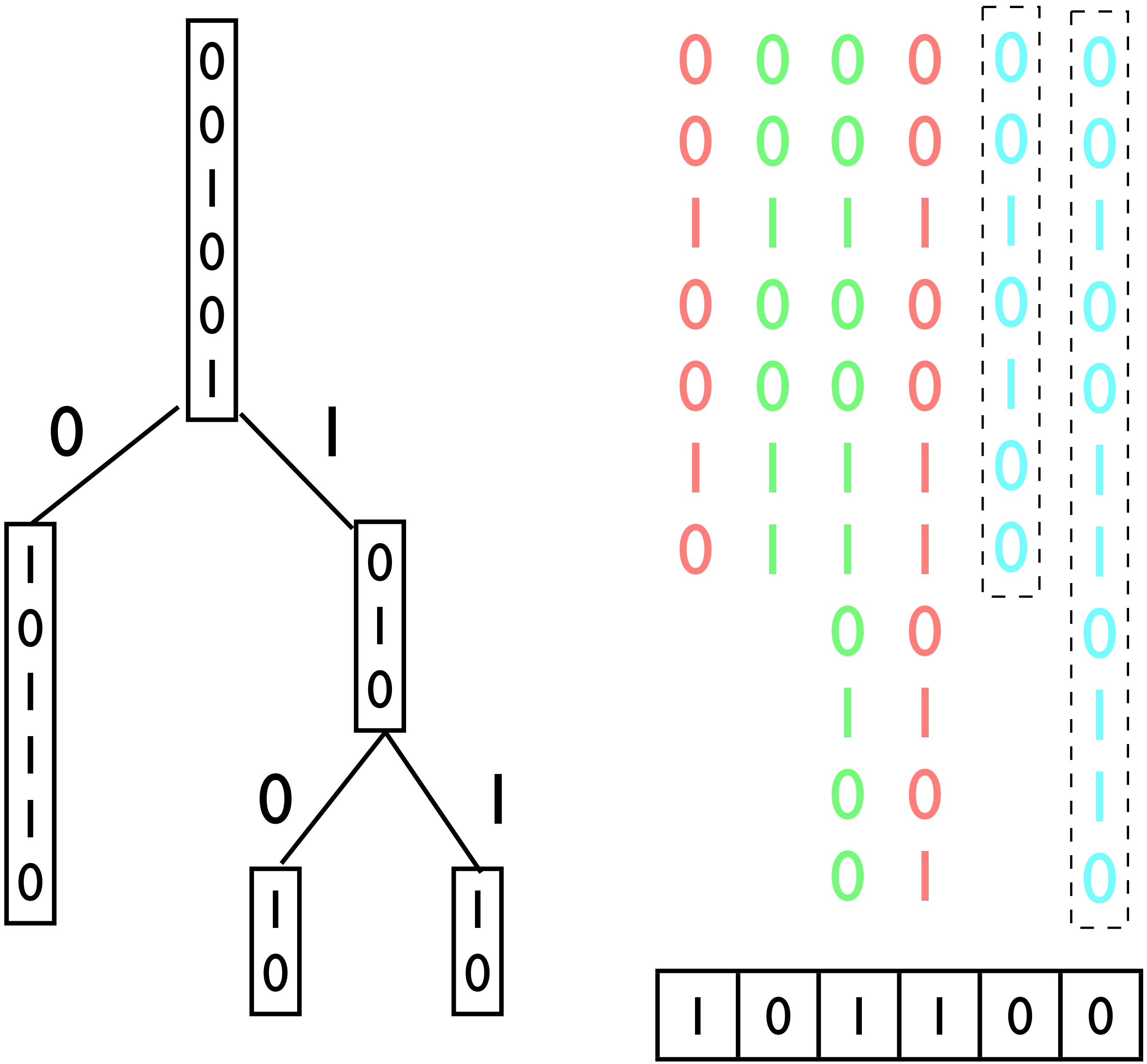
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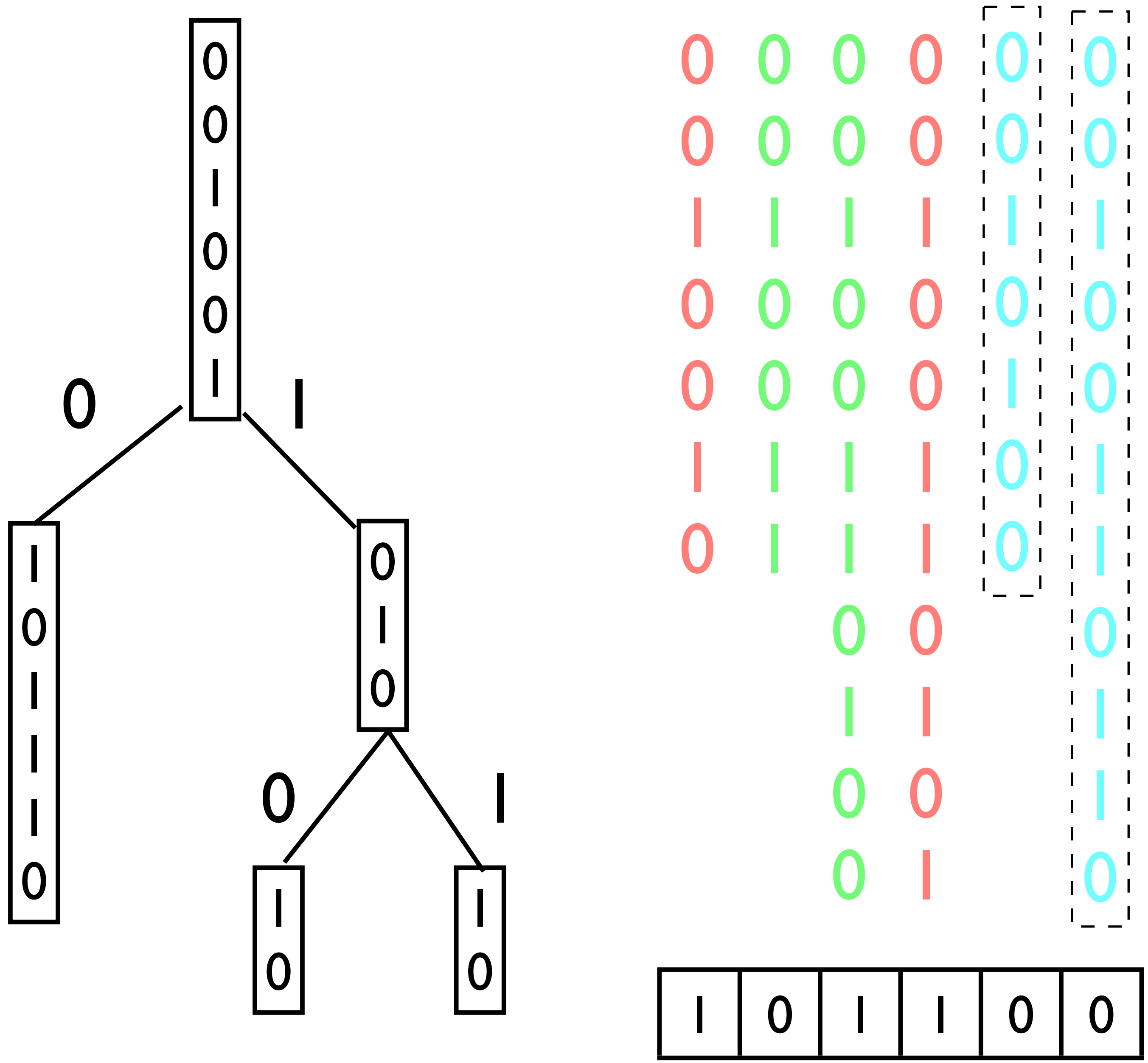

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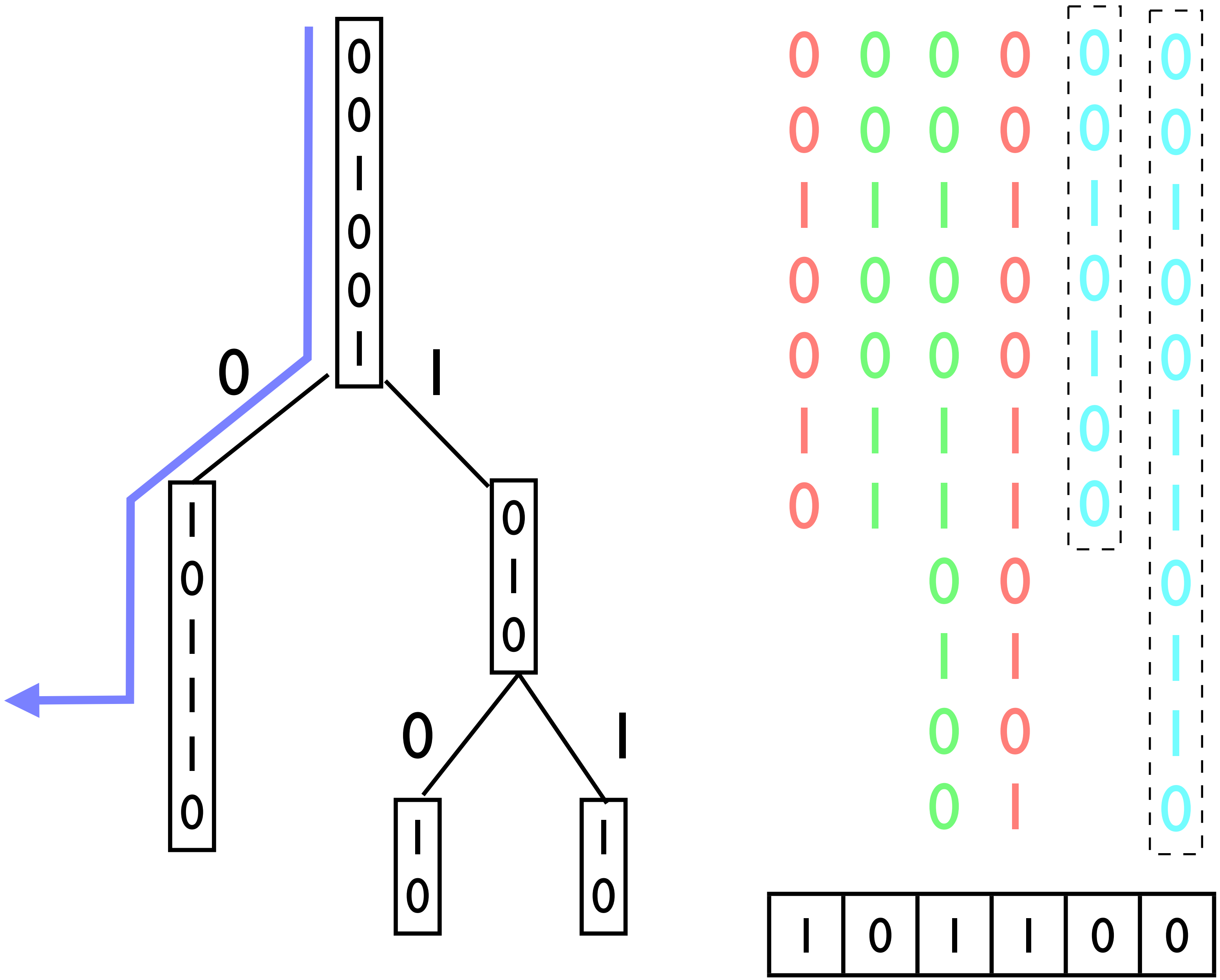

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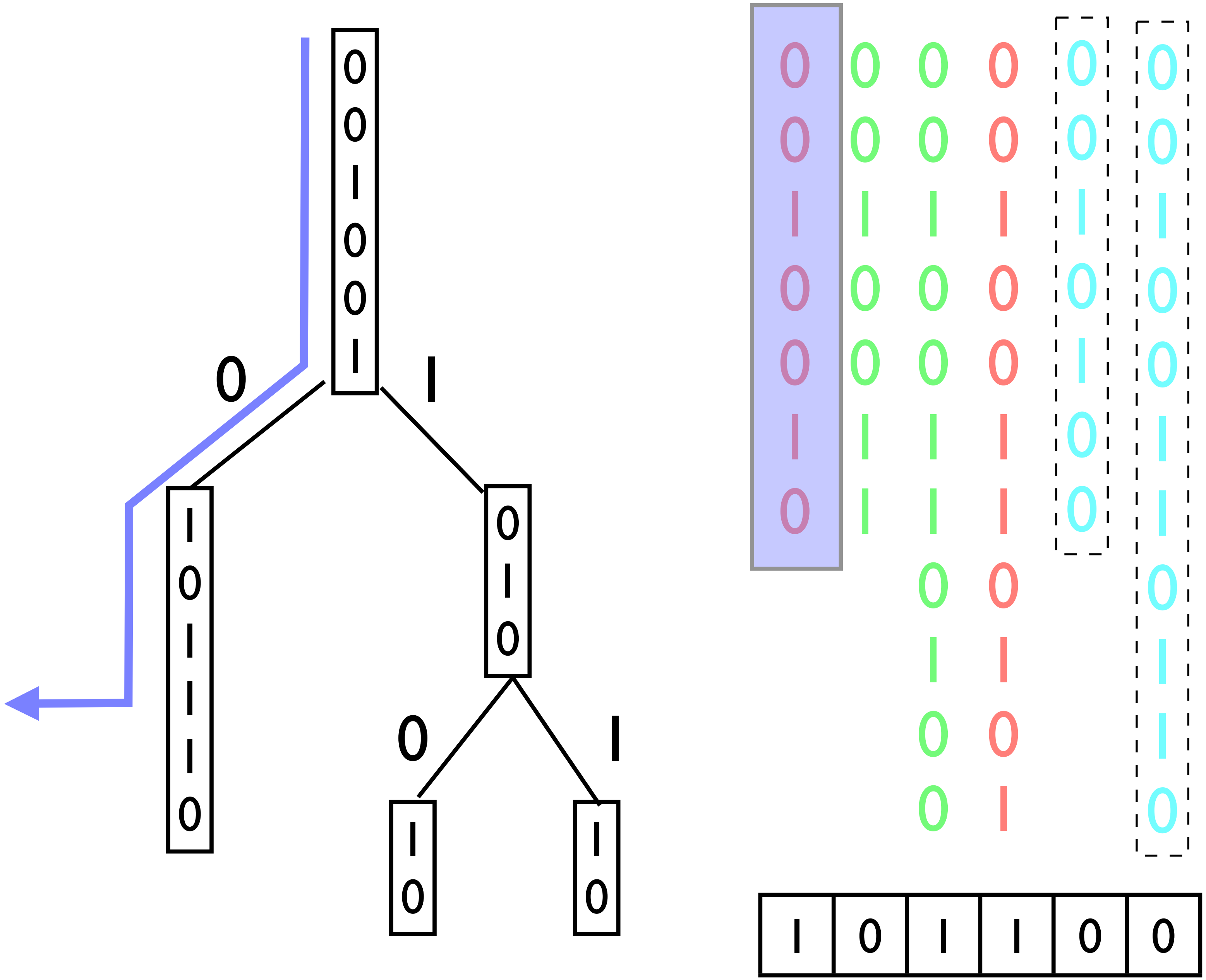

