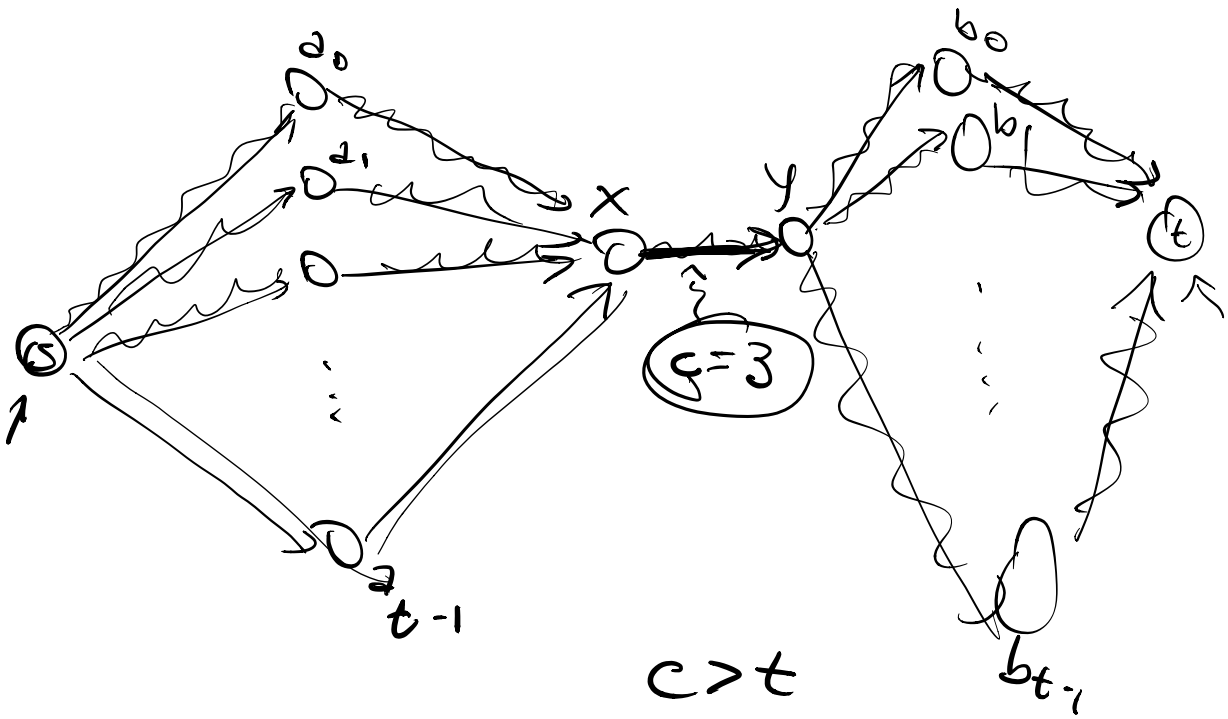
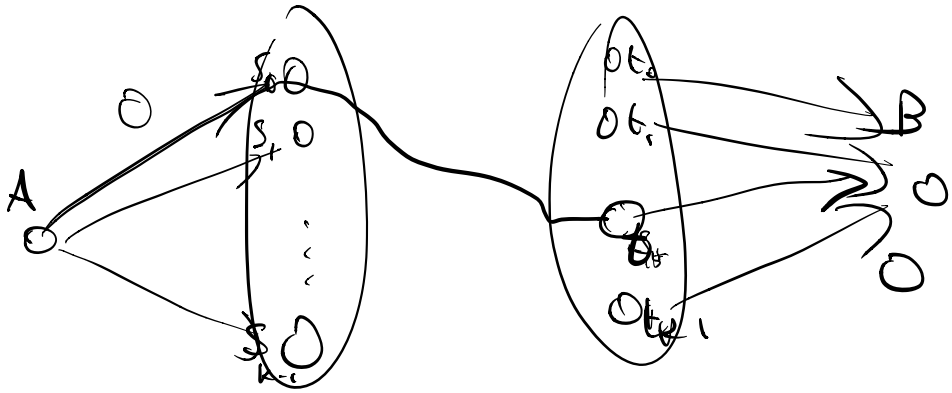

s
a_0
a_1
a_{t-1}
x
y
$c=3$
b_0
b_1
b_{t-1}
t
$c > t$


A
B

PL (PROGRAMMAZIONE LINEARE)

$$\min \quad 3x_1 + 7x_2 - 4x_3$$

$$\begin{cases} x_1 + x_2 \leq 3 \\ 3x_3 - x_1 \geq 7 \\ x_1 \geq 0 \\ x_2 \geq 0 \\ x_3 \geq 0 \end{cases}$$

