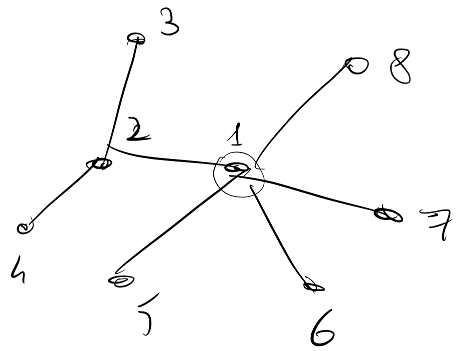
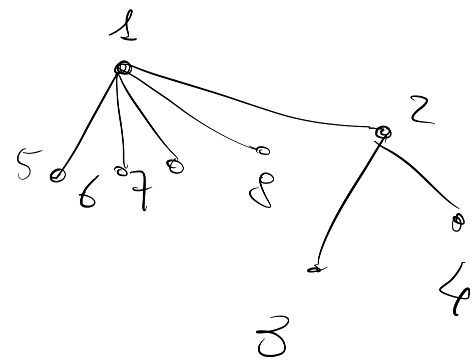
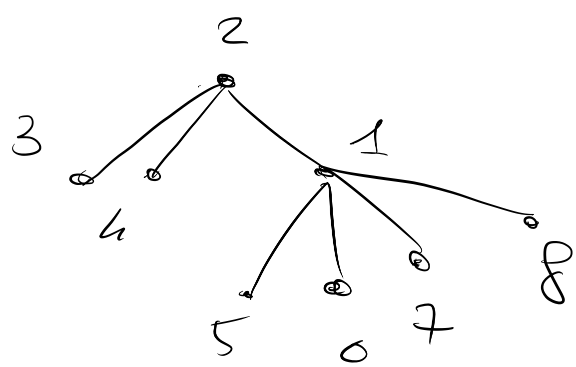
STRUTTURE SUCCINTE PER ALBERI BINARI



$\Rightarrow$

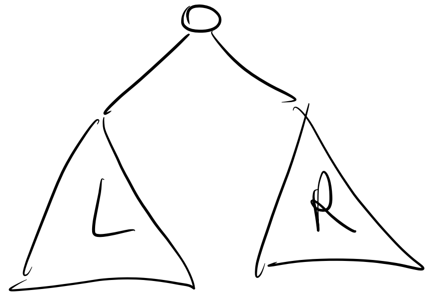


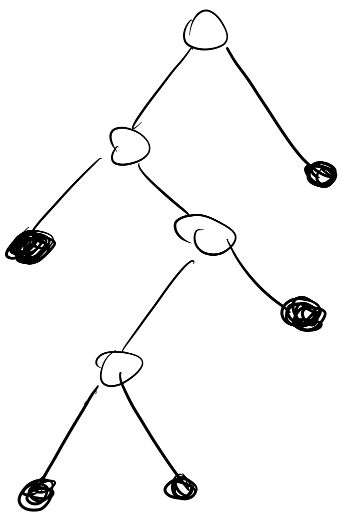


<u>Def</u>: Un albero binario è $\emptyset$
oppure una coppia (L, R)
di alberi binari

$\emptyset$ ●

(L, R)



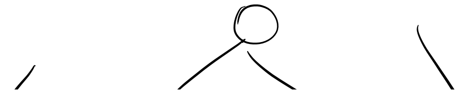


$((\emptyset, ((\emptyset, \emptyset), \emptyset)), \emptyset)$

Thm $\circledast$: $\#_{\text{esterni}}^{\text{nodi}} = \#_{\text{interni}}^{\text{nodi}} + 1$