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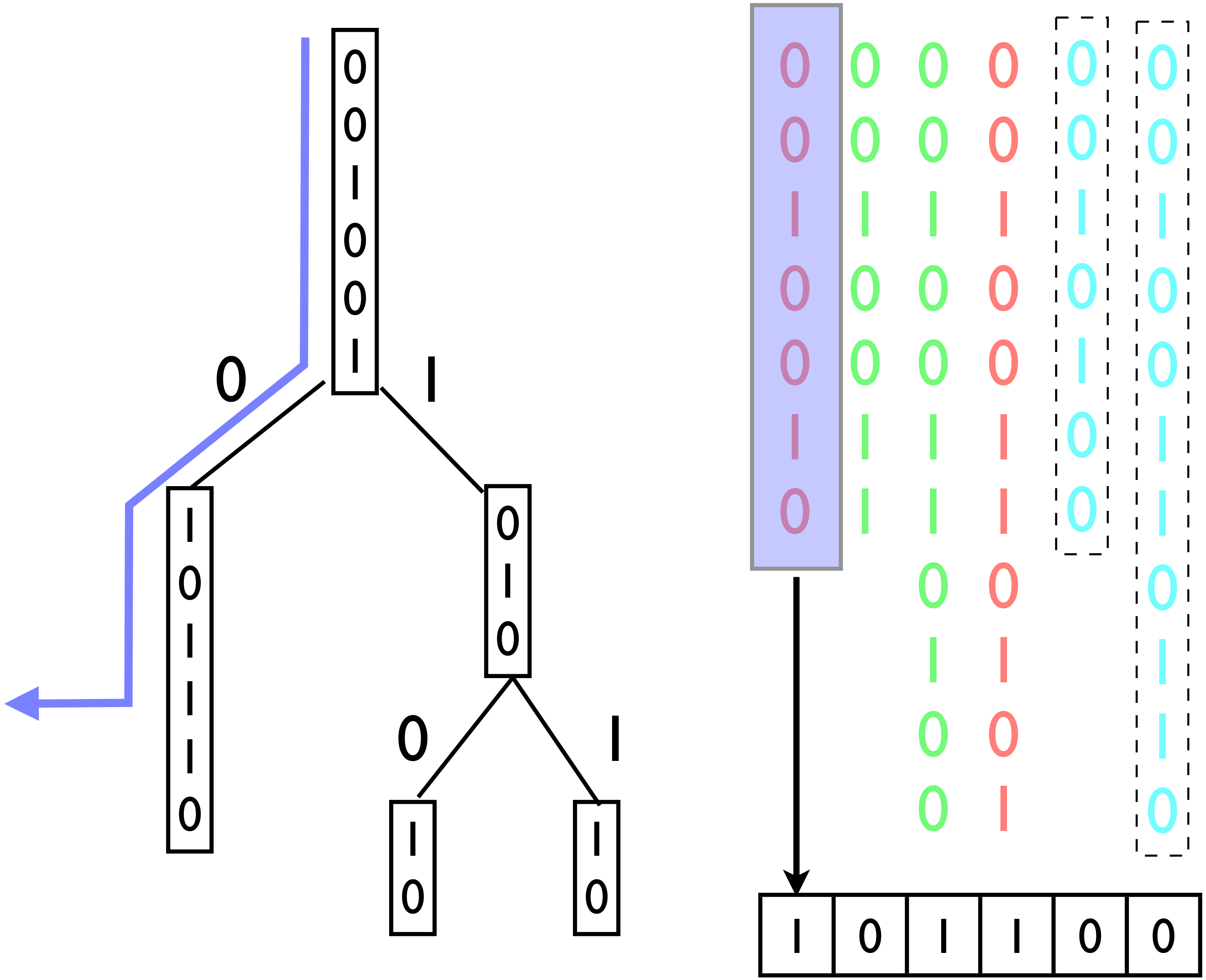

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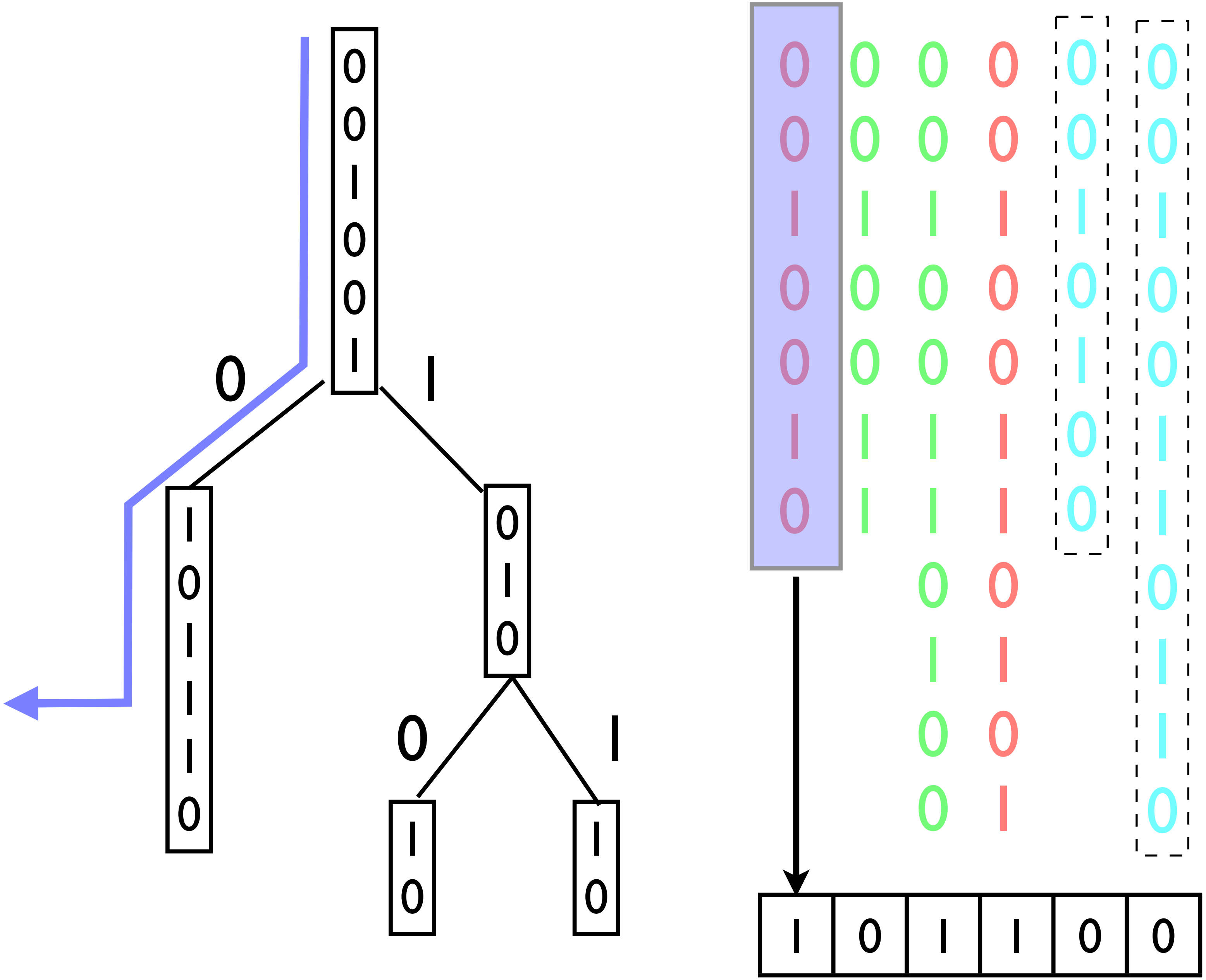

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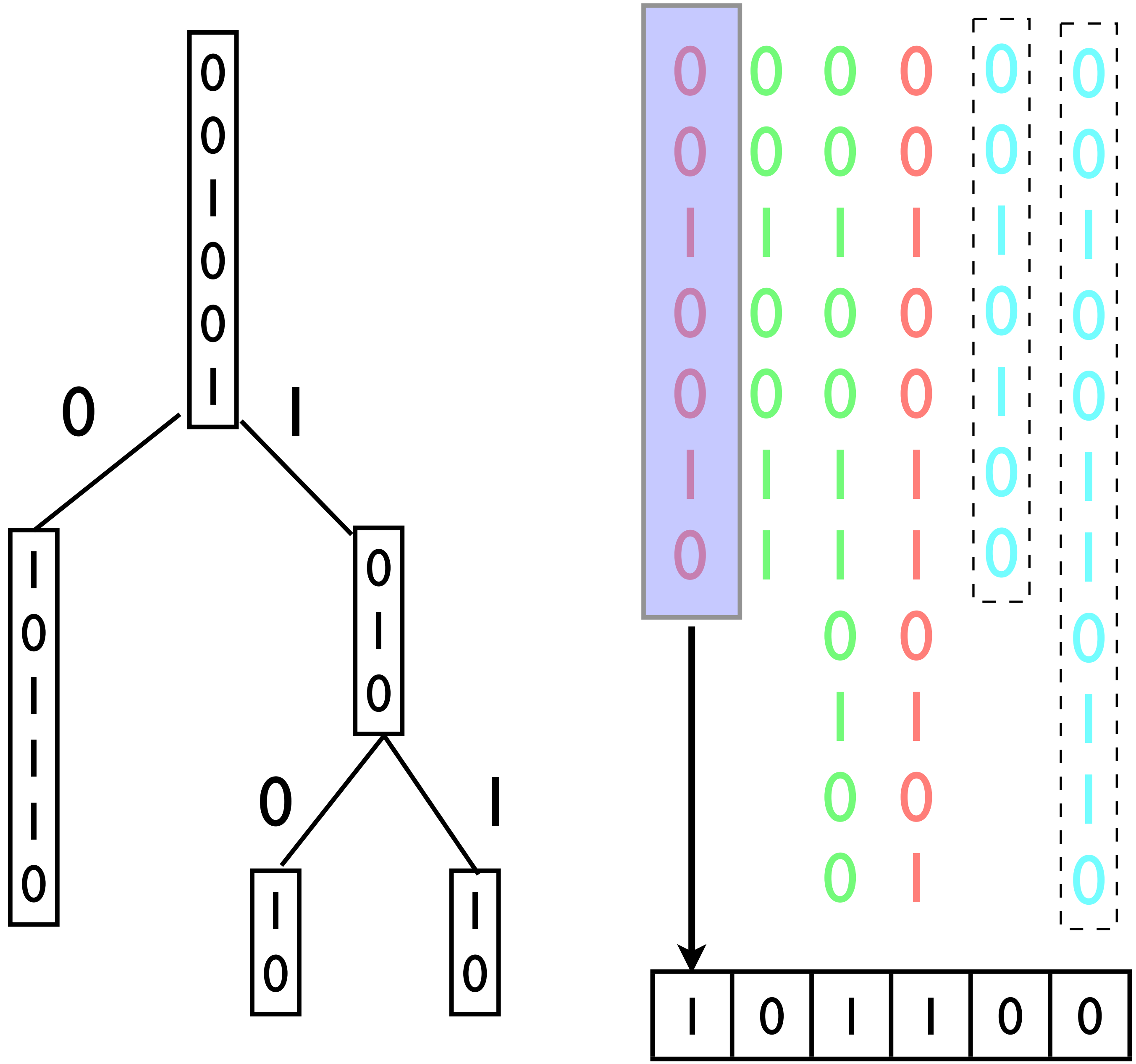

