ALGORITMI E COMPLESSITÀ

10:30 → 12:30

10:30 → 11:15

11:30 → 12:15

NOTAZIONI

INSIEMI NUMERICI

$\mathbb{N}$ $\mathbb{Z}$ $\mathbb{Q}$ $\mathbb{R}$

$x \in \mathbb{N}$ $\quad x = \{0, 1, \dots, x-1\}$

$0 \leq i < n$

$\underline{i \in n}$

$\underline{x}$ $\quad x_0, x_1, \dots, x_{n-1}$

$x_i \quad (i \in n)$

$|x| = x$

$0 = \emptyset$

$2 = \{0, 1\}$ $\qquad 2^*$

$$A^B = \{ f \mid f : B \to A \}$$

$$|A^B| = |A|^{|B|}$$

$$2^A = \{ f \mid f : A \to 2 \} = \mathcal{P}(A)$$

$$X \subseteq A$$

$$A^2 = \{ f \mid f : 2 \to A \}$$

$$2^{(2^*)} \qquad 2^{(2^*)}$$

$$\binom{A}{n} = \{X \subseteq A \mid |X| = n\}$$

$$\left|\binom{A}{n}\right| = \binom{|A|}{n}$$

$$\binom{x}{y} = \frac{x!}{y!\,(x-y)!}$$

$$\left|\binom{5}{2}\right| = \binom{|5|}{2} = \binom{5}{2} =$$

$$= \frac{5!}{2!\,(5-2)!} = \frac{5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot 1}{\cancel{2} \cdot 1 \cdot \cancel{3} \cdot \cancel{2} \cdot 1} =$$

$$= 10$$

$$\{a, b, c, d, e\}^2 = \{(a,a), (a,b), \dots\}$$

$$\frac{25-5}{2} = 10$$

PROBLEMA Π

1) Insieme di input $I_\Pi \subseteq 2^*$
2) Insieme di output $O_\Pi \subseteq 2^*$
3) Funzione
$$Sol_\Pi : I_\Pi \to 2^{O_\Pi} \setminus \{\emptyset\}$$

ESEMPIO

a - Decidere se un numero naturale è primo

$$I_\Pi = \mathbb{N}$$
$$O_\Pi = \{yes, no\}$$

b - Emettere il MCD fra due naturali positivi

$$I_\Pi = (\mathbb{N} \setminus \{0\})^2$$
$$O_\Pi = \mathbb{N}$$

c - SAT

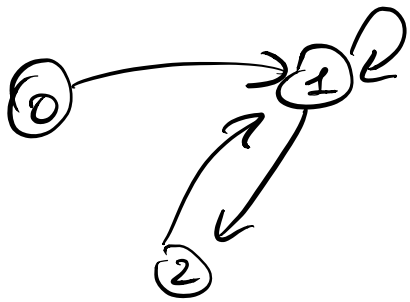
$$\underbrace{(x_1 \overset{\text{OR}}{\vee} \overset{\text{NOT}}{\neg} x_2)}_{\text{CLAUSOLE}} \overset{\text{AND}}{\wedge} (\overset{\text{LETTERALE}}{x_1} \overset{\text{OR}}{\vee} \overset{\text{LETTERALE}}{\overset{\text{NOT}}{\neg} x_7} \overset{\text{OR}}{\vee} x_2) \overset{\text{AND}}{\wedge} (\neg x_2 \vee x_4)$$

PROBLEMA

input: G grafo orientato

$x, y \in \mathbb{N}$

$7 \quad 12$



$$\begin{array}{c|ccc} & 0 & 1 & 2 \\ \hline 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 \end{array}$$

$n = 3$

$$\underbrace{010011010}_{2n^2}\,01\,\underbrace{\ldots\ldots}_{2\lceil \log x \rceil + 2}\,01\,\underbrace{\ldots\ldots}_{2\lceil \log y \rceil + 2}$$

