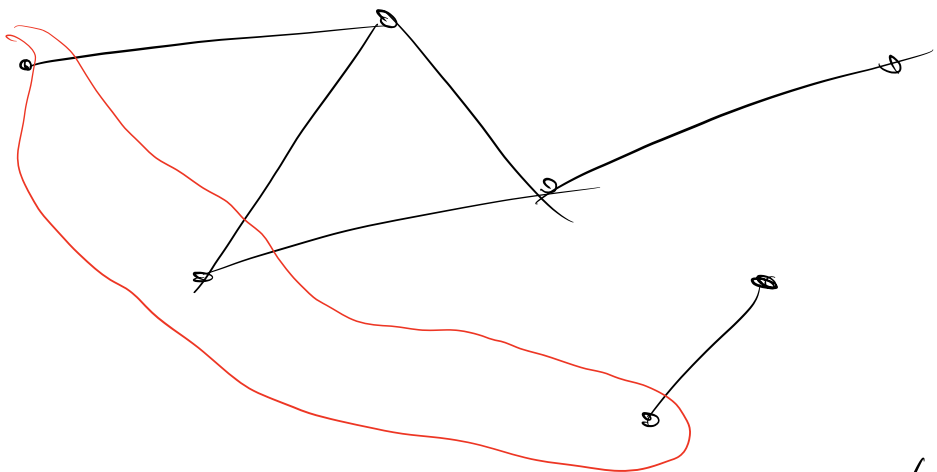
PROBLEMA DELL' INDEPENDENT SET E SUA INAPPROSSIMABILITÀ (ATTRAVERSO PCP)



INPUT : $G=(V,E)$ non orientato

OUTPUT : $X \subseteq V$ insieme indip. (cioè, t.c. $\forall i,j \in X \quad ij \notin E$)

FUNZ. OB. : $|X|$

TIPO: MAX

Teorema: Per ogni $\varepsilon > 0$,

> INDEPENDENT SET non è $(2-\varepsilon)$-approssimabile in tempo polinomiale (se $P \neq NP$).

Dim: $L \in NP\text{-completo}$

$L \in PCP[r(n), q]$ $\quad r(n) \in O(\log n)$, $q \in \mathbb{N}$

$z \in 2^*$ $\quad Q_z = 2^{r(|z|)}$ seq. bit random

$R \in R_z$ $\quad Q_z^R = 2^q$ risposte dell'oracolo

CONFIGURAZIONE (per z)