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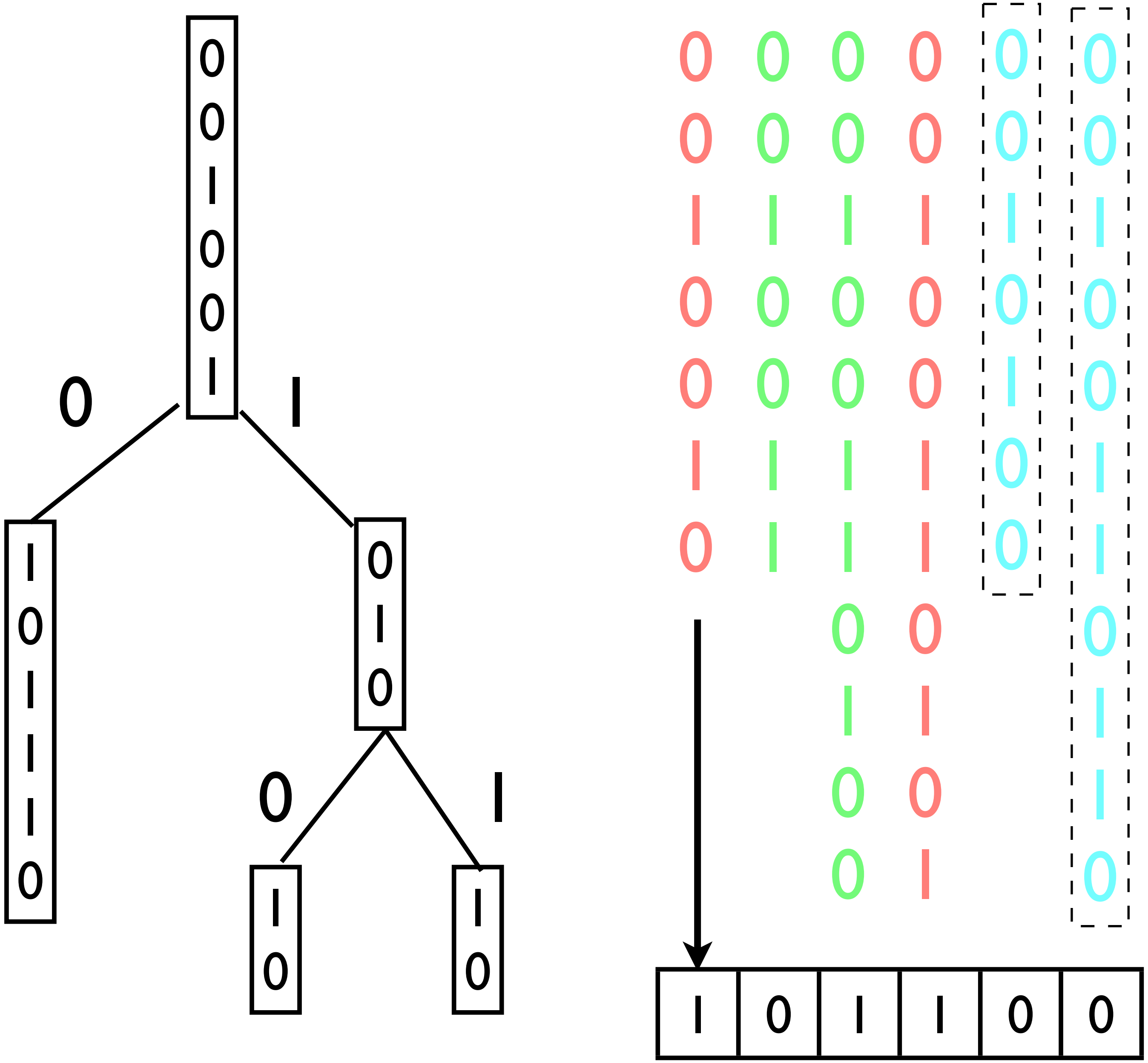

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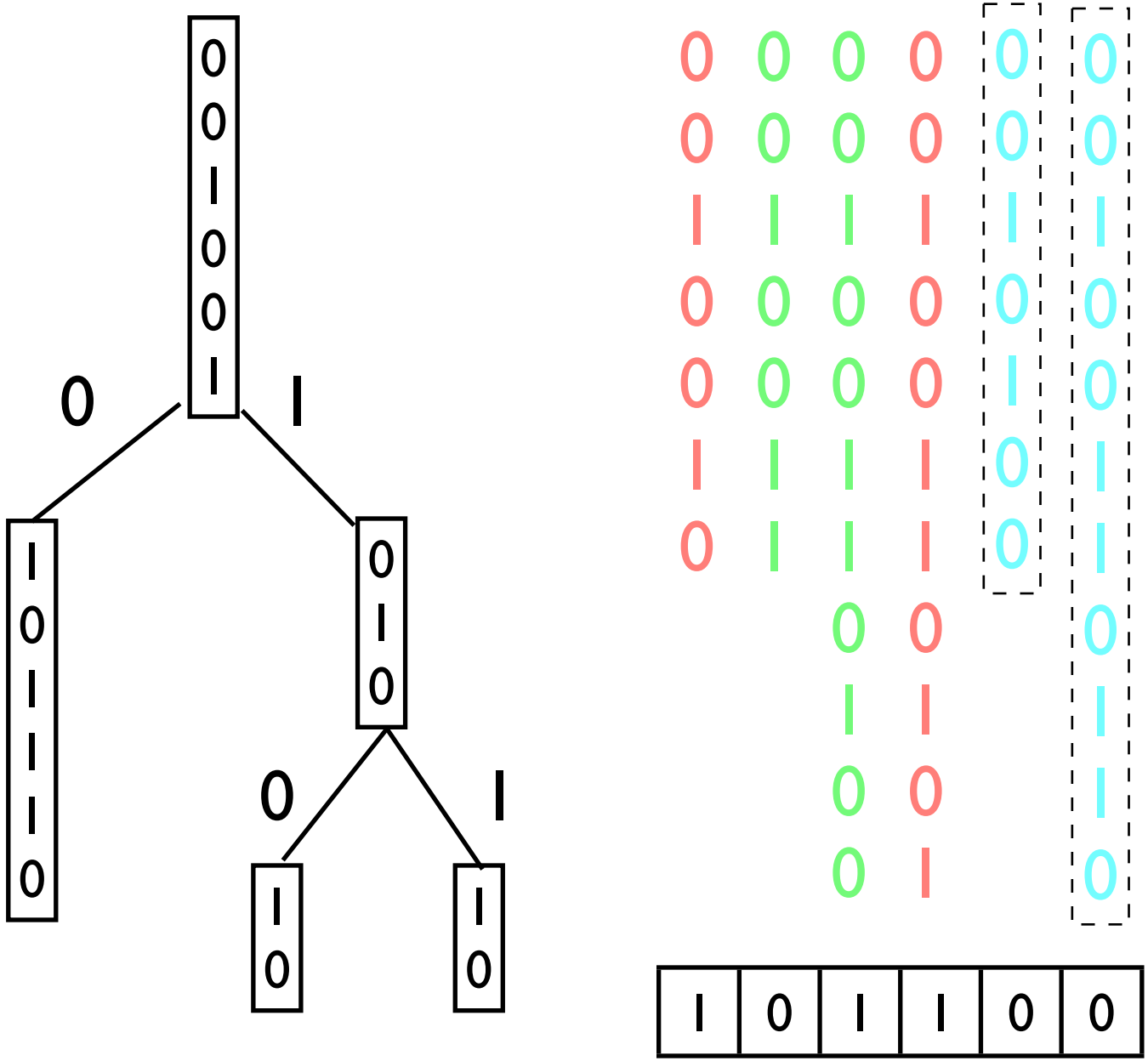

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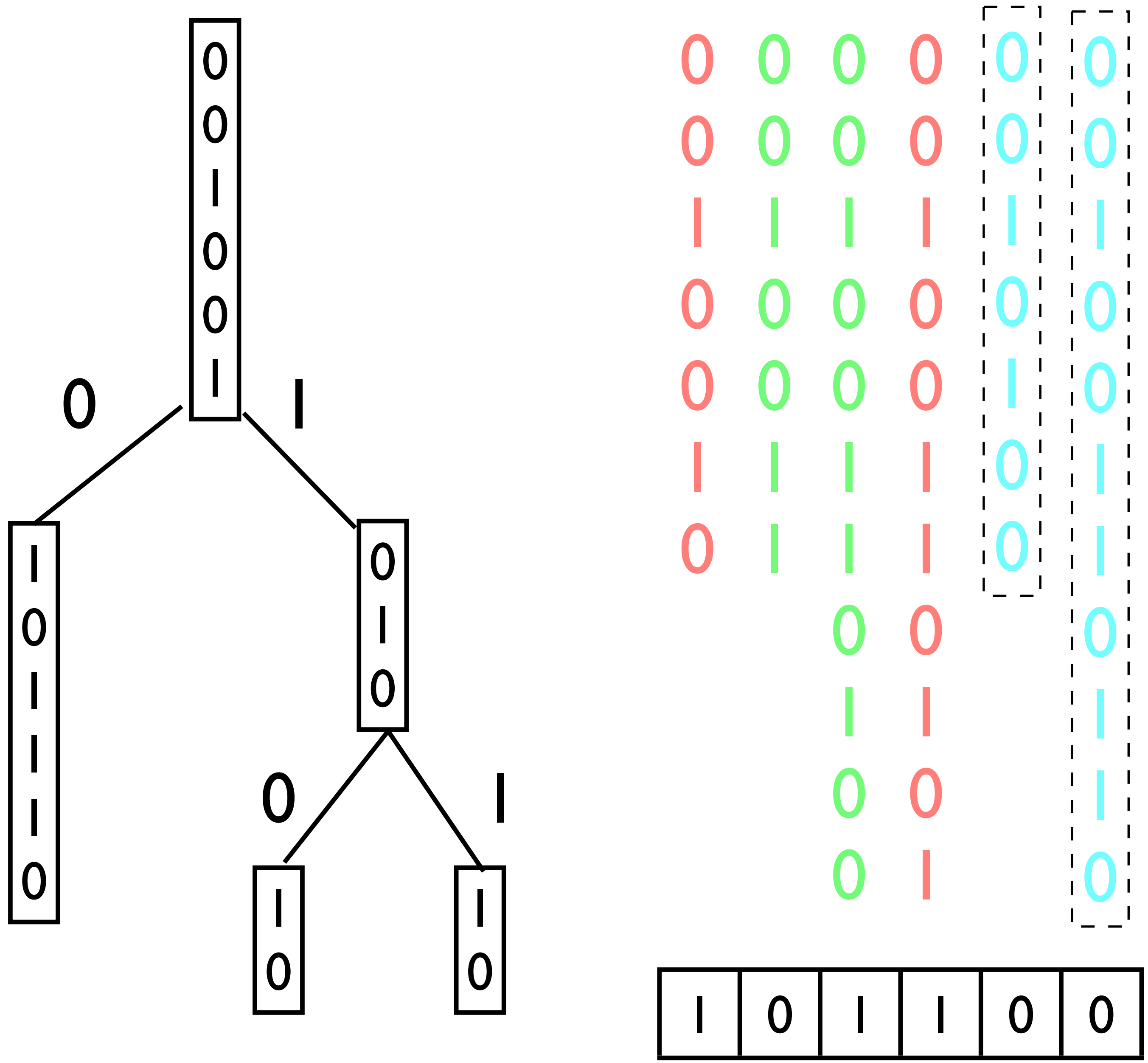
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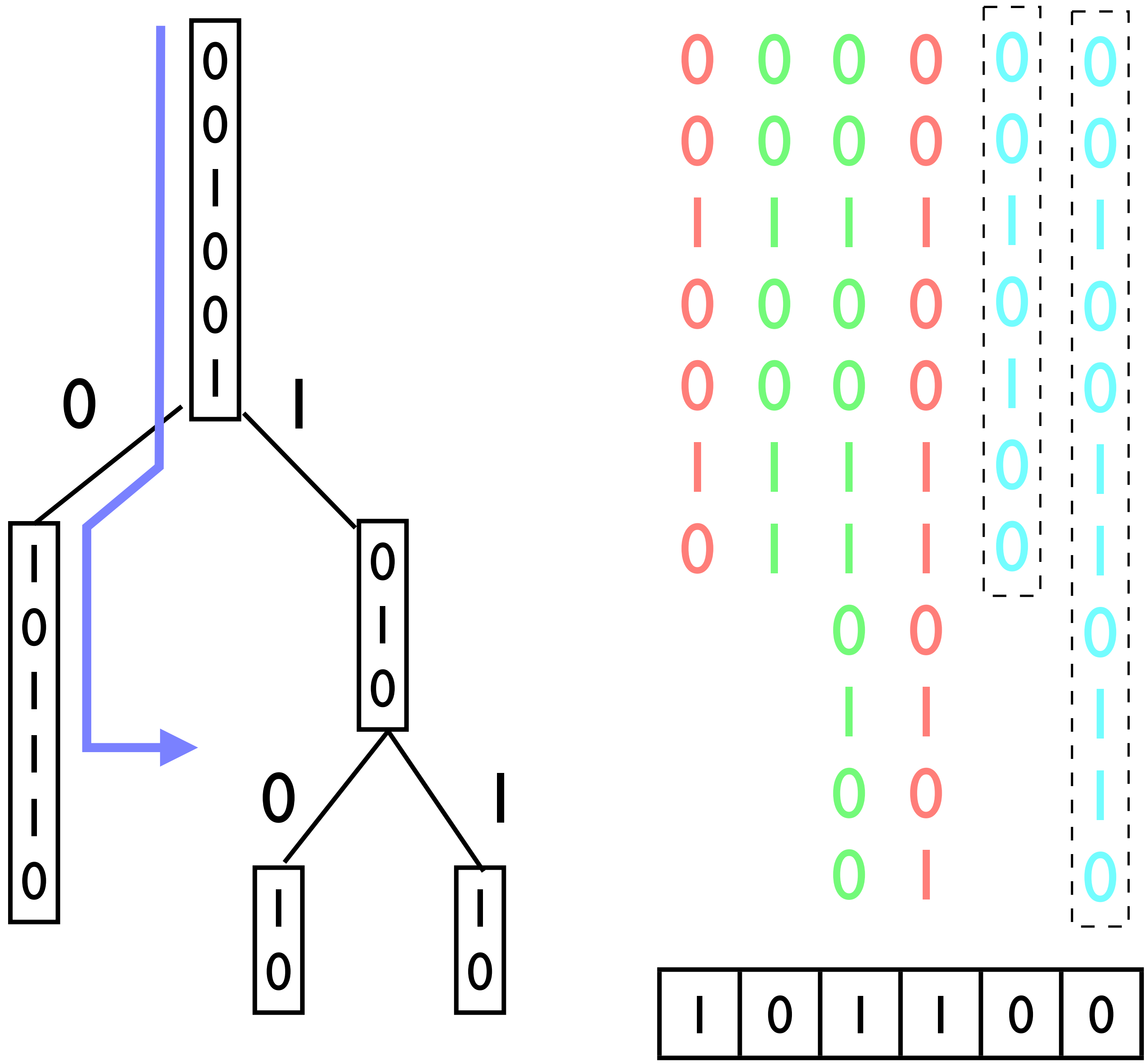