$+2$

$\rightarrow$

$0 \rightarrow 00$

$1 \rightarrow 11$

$$\underbrace{001100001111001100}_{\text{archi}}\,01$$

m

$$\underbrace{2\lceil \log n \rceil + 2}_{\boxed{n}} + \underbrace{2\lceil \log m \rceil + 2}_{\boxed{m}} + \underbrace{m(2\lceil \log n \rceil + 2 + 2\lceil \log n \rceil)}_{\boxed{\ldots\text{archi}\ldots}} +$$

$$\rightarrow + 2 + 2\lceil \log x \rceil + 2 + 2\lceil \log y \rceil$$

ALGORITMI PER Π

$$x \in I_\Pi \longrightarrow \boxed{A} \longrightarrow y \in O_\Pi$$

t.c. $y \in Sol_\Pi(x)$

complessità
- algoritmica
- strutturale

COMPLESSITÀ ALGORITMICA

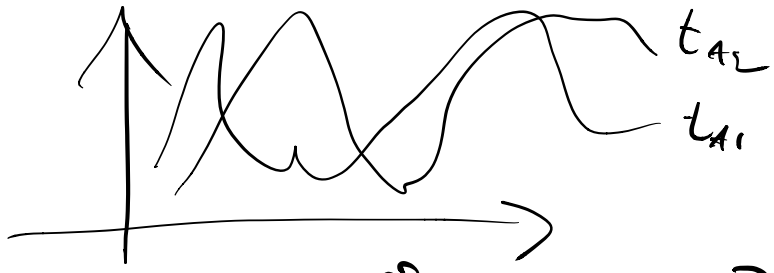
$$T_A : I_\Pi \to \mathbb{N}$$

$$t_A : \mathbb{N} \to \mathbb{N}$$

$$t_A(n) = \max_{\substack{x \in I_\Pi \\ |x| = n}} T_A(n)$$

$$t_A(200) = 1000000$$

$t_{A_1}, t_{A_2} : \mathbb{N} \to \mathbb{N}$



$$t_{A_1}(n) = 10^{7000} n^2 - 7n + 3$$

$$t_{A_2}(n) = \frac{2^n}{\log n}$$

$$\lim_{n \to \infty} \frac{t_{A_1}(n)}{t_{A_2}(n)} = 0$$

$O(n \log n)$ $\qquad$ $O(n^2)$

COMPLESSITÀ STRUTTURALE

Π $\quad t_{A_1}, t_{A_2}, t_{A_2}$

- Classe P

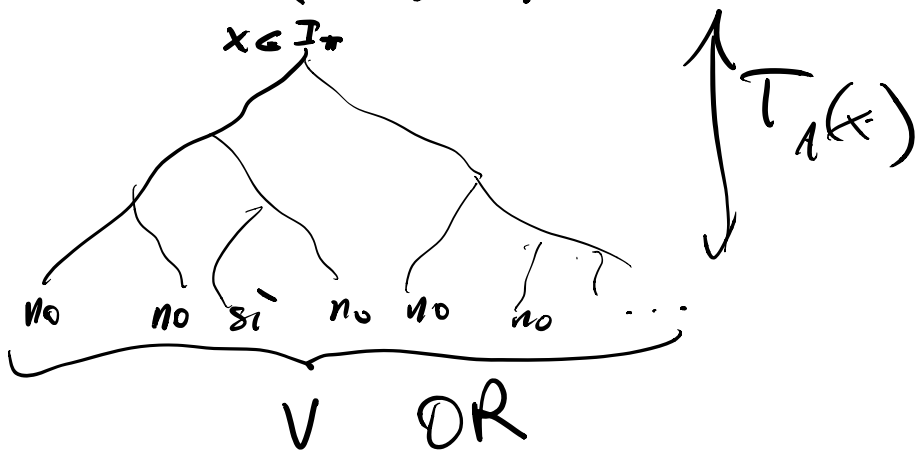
$P = \{ \Pi \mid \Pi$ è un problema di decisione e $\exists A$ per Π t.c. $t_A(n)$ è dominata da un polinomio $\}$

lower bound $\rightarrow$ $O(n^{1.15})$ $\quad \Pi \quad$ $O(n^{1.73})$ $\leftarrow A$ upper bound

- Classe NP

NPython

$x = ? = ?$



$NP = \{\Pi \mid \Pi$ è un problema di dec. ed $\exists A$ non-deterministico per Π t.c. $t_A(n)$ è olim. da un polinomio$\}$