Dim: • $est(\emptyset) = 1$ $\quad int(\emptyset) = 0$

$1 = 0 + 1$



• $\mathrm{est}(L, R) = \mathrm{est}(\triangle_L \ \triangle_R) =$

$$= \mathrm{est}(L) + \mathrm{est}(R) =$$

$$= \underbrace{\mathrm{int}(L) + 1 + \mathrm{int}(R)} + 1 =$$

$$= \mathrm{int}(L, R) + 1.$$

$\square$

$n = \#$ nodi interni

$\Rightarrow$ $2n+1$ nodi in tutto

<u>Thm</u>: Il numero di alberi binari con n nodi interni è

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

[NUMERO DI CATALANO]

$$x! \approx \sqrt{2\pi x}\left(\frac{x}{e}\right)^x ,$$

APPR. DI STIRLING

$$C_n = \frac{1}{n+1}\binom{2n}{n} =$$

$$= \frac{1}{n+1}\,\frac{(2n)!}{n!\,(2n-n)!} =$$

$$= \frac{1}{n+1}\,\frac{(2n)!}{(n!)^2} \approx \text{(Stirling)}$$

$$\approx \frac{1}{n+1}\,\frac{\sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n}}{2\pi n\left(\frac{n}{e}\right)^{2n}} =$$

$$= \frac{1}{n+1}\,\frac{1}{\sqrt{\pi n}}\,2^{2n} \approx \frac{4^n}{\sqrt{\pi n^3}}$$

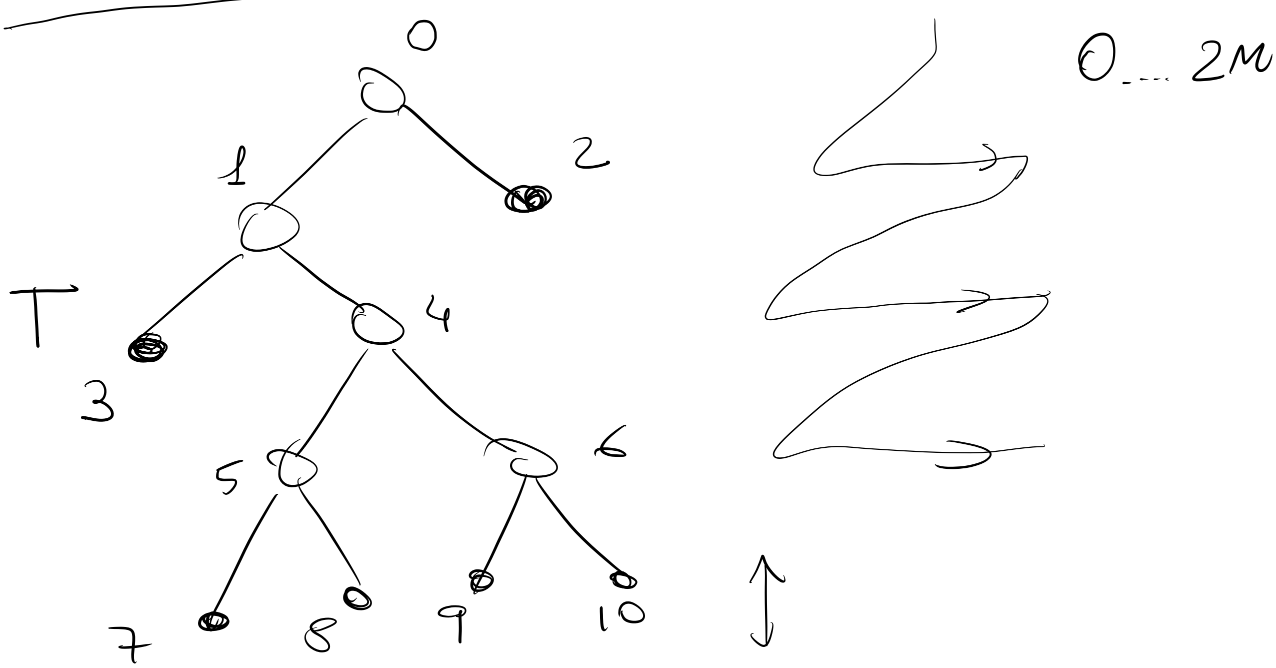
$$\Rightarrow \log C_n = n\log 4 - \frac{1}{2}\log\left(\pi n^3\right) =$$

