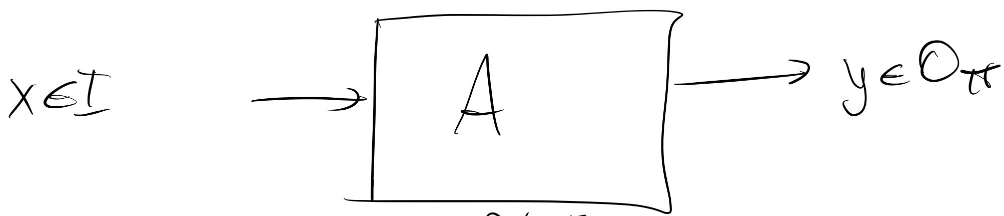
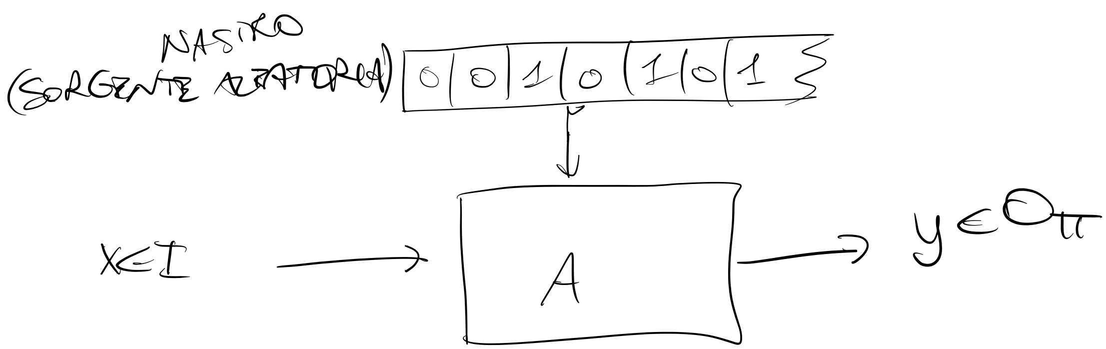
ALGORITMI PROBABILISTICI



