STRUTTURA DI CLARKE PER LA SELEZIONE

$\text{select}_{\underline{b}}(k)$ = posizione del k-esimo 1

$\boxed{\text{I LIVELLO}}$ $P[-]$

$P[-]$ memorizza le posizioni degli uni di posizione multipla di

$K = \log n \log\log n$

$P[i] = \text{select}_{\underline{b}}(i \cdot \log n \log\log n)$

$\boxed{\text{MEMORIA}}$

$$\frac{n}{\log n \log\log n} \log n = \frac{n}{\log\log n} = o(n)$$

$\boxed{\text{II LIVELLO}}$ $S[-]$

$\forall i.$ $r_i = P[i+1] - P[i] \geq \lg n \lg\lg n$

$(i+1)\underbrace{\lg n \lg\lg n}_{4 \cdot 17 = 68^\circ}$ $\quad i \underbrace{\lg n \lg\lg n}_{3 \cdot 17 = 51^\circ}$

$\boxed{\text{II A: CASO SPARSO}}$

$r_i \geq (\lg n \lg\lg n)^2$

$S[i]$ - In questo caso $S[i]$ contiene la lista esplicita delle posizioni intermedie (le posizioni di tutti gli uni fra quella di pos. $P[i]$ e quella di pos. $P[i+1]$) COME DIFF. DA $P[i]$

$\boxed{\text{MEMORIA}}$

$$(\lg n \lg\lg n) \lg r_i = \frac{(\lg n \lg\lg n)^2}{\lg n \lg\lg n} \lg r_i \leq$$

$r_i \leq n$

$$\leq \frac{r_i}{\lg n \lg\lg n} \lg r_i \leq$$

$$\leq \frac{r_i}{\cancel{\lg n} \lg\lg n} \cancel{\lg n} =$$

$$= \frac{r_i}{\lg\lg n}$$

$\boxed{\text{II B: CASO DENSO}}$

$$\lg n \lg\lg n \leq r_i < (\lg n \lg\lg n)^2$$

Memorizzo le posizioni multiple di

$$K'_i = \lg r_i \lg\lg n$$

$$S[i][j] = \text{select}_b\Big(\underbrace{i \lg n \lg\lg n}_{P[i]} + j \lg r_i \lg\lg n\Big) -$$

$\boxed{\text{MEMORIA}}$

$$\frac{\log n \log\log n}{\log r_i \log\log n} \log r_i \leq$$

$$\leq \frac{r_i}{\cancel{\log r_i} \log\log n} \cancel{\log r_i} =$$

$$= \frac{r_i}{\log\log n}$$

$\boxed{\text{VALUTAZIONE COMPL. MEMORIA PER } S[\cdot]}$

$$\leq \sum_i \frac{r_i}{\log\log n} = \sum_i \frac{P[i+1] - P[i]}{\log\log n} =$$

$$= \frac{P[n] - P[0]}{\log\log n} \leq \frac{n}{\log\log n} = o(n)$$

III LIVELLO

Solo per gli i del caso II B
cioè

(*) $r_i < (\log n \log\log n)^2$

Ricordiamo che $S[i]$ contiene solo le posiz. multiple di $\log r_i \log\log n$.

$\forall j$ $\quad \overline{r_j^i} = S[i][j+1] - S[i][j]$

Osserv.
- $\overline{r_j^i} \geq \log r_i \log\log n$
- $\overline{r_j^i} < (\log n \log\log n)^2$

III A: CASO SPARSO

$$\overline{r_j^i} \geq \log \overline{r_j^i} \log r_i (\lg\lg n)^2$$

Memorizzo in $T[i][j]$ la lista delle posizioni di tutti gli 1 (come differenza di $S[i][j]$).

$\boxed{\text{MEMORIA}}$

$$(\log r_i \lg\lg n) \log \overline{r_j^i} =$$

$$= \frac{\lg r_i \, (\lg\lg n)^2 \, \lg \overline{r_j^i}}{\lg\lg n} \leq$$

$$\leq \frac{\overline{r_j^i}}{\lg\lg n} = \frac{S[i][j+1] - S[i][j]}{\lg\lg n}$$

IN TUTTO:

$$\sum_j \frac{S[i][j+1] - S[i][j]}{\lg\lg n} =$$

$$= \frac{P[i+1] - P[i]}{\lg\lg n} \leq \frac{n}{\lg\lg n} =$$

$$= o(n)$$

$\boxed{\text{III A : CASO DENSO}}$

$$\overline{r_j^i} < \lg \overline{r_j^i} \, \lg r_i \, (\lg\lg n)^2$$

Fra $s[i][j]$ e $s[i][j+1]$
ci sono pochi zeri.
→ Four-Russians trick

a) $\log \overline{r_j^i} \le \lg r_i \le$ (*)
$\le \lg\left(\left(\lg n \lg\lg n\right)^2\right) =$
$= 2\lg\lg n + 2\lg\lg\lg n \le$
$\le 4\lg\lg n$

b) $\overline{r_j^i} \le \underbrace{\lg \overline{r_j^i}}_{\le 4\lg\lg n \ (a)} \underbrace{\lg r_i}_{\le 4\lg\lg n \ (a)} (\lg\lg n)^2 \le$
$\le 16(\lg\lg n)^4$