$s_1 \leftarrow$ <00101, <2: 0 17: 1 32: 1 4: 0>

$R \in R_z$ $\quad q$ query

No $z \leftarrow$ <00101, <2: 1 17: 1 32: 0 4: 0>

$a \leftarrow \langle 01011, \langle 3:0$ $10:1$ $6:0$ $7:1$

$\mathcal{A}_z$ = configurazioni accettanti per z

$|\mathcal{A}_z| \leq 2^{|r(z)|} \cdot 2^q = 2^{\alpha(q,|z|)} 2^q =$

$G_z = (\mathcal{A}_z, \dots)$ $= Q(|z|)$

$\langle R, \langle i_1^R : v_1, \dots, i_q^R : v_q \rangle \rangle$

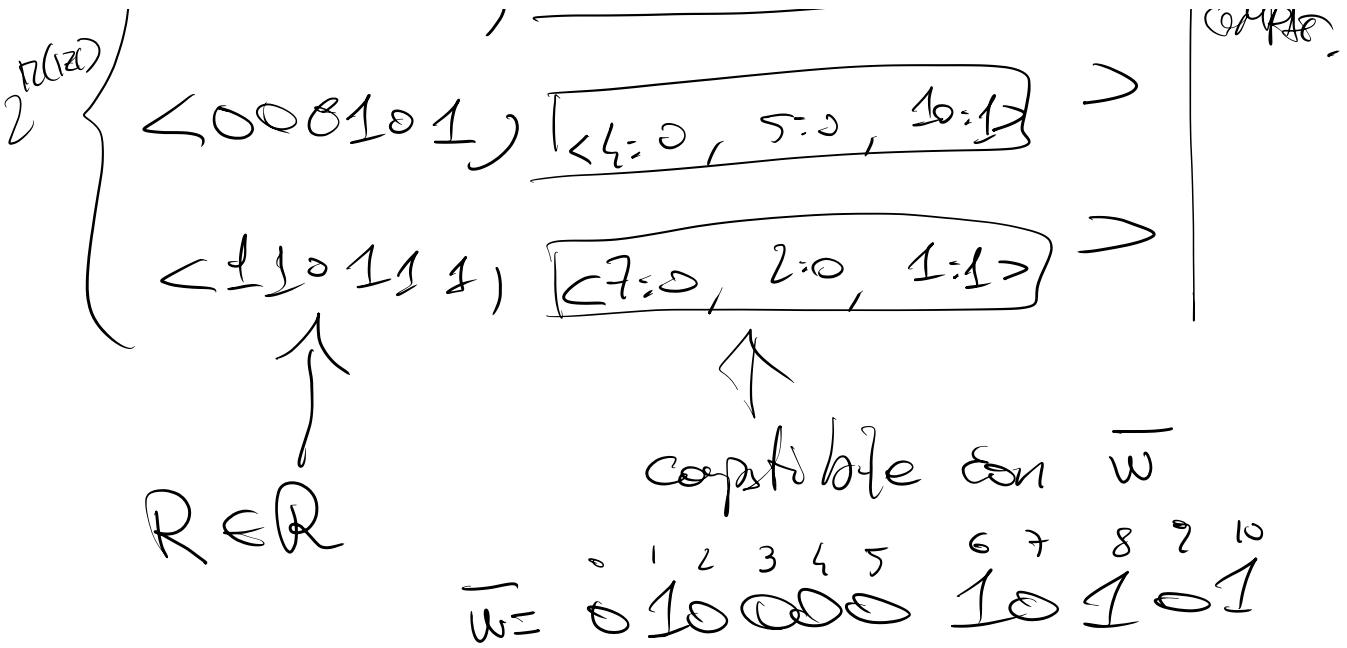
incompatib. $R = R'$ $\vee$ $\exists k, k'$ t.c. $i_k^R = i_{k'}^{R'}$ e $v_k = v'_{k'}$

$\langle R', \langle i_1^{R'} : v_1', \dots, i_q^{R'} : v_q' \rangle \rangle$

Fatto 1: Se $z \in L$, G_z ha un insieme indipendente di card. $\geq 2^{r(|z|)}$

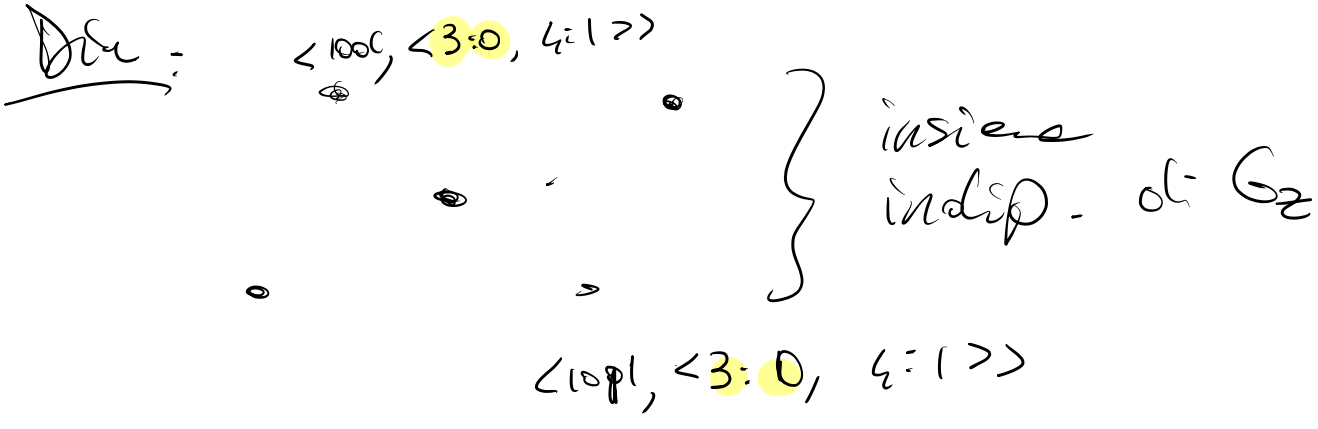
Dim: $\exists \bar{w} \in 2^q$ t.c. il verificatore accetta con prob. 1

$|\langle 000001, \boxed{\langle 3:0, 5:0, 8:1 \rangle} \rangle|$ ...