ALGORITMO DETERMINISTICO

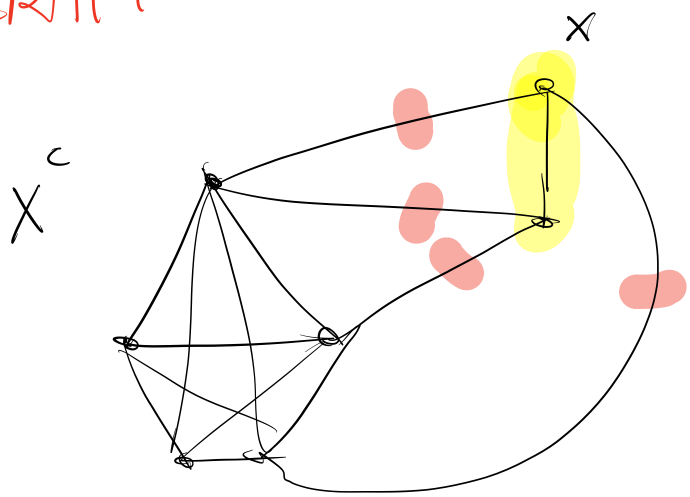


ALGORITMO PROBABILISTICO

$$P[Y=y \mid X=x]$$

Algoritmi
- $\boxed{\text{Monte-Carlo}}$
- Las Vegas

TAGLIO MINIMO DI GRAFI



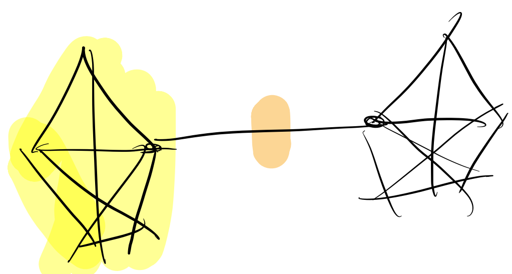
G

$X \subseteq V$

$X \neq \emptyset$

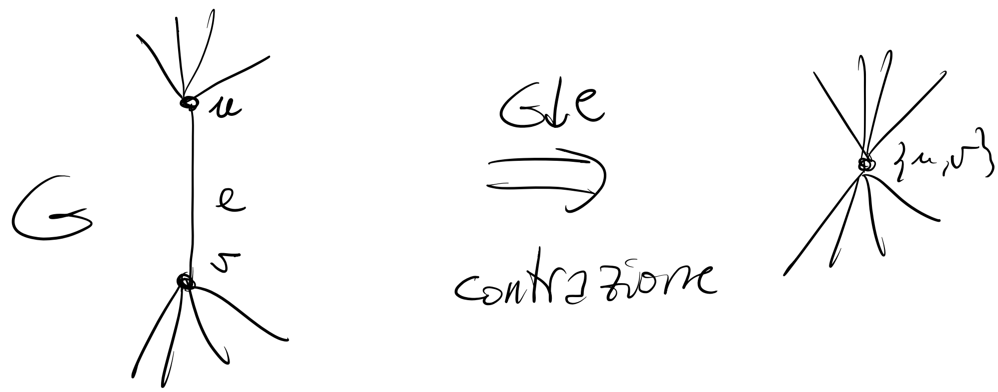
$X \neq V^c$

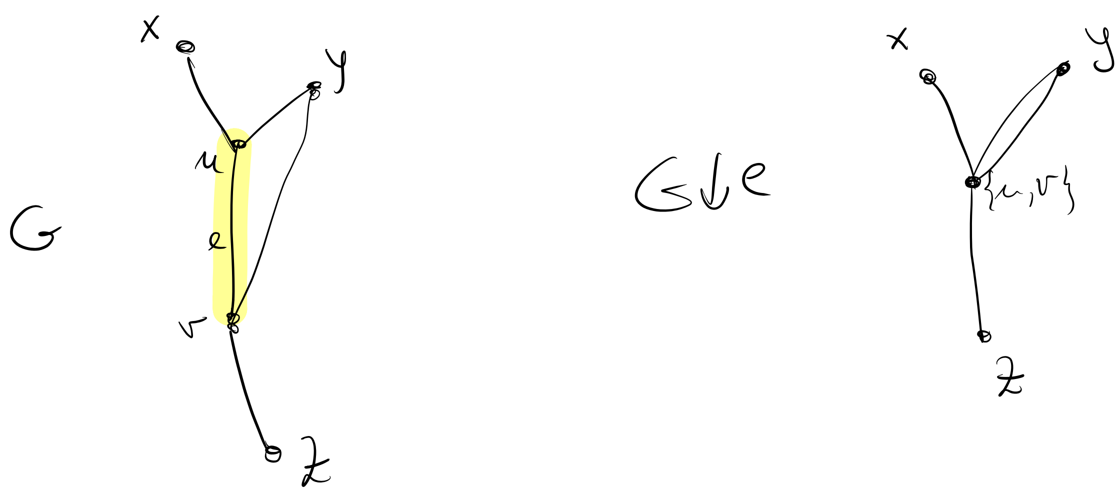
G

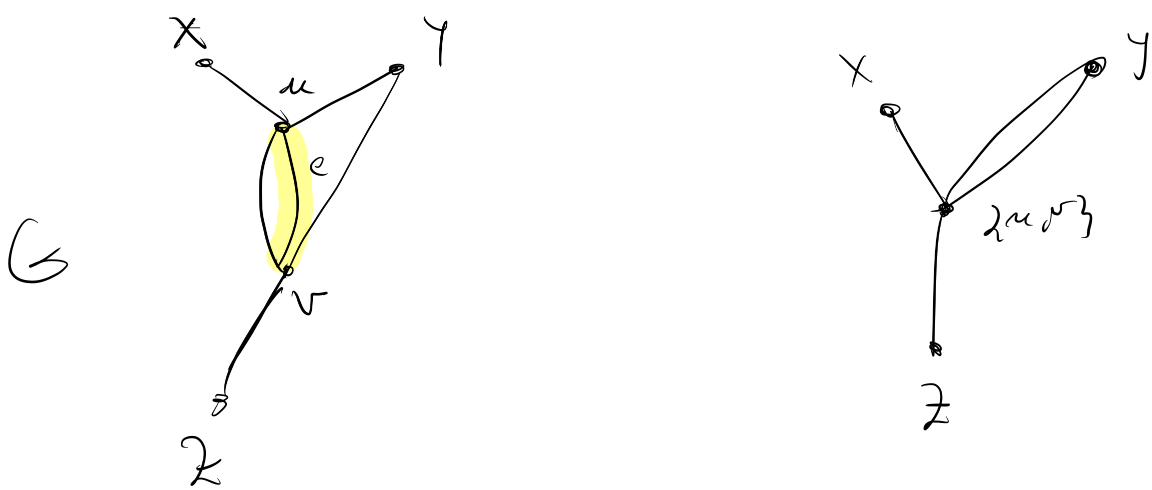


$d_{MIN}(G) = 4$

ALGORITMO DI KARGER







ALGORITMO DI KARGER

INPUT: $G = (V, E)$