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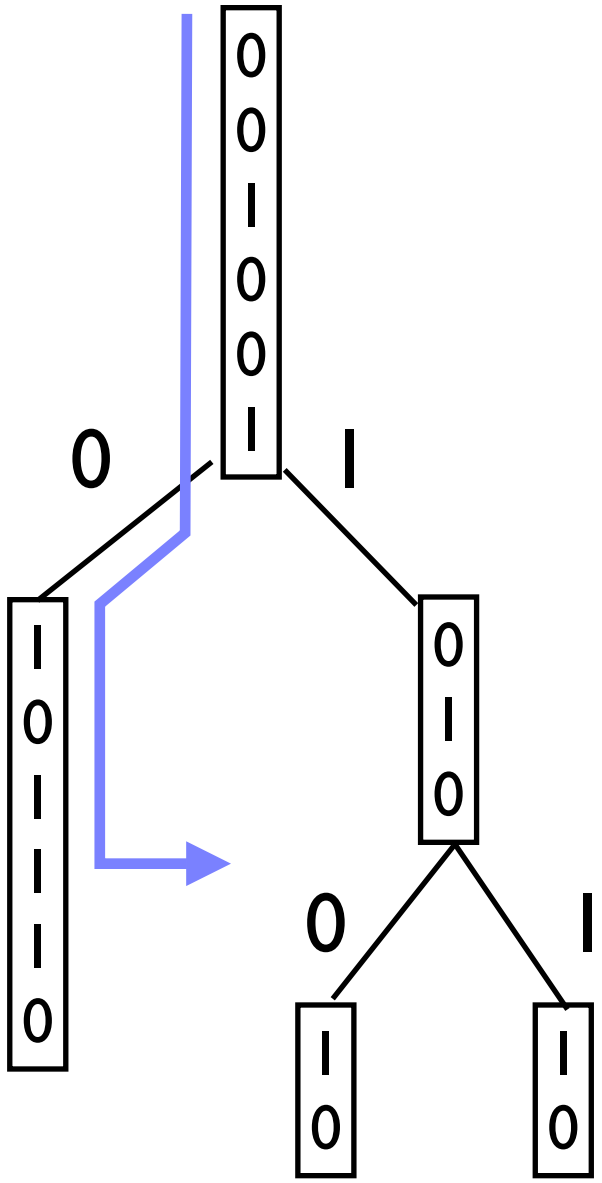
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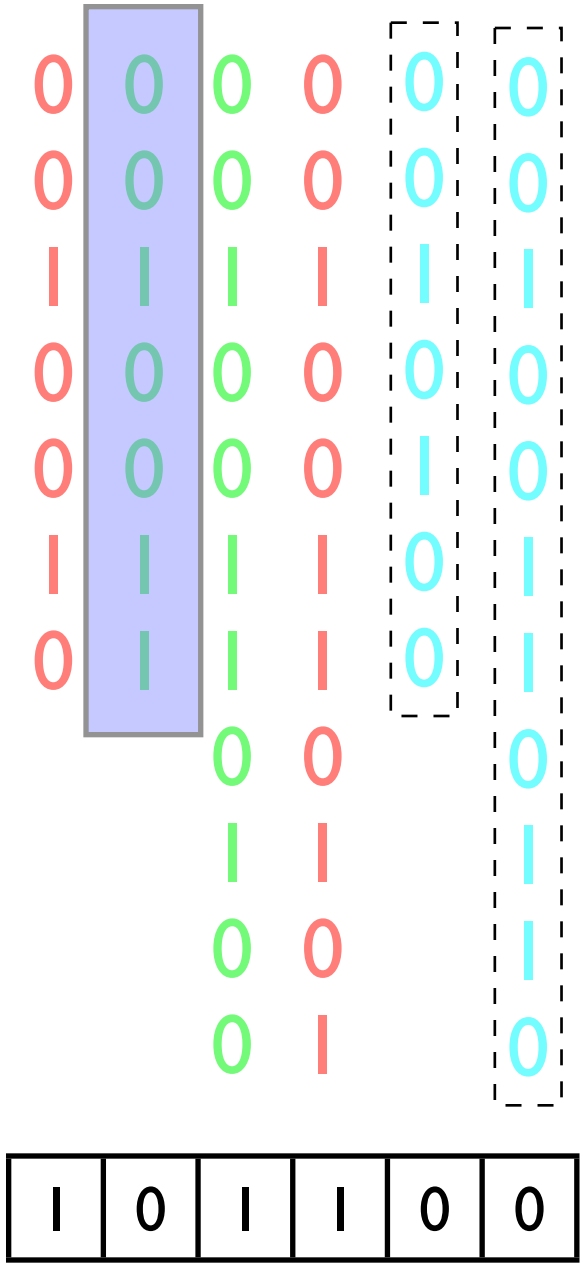
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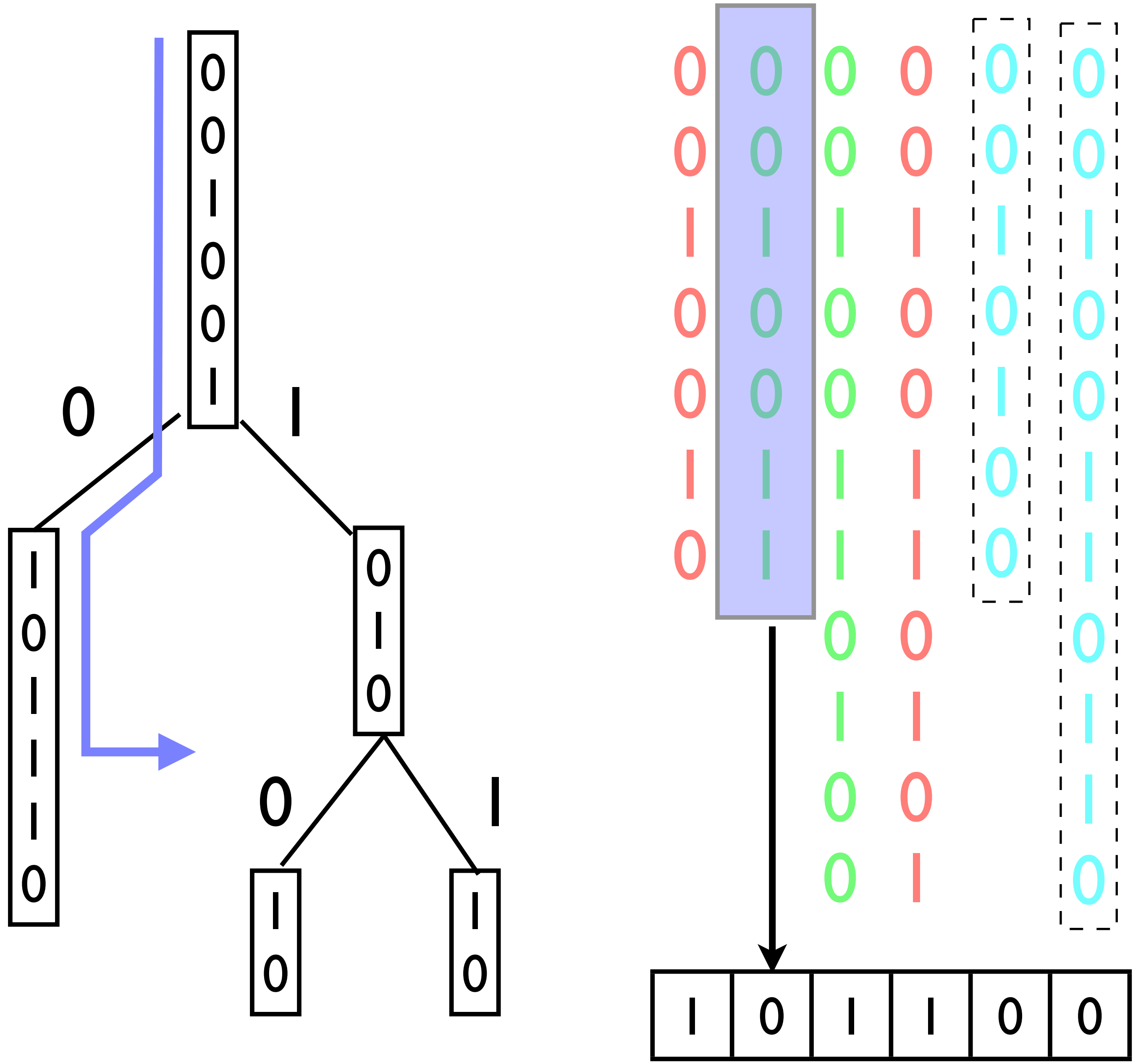

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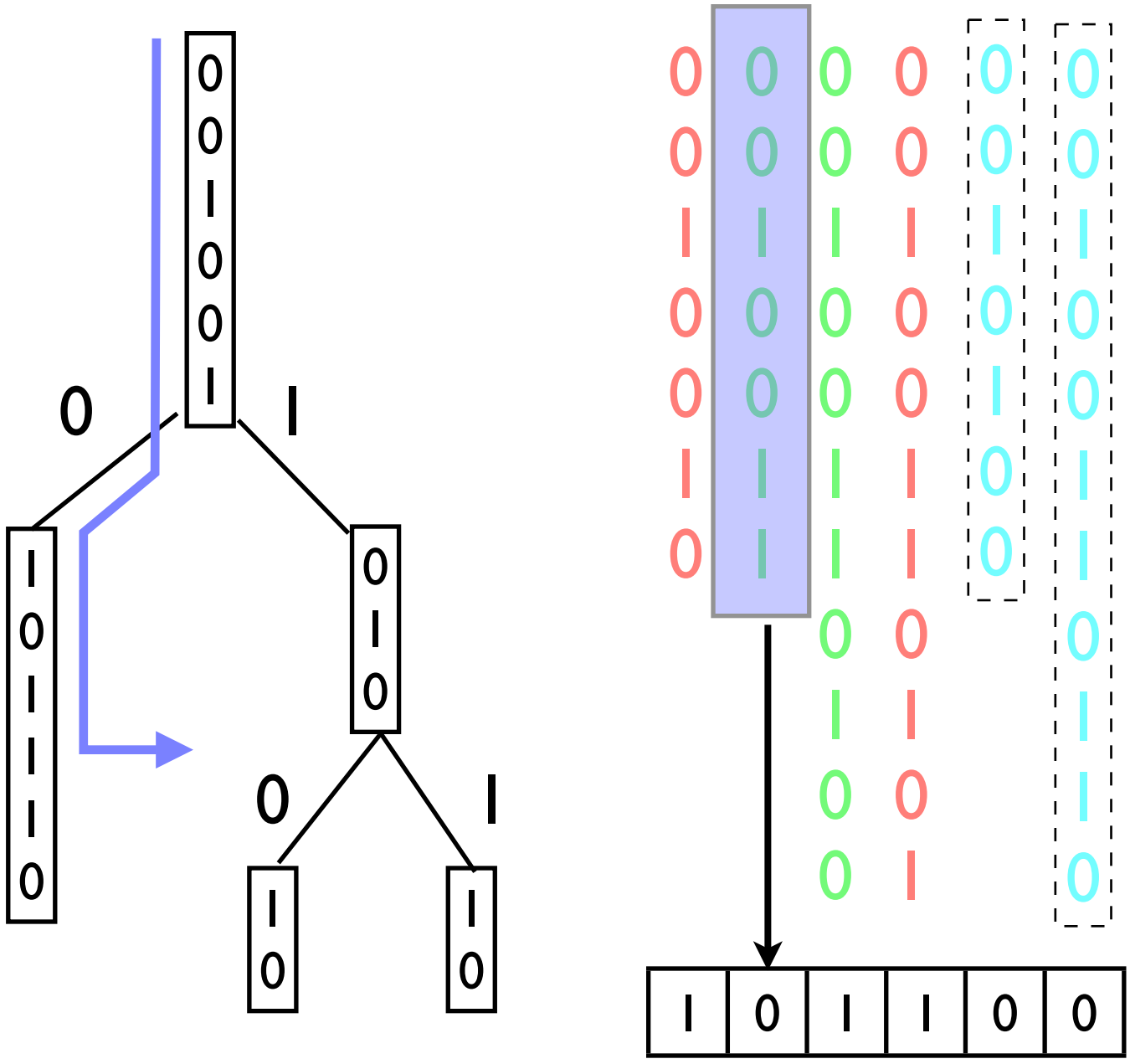

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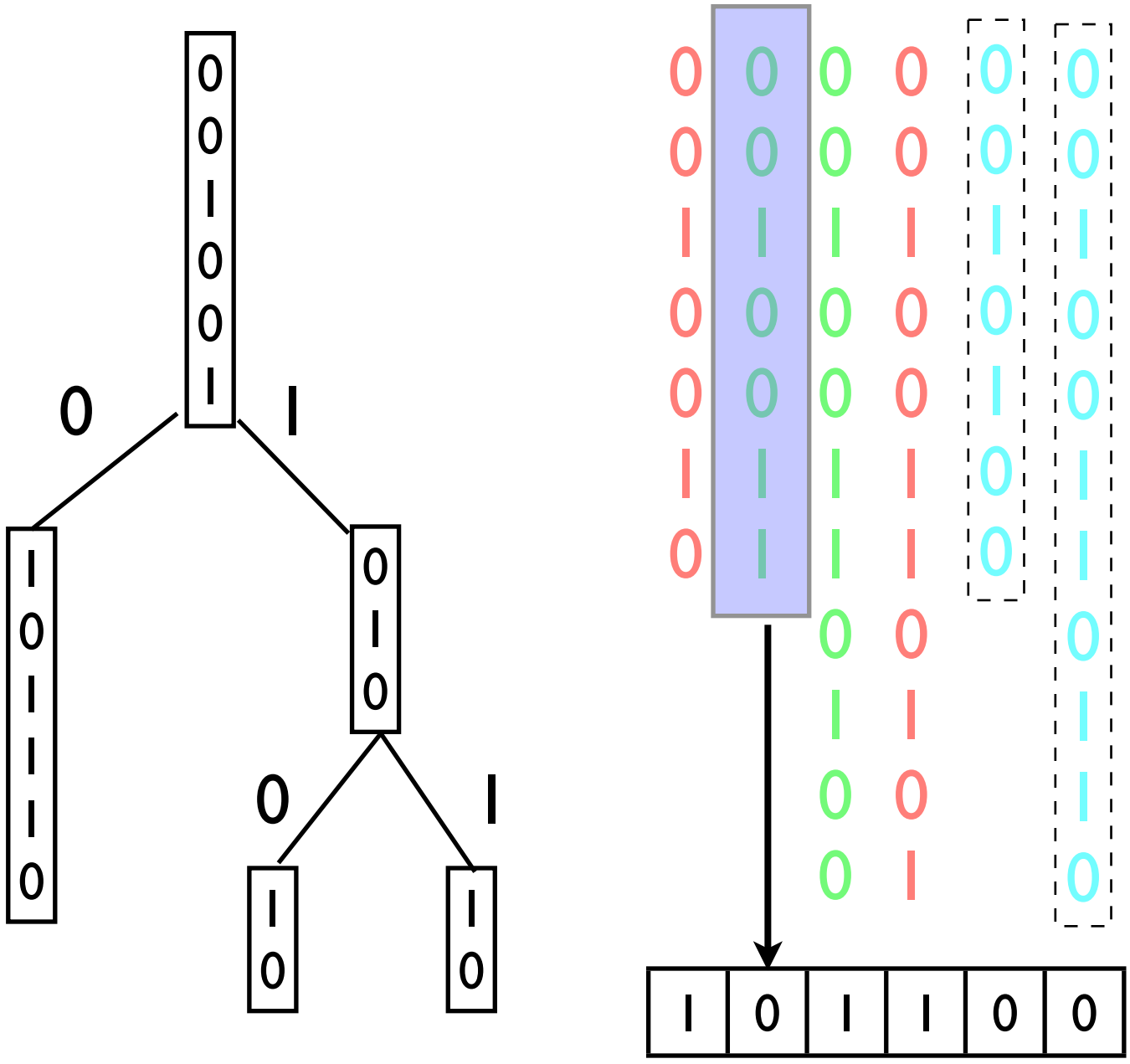