$$A\underline{x} \geq \underline{b}$$

LP

INPUT: $A \in \mathbb{Q}^{m \times n}$ $\quad \underline{b} \in \mathbb{Q}^m$

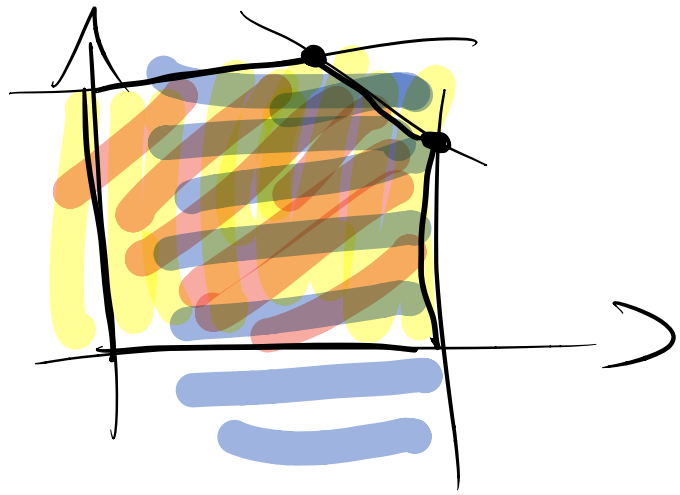
$\underline{c} \in \mathbb{Q}^n$

SOLUZIONE AMMISSIBILE: $\underline{x} \in \mathbb{R}^n$ t.c. $A\underline{x} \geq \underline{b}$

COSTO: $\underline{c}^T \cdot \underline{x}$

TIPO: MIN

$LP \in PO$



$n=2$

ILP

INPUT: $A \in \mathbb{Q}^{m \times n}$ $\quad \underline{b} \in \mathbb{Q}^{m}$

$\underline{c} \in \mathbb{Q}^{n}$

SOLUZIONE AMMISSIBILE:

$\underline{x} \in \mathbb{Z}^{n}$ t.c. $A\underline{x} \geq \underline{b}$

COSTO: $\underline{c}^{T} \cdot \underline{x}$

TIPO: MIN

$ILP \in NPO$-completo

Teorema: $\widehat{\text{VERTEX COVER}} \leq_p \widehat{\text{ILP}}$

Dim:

Π istanza di $\widehat{\text{VertexCover}}$: $(G=(V,E), \langle w_i \rangle_{i \in V}, W)$

$\exists X \subseteq V$ t.c. $\forall xy \in E$ $x \in X$ o $y \in X$

e $\sum_{x \in X} w_x \leq W$

$\tilde{\Pi}$ istanza di ILP:

- x_i $\quad i \in V$
- VINCOLI

$(2n+m)$
$$\begin{cases} x_i \geq 0 & \forall i \in V \\ x_i \leq 1 & \forall i \in V \\ x_i + x_j \geq 1 & \forall ij \in E \end{cases}$$

- FUNZ. OBIETTIVO $\quad$ VINCOLO W

$$\sum_{i \in V} w_i x_i$$

- TIPO: MIN

$\bar{x}_i$ è soluzione di $\bar{\pi}$
ammissibile

$$\bar{X} = \{ i \mid \bar{x}_i = 1 \}$$

$\forall ij \in E \qquad i$ o $j \in \bar{X}$ $\square$

ISTANZA DI VERTEX COVER

Π

INPUT $G=(V,E)$, $\langle w_i \rangle_{i \in V}$

SOL AMM $\exists X \subseteq V$. t.c. $\forall xy \in E$ $x \in X$ o $y \in X$

FUNZ. OB - $\sum_{x \in X} w_x$

$\Downarrow$

ISTANZA DI PL

Π'

- x_i $\quad i \in V$
- VINCOLI

$$\begin{cases} x_i \geq 0 & \forall i \in V \\ x_i \leq 1 & \forall i \in V \\ x_i + x_j \geq 1 & \forall ij \in E \end{cases}$$

- FUNZ. OBIETTIVO

$$\sum_{i \in V} w_i x_i$$

- TIPO: MIN

OTTIMO

x_i^*

w^*

$\Uparrow$

Π'_{INT} [$\boxed{\text{OTTIMO } \tilde{x}_i \quad \tilde{w}}$

$$\tilde{w} \geq w^*$$

$$x_i = \begin{cases} 1 & \text{se } x_i^* \geq \frac{1}{2} \\ 0 & \text{se } x_i^* < \frac{1}{2} \end{cases} \qquad i \in V$$

x_i è ammissibile per Π_{INT}

DIMOSTRIAMO CHE

$$\forall ij \in E \qquad x_i + x_j \geq 1 \equiv \begin{matrix} x_i = 1 \\ \vee \\ x_j = 1 \end{matrix}$$

SAPENDO CHE

$$\forall ij \in E \qquad x_i^* + x_j^* \geq 1$$

Per assurdo, supponiamo $x_i = x_j = 0$

$$x_i^* + x_j^* < \frac{1}{2} + \frac{1}{2} = 1$$

$$\sum_{i \in V} w_i r_i \leq 2 \underbrace{\sum_{i \in V} w_i x_i^*}_{\leq 2\tilde{w}} = 2w^*$$

fodie

$r_i \leq 2x_i^*$