Fatto 2: Se $z \notin L$, ogni insieme indipendente di G_z ha card. $\leq 2^{x(|z|)-1}$

Dim:



$\Rightarrow$ COSTRUIRE W

0 1 2 3 4 5 6

ADT (Abstract Data Types)

ADT: Stack<T>

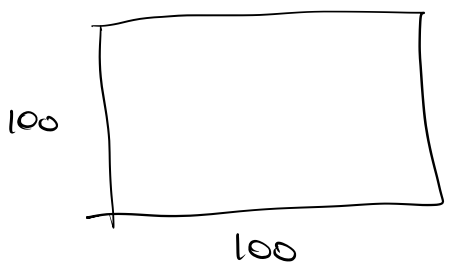
primitive
- boolean isEmpty()
- T top()
- void pop()
- void push(T x)

- ∀S
  S.push(x).top() == x
- ∀S
  S.isEmpty() == S.push(x).pop().isEmpty()

ADT ⟶ IMPLEMENTAZIONI

INFORMATION - THEORETICAL LOWER BOUND

- Per codificare v valori servono in media $\log_2 v$ bit.



2^{10000} $\left\{ \log 2^{10000} = 10000 \right.$

- $x_1, x_2, \ldots, x_v$

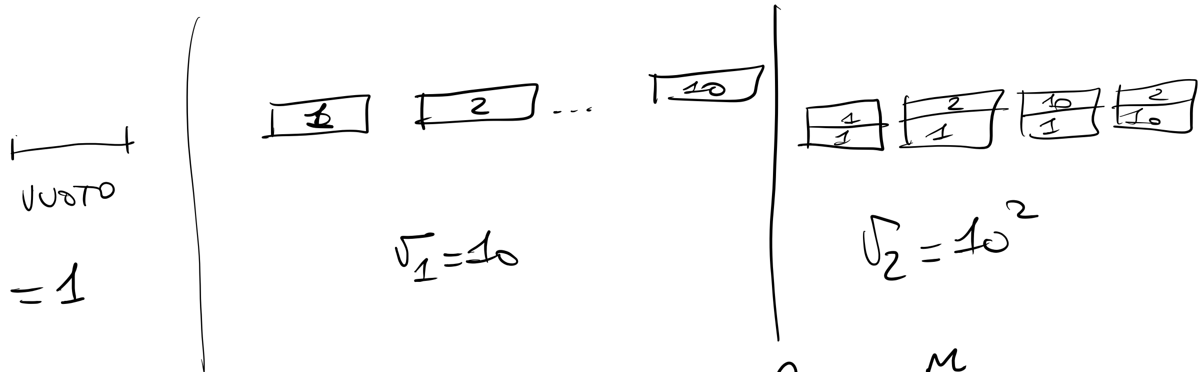
x_i = #bit necessari per rappresentare l'i-esimo valore

$$\geq \frac{x_1 + x_2 + \ldots + x_v}{v} \geq \boxed{\log_2 v}$$

$[\log_2 v,\ 1+\log_2 v)$

- Immaginiamo un ADT con V_n VALORI DI TAGLIA n

- STACK <{1,...,10}>



$$V_n = 10^n \rightarrow \log 10^n = n \log 10 \approx \boxed{4n}$$

$$\boxed{Z_n = (\log_2 10)\, n \text{ bit}}$$ INF. THEORETICAL LOWER BOUND

$D_n \geq Z_n$ IMPLEMENTAZIONE CHE USA D_n BIT

Un'implementazione dell'ADT è chiamata

(proprietà eff. come su str. "naturale")

1) implicita se $D_n = Z_n + O(1)$
2) succinte se $D_n = Z_n + o(Z_n)$
3) compatte se $D_n = O(Z_n)$