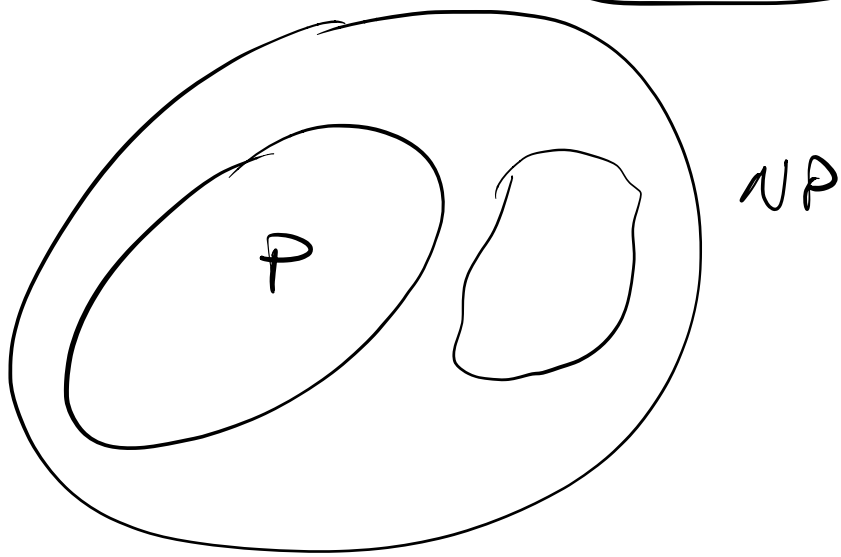
$SAT \in NP$

$SAT \overset{?}{\in} P$

$P \subseteq NP$

$P = NP$

$P \subset NP$



$\boxed{\text{RIDUCIBILITÀ IN TEMPO POLINOMIALE}}$ $\leq_p$

$\Pi_1 \leq_p \Pi_2$ <u>sse</u>

$\exists f : 2^* \to 2^*$ t.c.

1) f è calcolabile in tempo pol.

2) $\forall x \in I_{\Pi_1}$. $f(x) \in I_{\Pi_2}$

e $sol_{\Pi_1}(x) = sol_{\Pi_2}(f(x))$

<u>Lemma</u>: Se $\Pi_1 \leq_p \Pi_2$ e $\Pi_2 \in P$

allora $\Pi_1 \in P$.

Π è <u>NP-completo</u> sse

$\forall \Pi' \in NP.$

$\Pi' \leq_p \Pi$ e $\Pi \in NP$

<u>Teorema</u> (Cook): SAT è NP-completo.

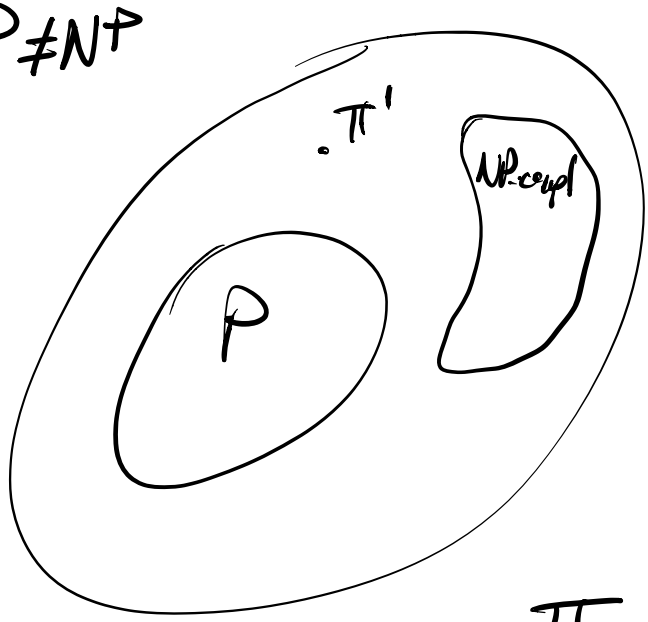
<u>Corollario</u>: $\Pi_1 \leq_p \Pi_2 \in NP$ e Π_1 è NP-completo, anche Π_2 è NP-completo

<u>Dim</u>: $\Pi' \in NP$

$\Pi' \leq_p \Pi_1 \leq_p \Pi_2$. $\square$

Π $\qquad$ $\underline{SAT \leq_p \Pi}$

se $P \neq NP$



NP

π NP-completo

Per assurdo
t.c. $\pi \in P$

$\pi' \leq_p \pi$

Garey - Johnson

COLORABILITY $\leq_p$ SAT