- Four-Russians trick

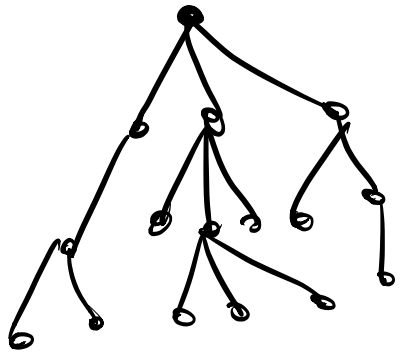
$\le \underbrace{2^{\overline{r_j^i}}}_{\text{n° tabelle}} \underbrace{\overline{r_j^i}}_{\text{n° righe}} \underbrace{\log \overline{r_j^i}}_{\text{n° bit per riga}} \overset{(b)\ (\text{pre valide})}{\le}$

$\le 2^{16(\lg\lg n)^4} \cdot 16(\lg\lg n)^4 \cdot$
$\log\left(16(\lg\lg n)^4\right) =$

$$= (\log \log n)^{4 \cdot 16} \cdot 16 (\log \log n)^4$$

$$\log \left(16 (\log \log n)^4\right) = o(n)$$

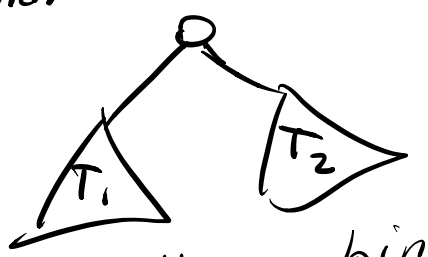
STRUTTURE SUCCINTE PER ALBERI BINARI



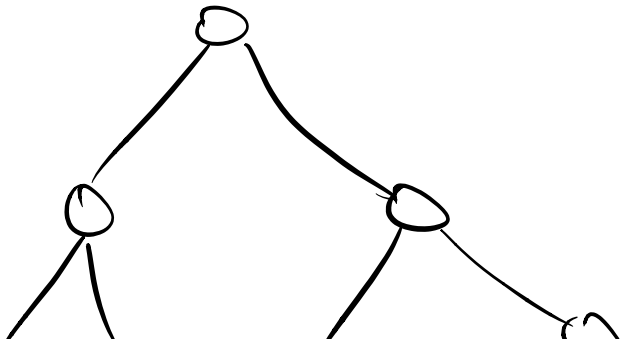
ALBERO BINARIO

1) • è un albero binario

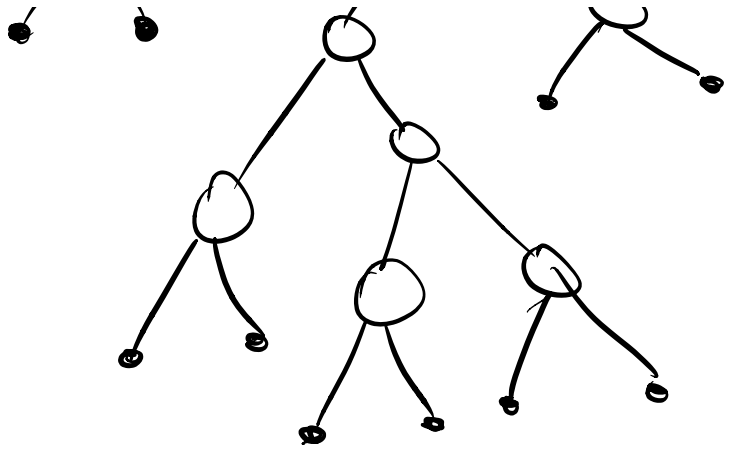
2) se T_1 T_2 sono alberi binari

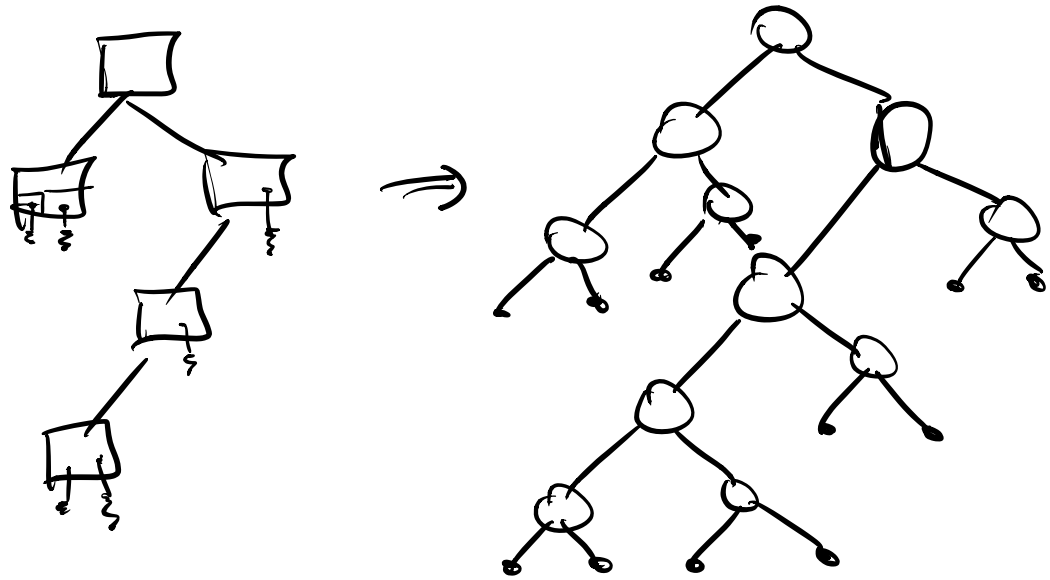


è un albero binario



- FOGLIE
- NODI ESTERNI
- NODI INTERNI





Teorema: In ogni albero binario

$$|E| = 1 + |I|$$

e quindi

$$n = 2|E| - 1 = 2|I| + 1$$

E = ins. nodi esterni

$\bar{\mathbb{R}}$ = " " interno

n = numero complessivo di nodi