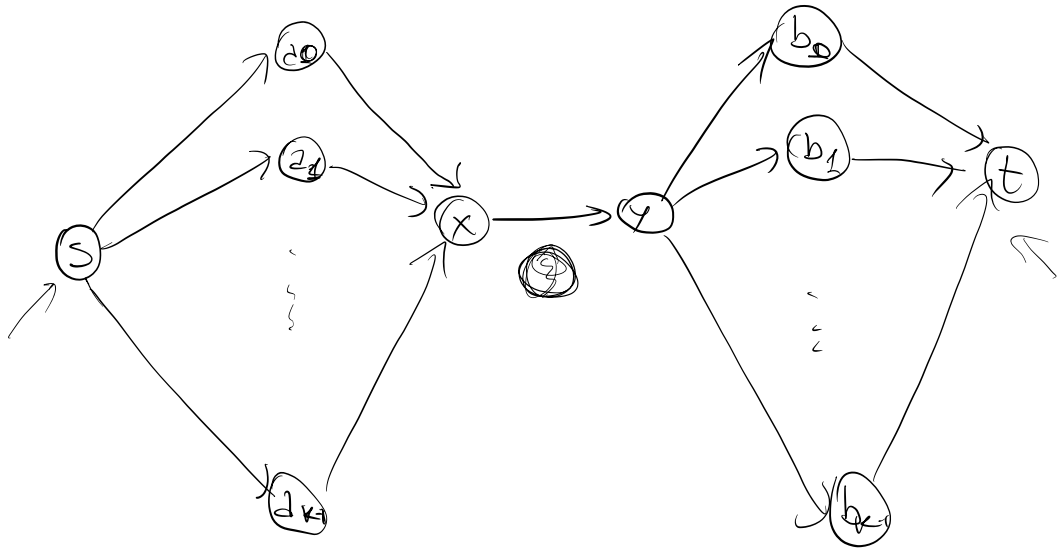

s
a_0
a_1
a_{k-1}
x
y
b_0
b_1
b_{k-1}
t

VERTEX COVER MEDIANTE ARROTONDAMENTO

$\boxed{\text{PROBLEMA LP}}$

INPUT: $A \in \mathbb{Q}^{m \times n}$, $\underline{b} \in \mathbb{Q}^m$, $\underline{c} \in \mathbb{Q}^n$

SOL. AMMISSIBILI: $\underline{x} \in \mathbb{Q}^n$

t.c. $A\underline{x} \geq \underline{b}$

FUNZ. OBIETTIVO: $\underline{c}^T \underline{x}$

TIPO: MIN

$\boxed{\text{PROBLEMA ILP}}$

INPUT: $A \in \mathbb{Q}^{m \times n}$, $\underline{b} \in \mathbb{Q}^m$, $\underline{c} \in \mathbb{Q}^n$

SOL. AMMISSIBILI: $\underline{x} \in \mathbb{Z}^n$

t.c. $A\underline{x} \geq \underline{b}$

FUNZ. OBIETTIVO: $\underline{c}^T \underline{x}$

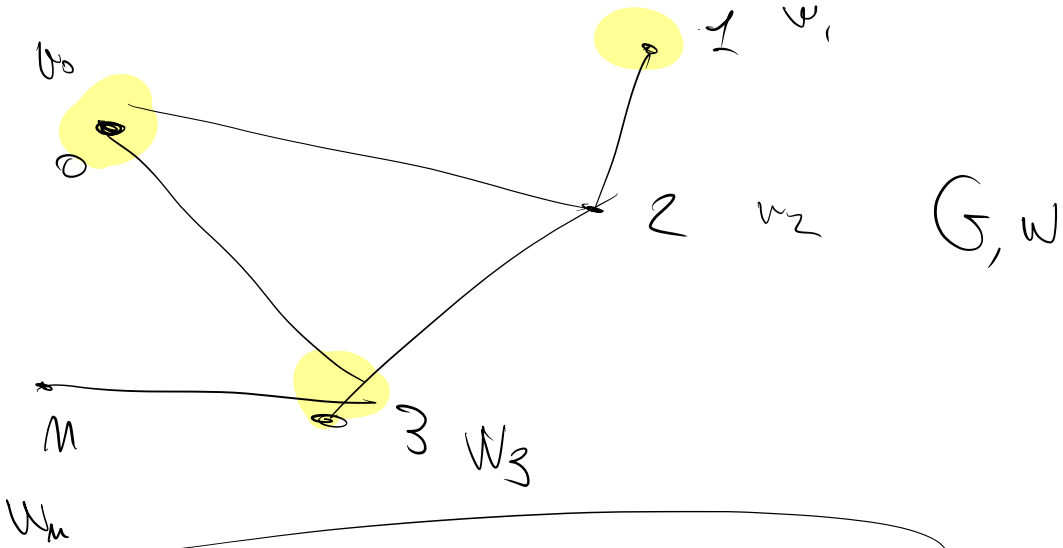
TIPO: MIN

$\boxed{LP}$

- Risolubile in tempo polinomiale in (n) [1981, 1984] [metodo del punto interno di Karmarkar]
- Dantzig

$LP \in PO$

$\boxed{ILP} \in NPO$ - completo



$$
\boxed{
\begin{cases}
- \; x_i \in \mathbb{Z} & i \in V \\
\left.\begin{array}{l} x_i + x_j \geq 1 \\ x_i \geq 0 \\ x_i \leq 1 \end{array}\right\} & \begin{array}{l} \forall ij \in E \\ \forall i \in V \end{array} \\
\sum_{i \in V} w_i x_i
\end{cases}
}
\quad ILP(G,w)
$$

$$\boxed{\begin{array}{l} - \; x_i \in \mathbb{Q} \quad i \in V \\ \left\{ \begin{array}{ll} x_i + x_j \geq 1 & \forall ij \in E \\ x_i \geq 0 & \forall i \in V \\ x_i \leq 1 & \end{array} \right. \\ \sum_{i \in V} w_i x_i \end{array}}$$

(G, w)

$\downarrow$

$LP(G,w)$

$\downarrow$

$\underline{x}^*$

<u>Lemma 1</u>: $w^*_{LP} \leq w^*_{ILP}$

$\underline{\pi}$ $\quad \pi_i = \begin{cases} 1 & \text{se } x_i^* \geq \frac{1}{2} \\ 0 & \text{altrimenti} \end{cases}$

<u>Lemma 2</u>: $\underline{\pi}$ è una sol. ammissibile di ILP.

<u>Dim</u>: $\forall ij \in E \qquad \pi_i + \pi_j \geq 1$

Per assurdo $\exists ij \in E$ t.c. $r_i + r_j < 1$.

$$\Rightarrow r_i = r_j = 0$$

$$\Rightarrow x_i^* < \frac{1}{2} \quad e \quad x_j^* < \frac{1}{2}$$

$$\Rightarrow \underbrace{x_i^* + x_j^* < 1}_{\text{IMPOSS.}}$$

$\square$

$$r_i \leq 2x_i^* \begin{cases} r_i = 0 & \text{ovvia} \\ r_i = 1 \Rightarrow x_i^* \geq \frac{1}{2} & 2x_i^* \geq 1 = r_i \end{cases}$$

<u>Lema 2</u>: $\sum_{i \in V} w_i r_i \leq 2 \underbrace{\sum_{i \in V} w_i x_i^*}_{} =$

$$= 2 W_{LP}^* \underset{\text{Lema 1}}{\leq} 2 W_{IKP}^*$$