$$= 2n - \frac{3}{2}\log n - \frac{1}{2}\log \pi$$

$= 2n + O(\log n)$

Corollario: Per memorizzare alberi binari con n nodi interni servono

$$Z_n = 2n + O(\log n)$$



b

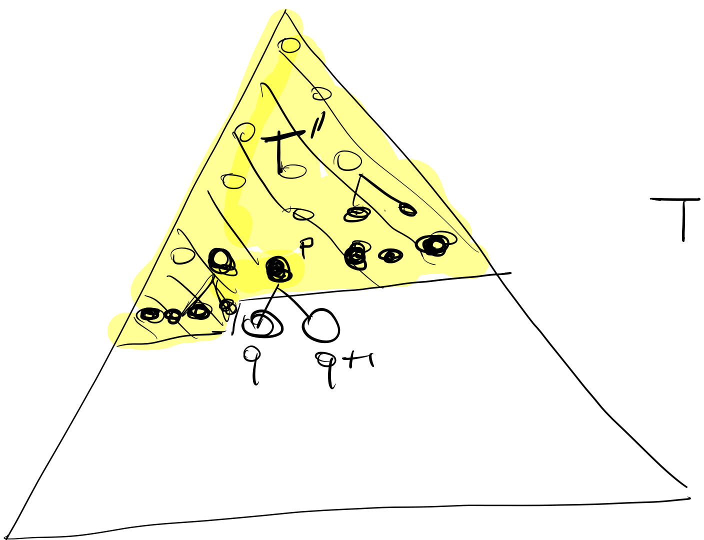
0	1	2	3	4	5	6	7	8	9	10
1	1	0	0	1	1	1	0	0	0	0

$2n+1$

1) LUNGH. $2n+1$

2) 1 se il nodo interno

$p \rightsquigarrow q$



(Thm ⊛)

$T' =$

$q = \#\text{nodi di } T' + 1 =$

$= 2\,\underbrace{\#\text{nodi interni di}}$

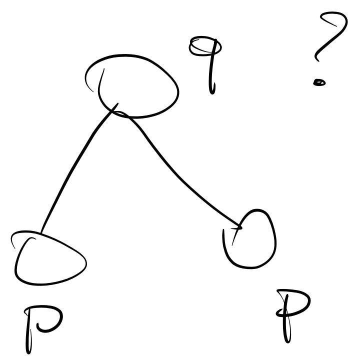
$= 2\left(\#\text{nodi interni di } T \;\; {<}\, p\right) + 1 =$

$= 2\left(\#\text{uni dentro } \underline{b} \text{ di indice } < p\right) + 1 =$

$= 2\,\text{rank}_{\underline{b}}(p) + 1$

$$\boxed{\text{is-leaf}(p) = [b_p == 0]}$$

$$\boxed{\begin{aligned}
&\text{left-child}(p) = 2\,\text{rank}_{\underline{b}}(p) + 1\\
&\text{right-child}(p) = 2\,\text{rank}_{\underline{b}}(p) + 2\\
&\text{parent}(p) = \text{select}_{\underline{b}}\left(\left\lfloor \frac{p}{2} - \frac{1}{2} \right\rfloor\right)
\end{aligned}}$$



q t.c.

$$\begin{cases} \bullet\ 2\,\text{rank}_{\underline{b}}(q) + 1 = p & \leftarrow \begin{cases} p \text{ figlio sx} \\ q \text{ padre} \end{cases} \\ \bullet\ 2\,\text{rank}_{\underline{b}}(q) + 2 = p & \leftarrow \begin{cases} p \text{ figlio dx} \\ q \text{ padre} \end{cases} \end{cases}$$

$$\begin{cases} \text{rank}_{\underline{b}}(q) + \frac{1}{2} = \frac{p}{2} \\ \text{rank}_{\underline{b}}(q) + 1 = \frac{p}{2} \end{cases}$$

$$\begin{cases} \mathrm{rank}_{\underline{b}}(q) = \dfrac{p}{2} - \dfrac{1}{2} \\[2ex] \mathrm{rank}_{\underline{b}}(q) = \dfrac{p}{2} - \underline{b} \end{cases}$$

$$\Updownarrow$$

$$\mathrm{rank}_{\underline{b}}(q) = \left\lfloor \frac{p}{2} - \frac{1}{2} \right\rfloor$$

$$\mathrm{select}_{\underline{b}}(\mathrm{rank}_{\underline{b}}(q)) = \mathrm{select}_{\underline{b}}\left(\left\lfloor \frac{p}{2} - \frac{1}{2} \right\rfloor\right)$$

$$q = \mathrm{select}_{\underline{b}}\left(\left\lfloor \frac{p}{2} - \frac{1}{2} \right\rfloor\right)$$