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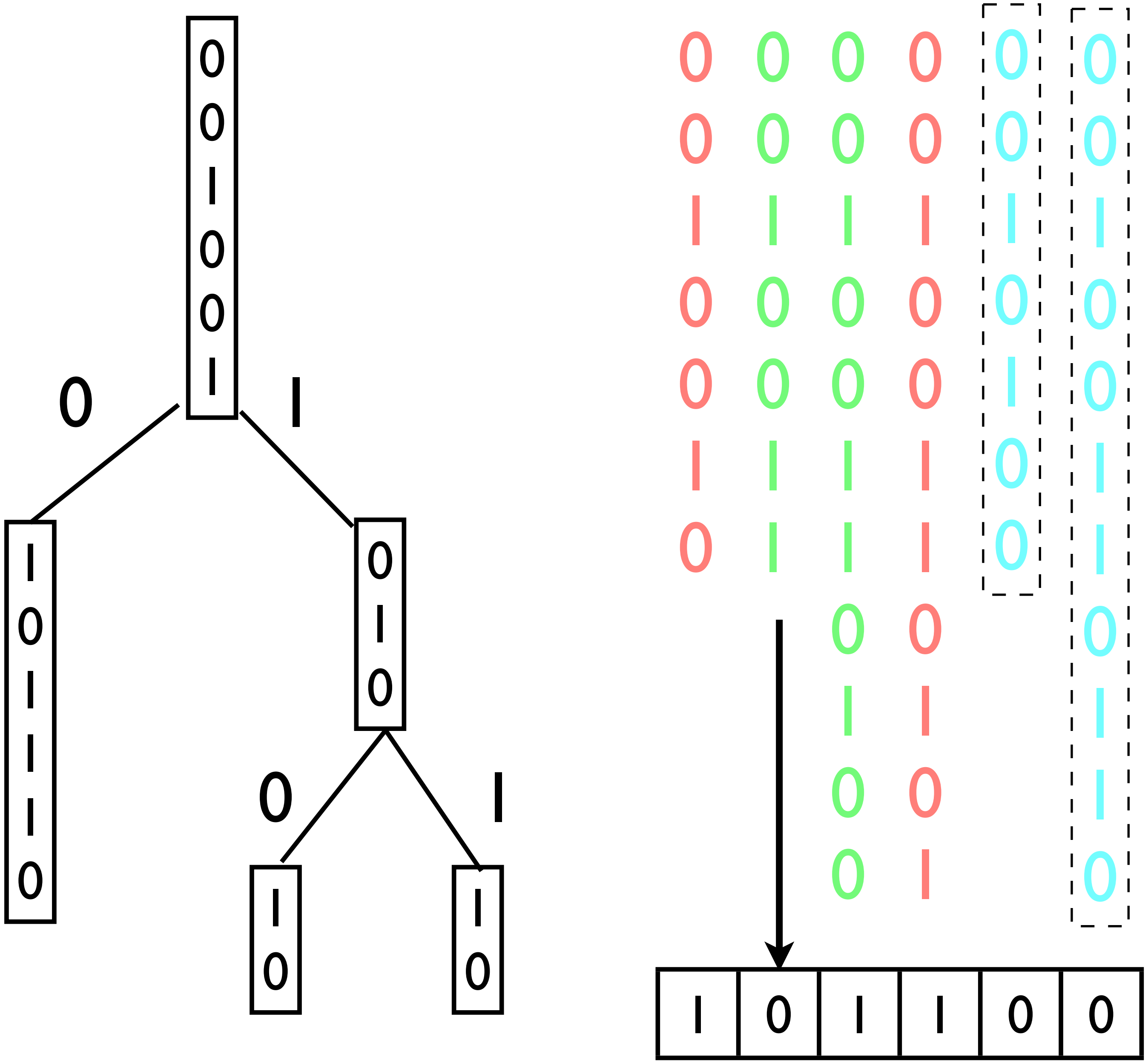