1) $Z_n + 3$ 2) $Z_n + \log Z_n$ 3) $Z_n + \sqrt{Z_n}$

RANGO E SELEZIONE

ADT definiti da $\underline{b} \in 2^n$

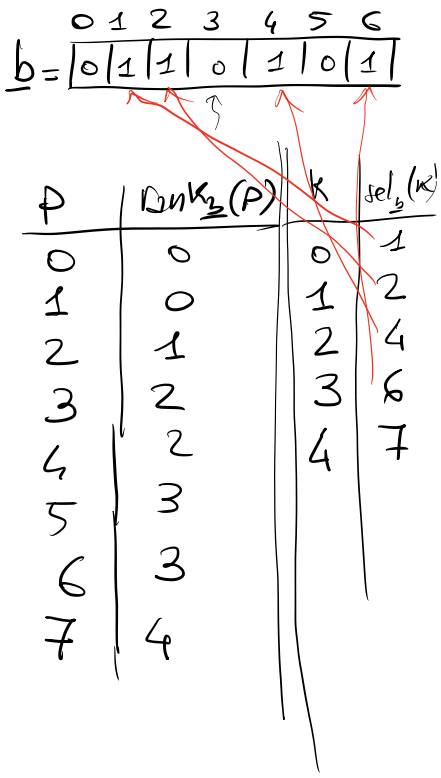
$$\left.\begin{matrix} rank_{\underline{b}} \\ select_{\underline{b}} \end{matrix}\right\} : \mathbb{N} \longrightarrow \mathbb{N}$$

1) $\forall p \leq n$

$$rank_{\underline{b}}(p) = |\{i \mid i < p \text{ e } b_i = 1\}|$$

2) $\forall k \leq$

$$select_{\underline{b}}(k) = \max\{p \mid rank_{\underline{b}}(p) \leq k\}$$



p	$rank_{\underline{b}}(p)$	k	$sel_{\underline{b}}(k)$
0	0	0	1
1	0	1	2
2	1	2	4
3	2	3	6
4	2	4	7
5	3		
6	3		
7	4		

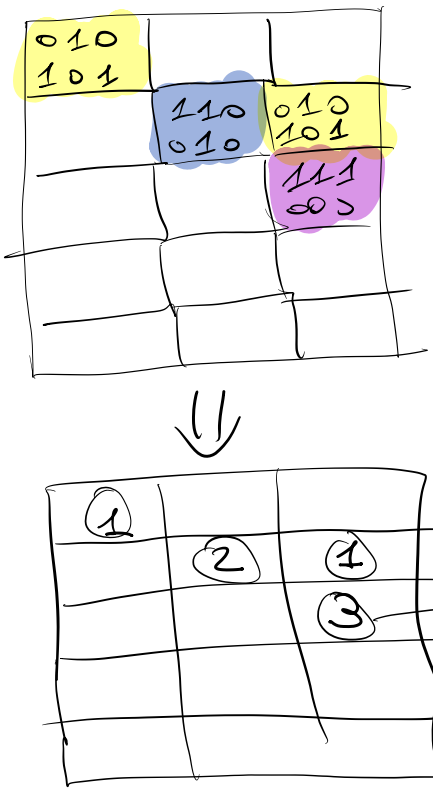
a) $\forall k \quad rank(select(k)) = k$

b) $\forall p \quad select(rank(p)) \geq p$] PERMETTE DI DEDURRE b

è = sse $b_p = 1$

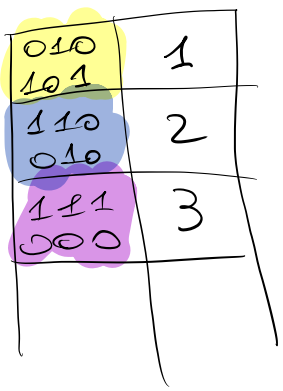
STRUTTURA DI JACOBSON PER IL RANGO

IDEA Usa il "Four-Russians trick"!



MATRICE BINARIA

TABELLA



2^6 righe

$\log 2^6$ bit $= 6$ bit