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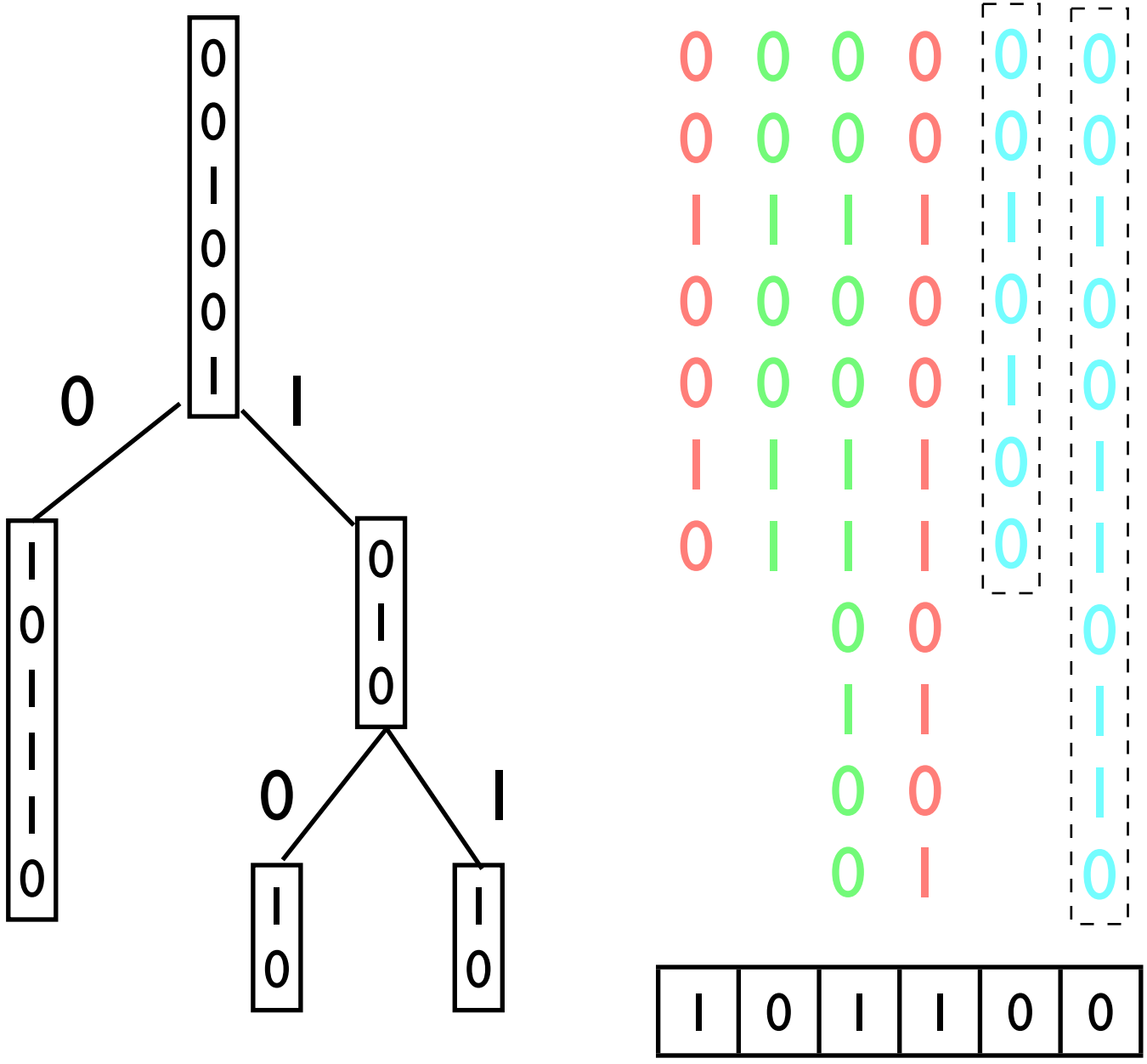
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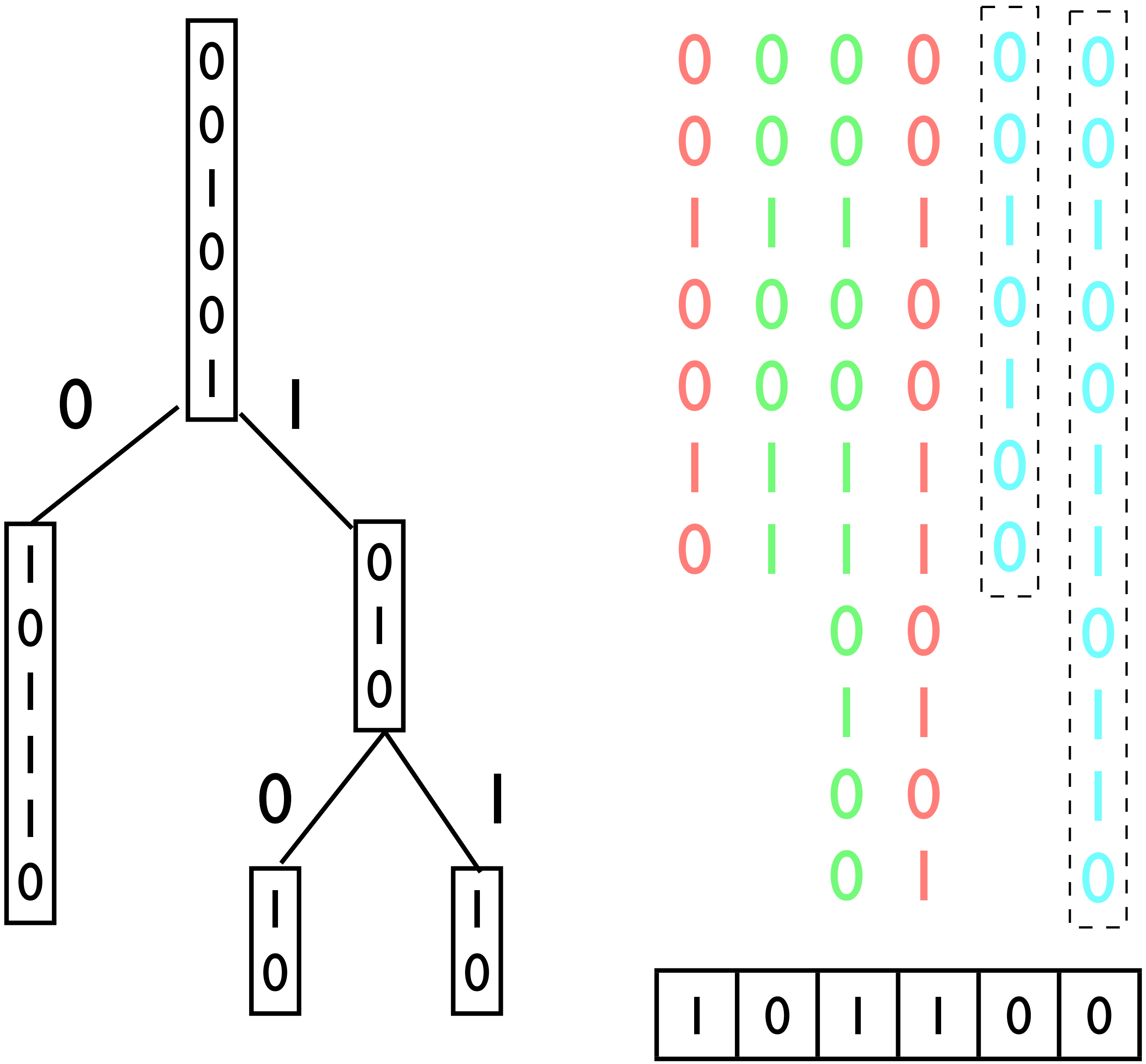
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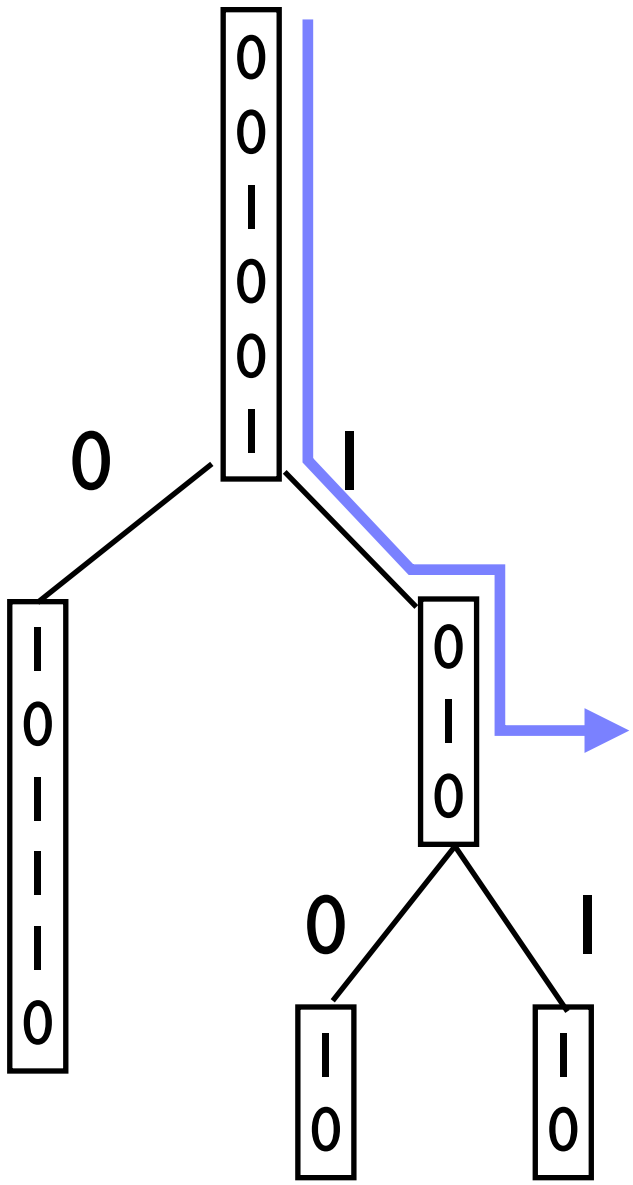
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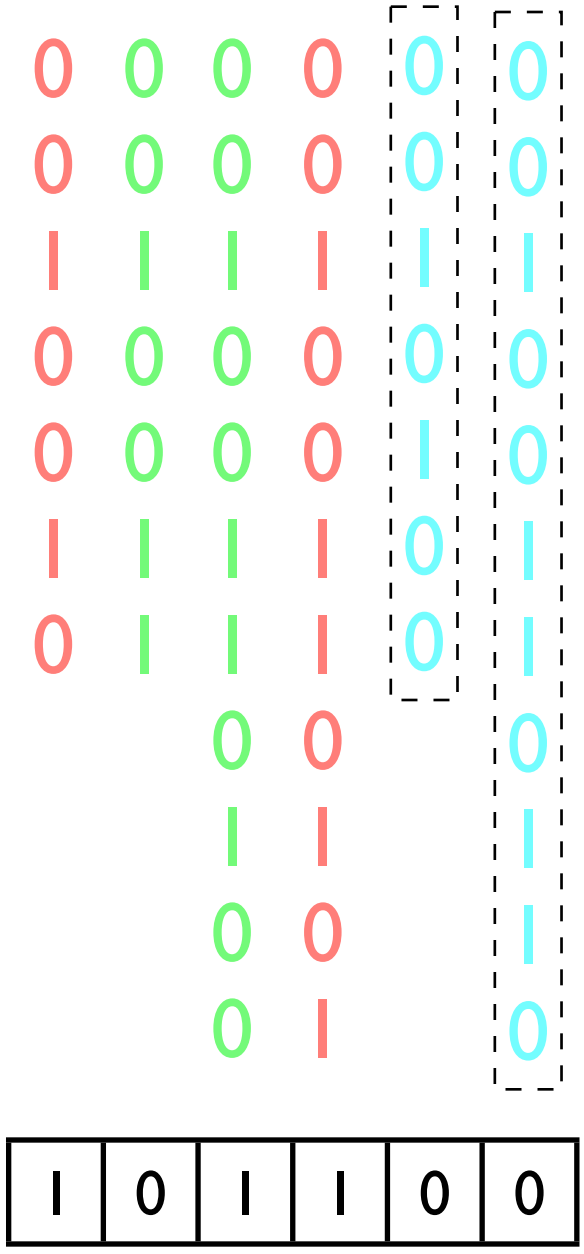
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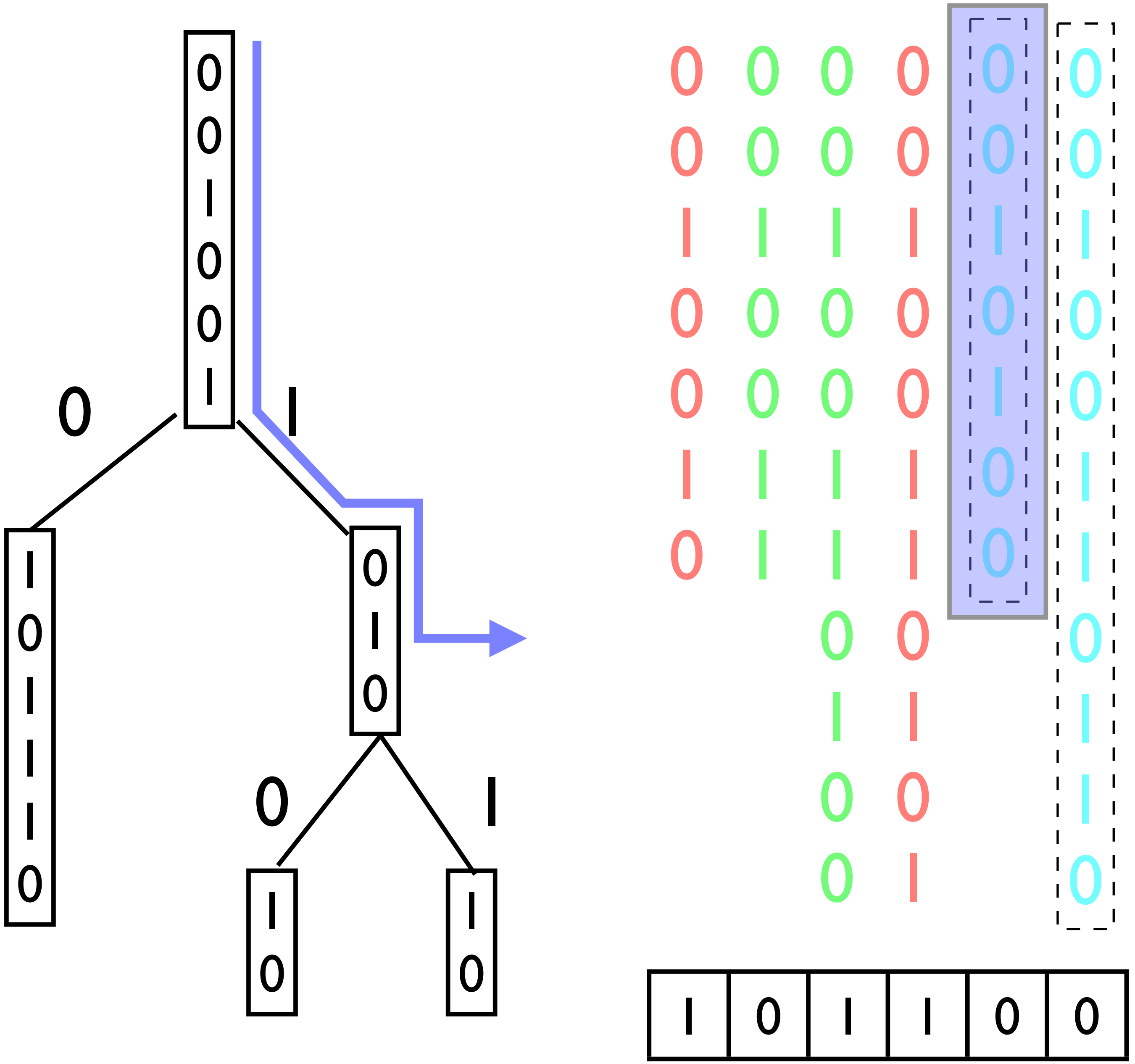
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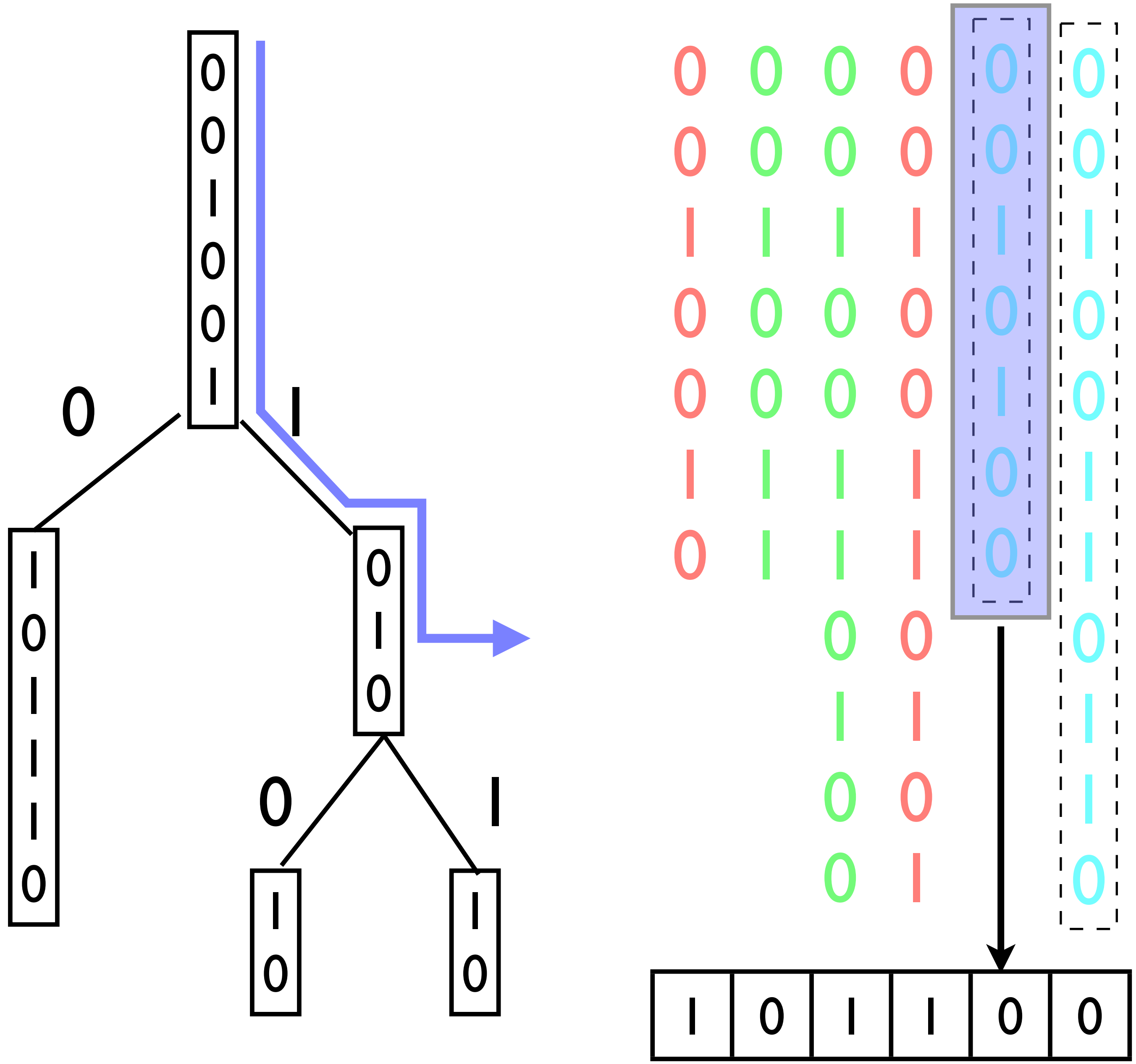
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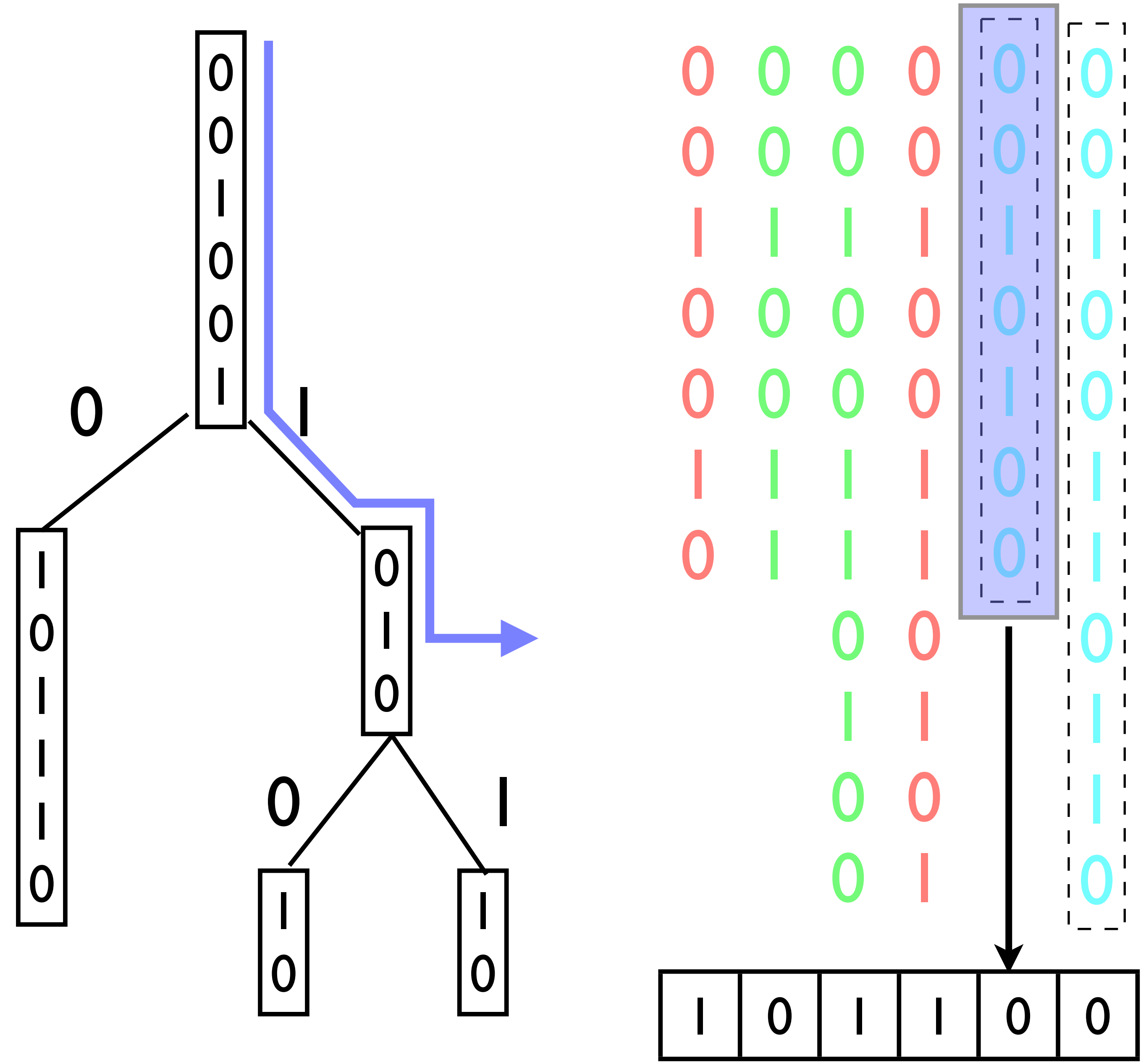
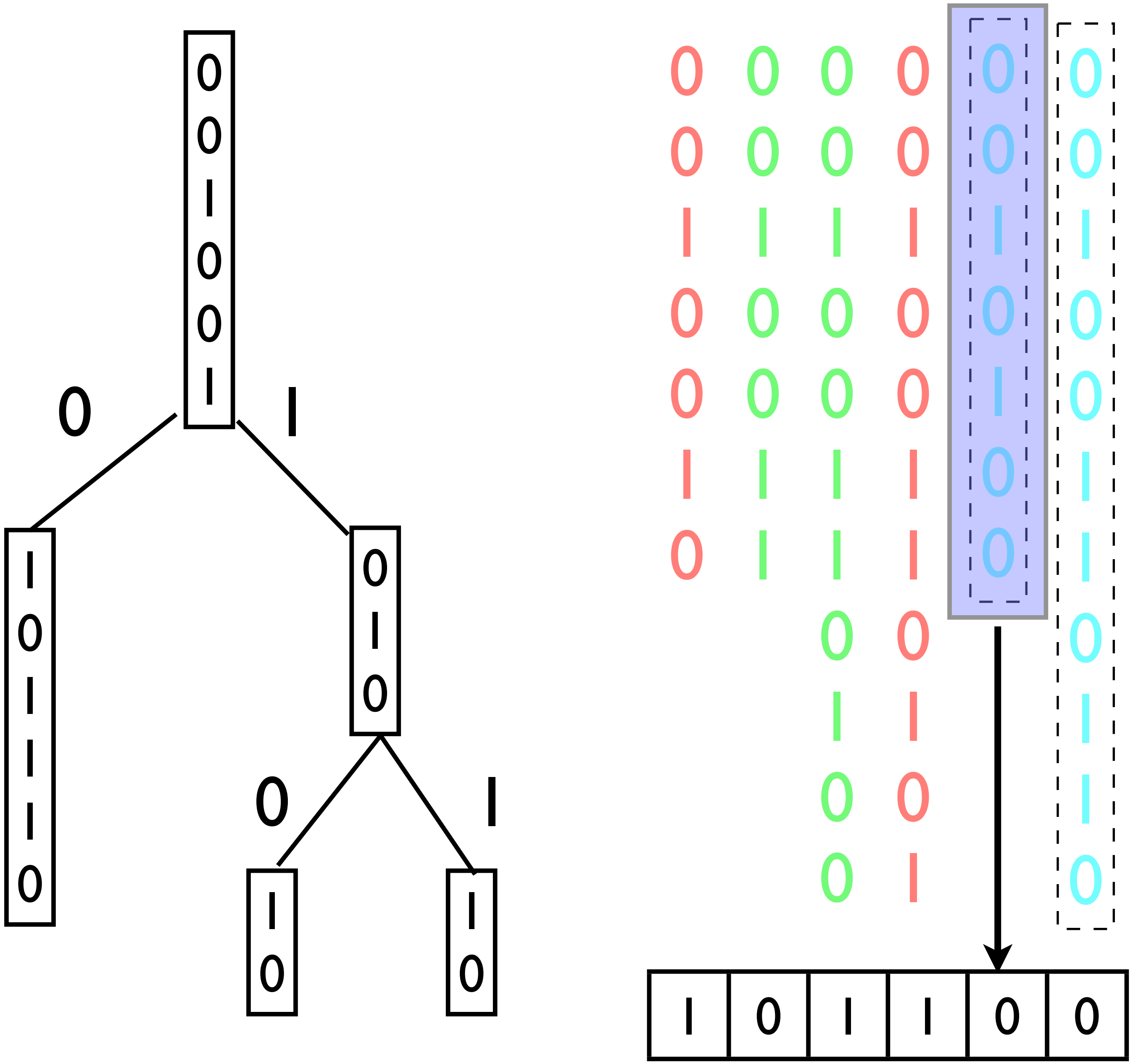
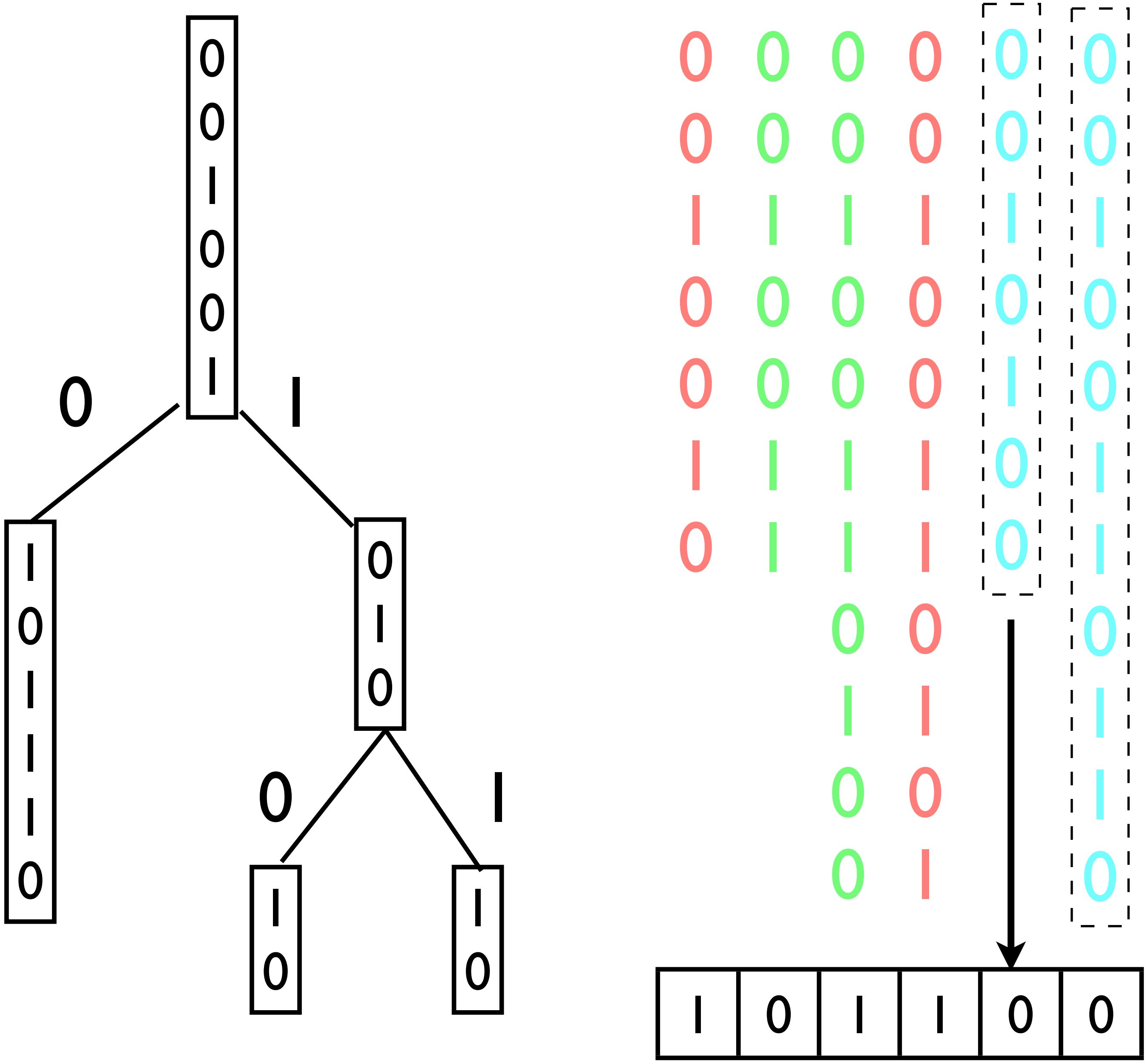


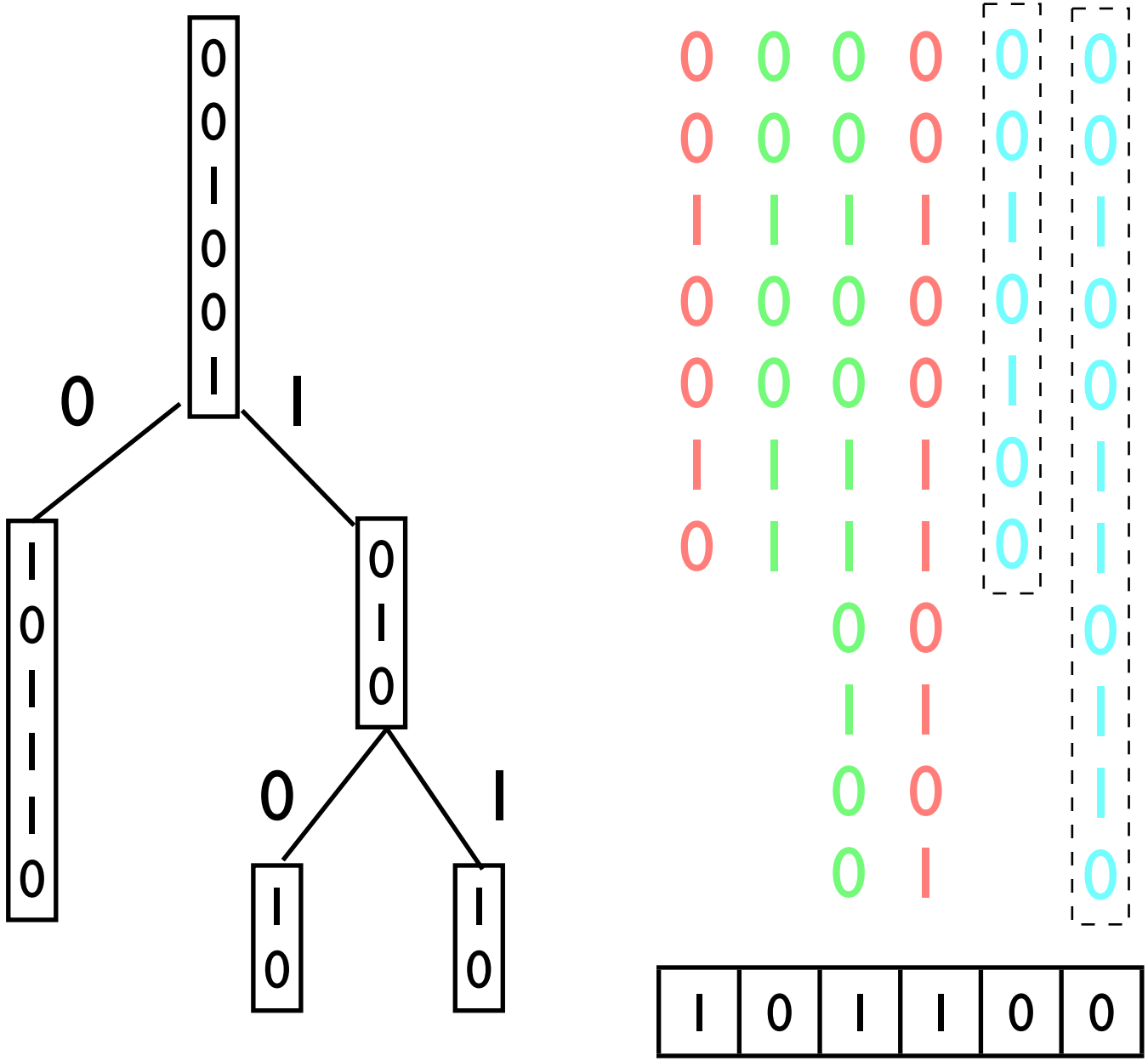
Rank 3 ⇒ bucket 3!



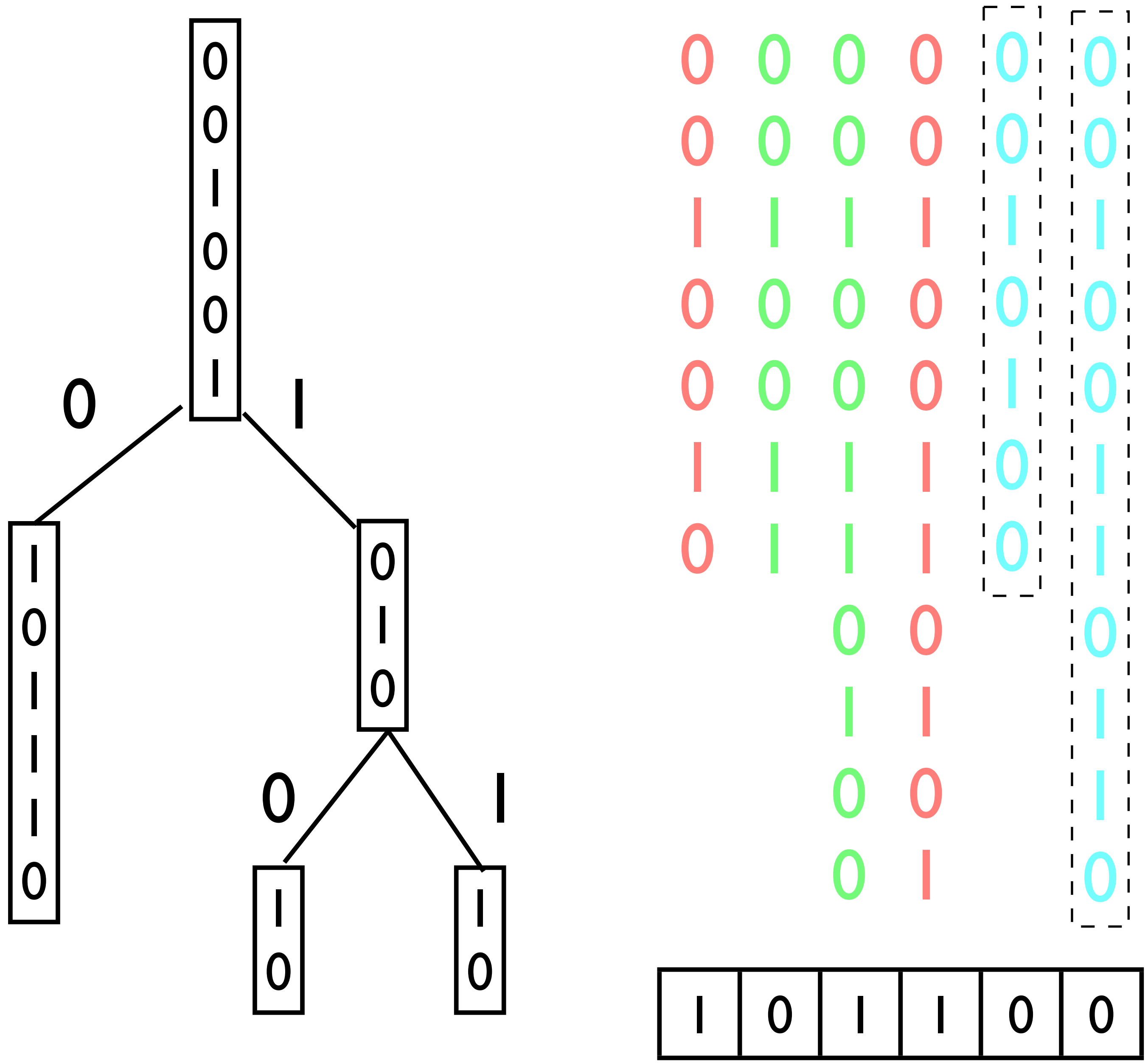
Rank 3 ⇒ bucket 3!



Rank 3 ⇒ bucket 3!



Rank 3 ⇒ bucket 3!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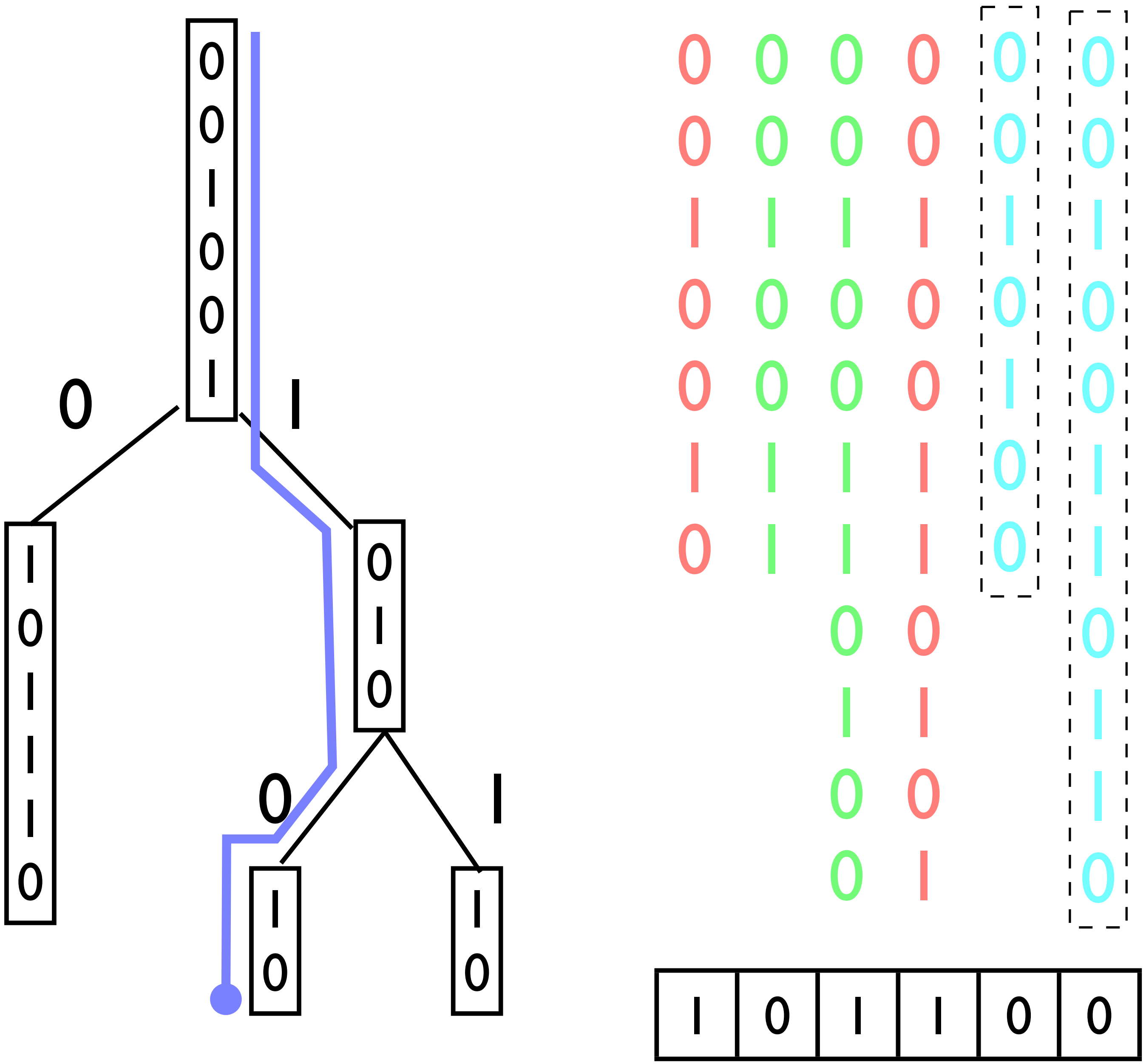

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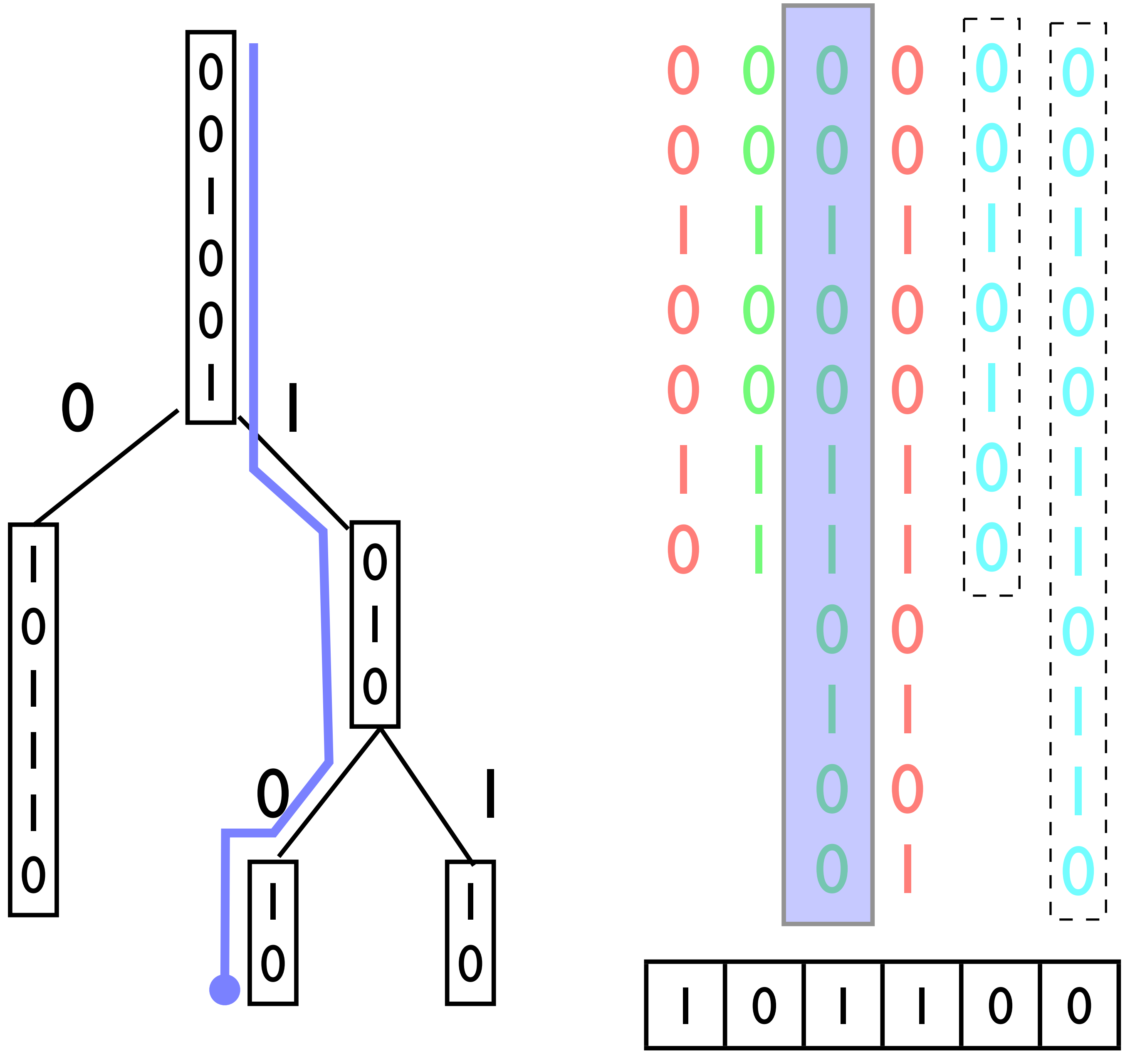

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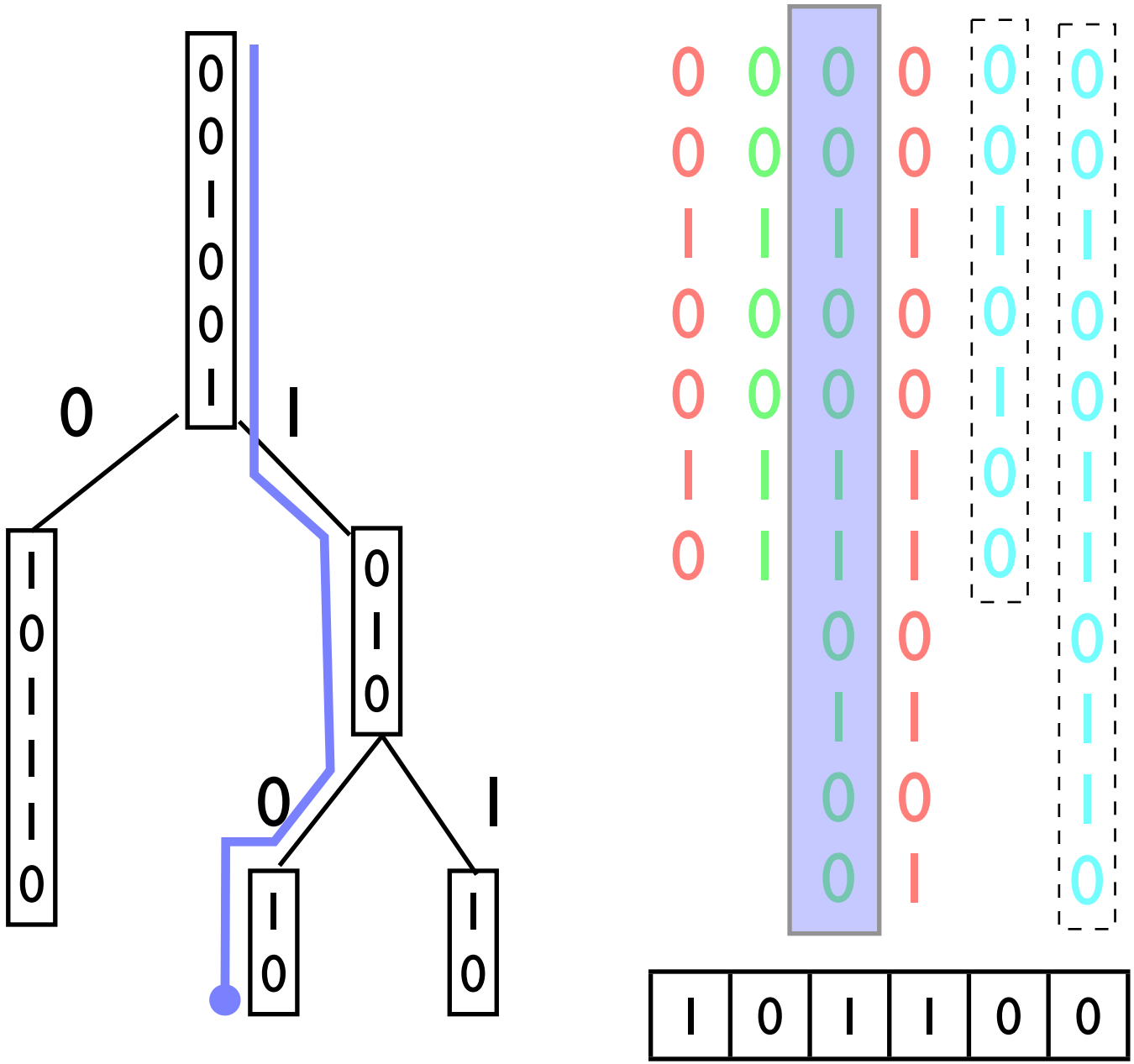

Rank 1 ⇒ bucket 1!

Missing Pieces

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- We use signatures, not paths, so there will be mistakes: but with log w-sized signatures mistakes are so few that they can be stored in a *dictionary relative to* S in space $O(n)$
- All in all, by using buckets of size log w we can do monotone minimal perfect hashing in time $O(\log w)$ using $O(n \log \log w)$ bits (i.e., constant cost per key to all practical purposes)

Conclusions

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- Z-fast tries are the first log *w*-time trie-like structure with (true) linear space occupancy
- Many applications are possible (e.g., Bee-trees, where internal node indexing is made using z-fast tries)

Thanks!

BTW, all this is implemented and working. Please visit

`http://sux4j.dsi.unimi.it/`