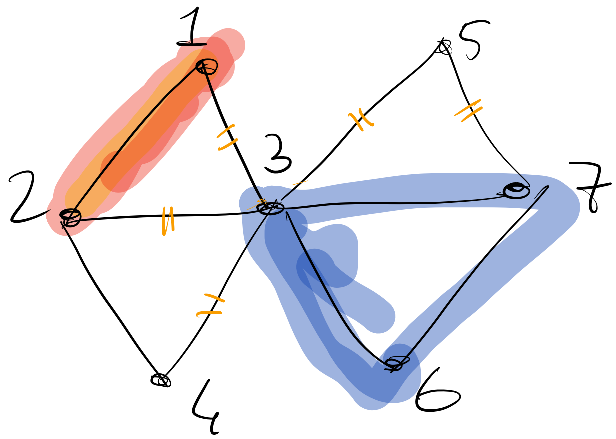
1) Se G non è connesso, cuetti una qualunque componente connessa
2) Altrimenti

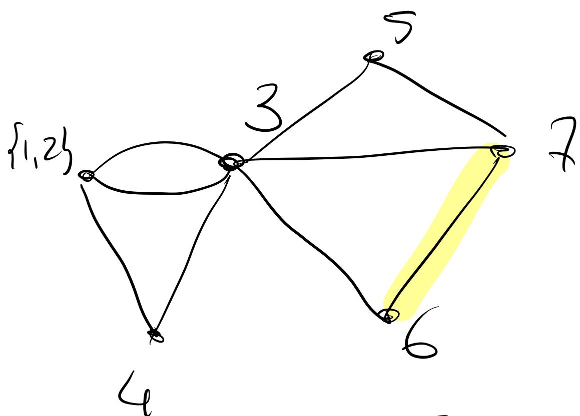
   while $|V| > 2$

   scegli un lato a caso e

   $G \leftarrow G \downarrow e$

3) Output uno dei due vertici (classe di equiv.)


1
2
3
4
5
6
7


{1,2}
3
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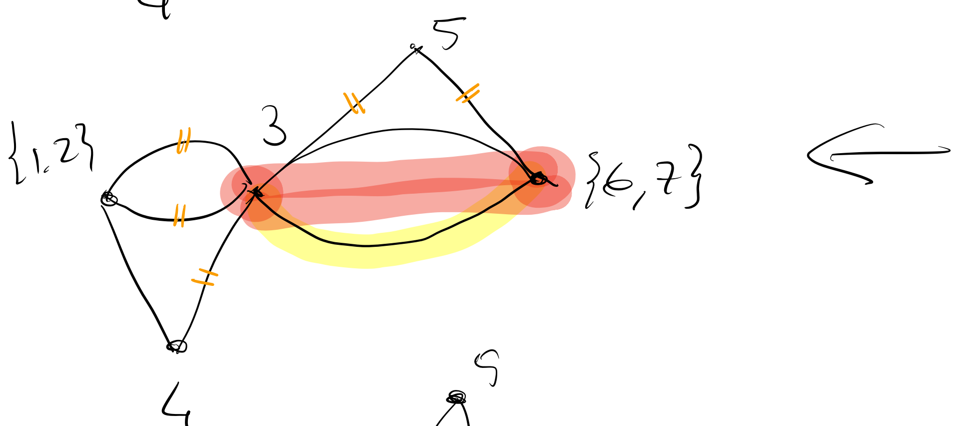
{1,2}
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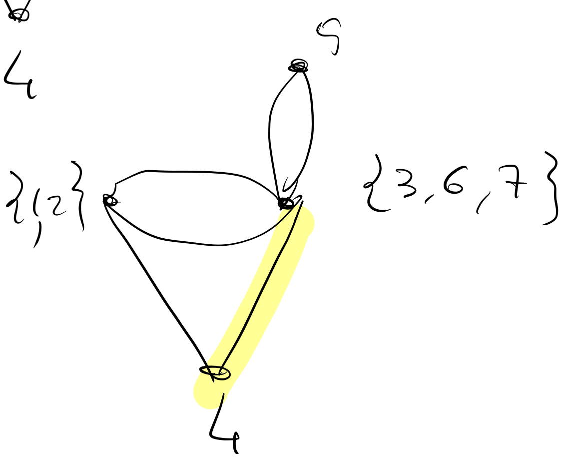
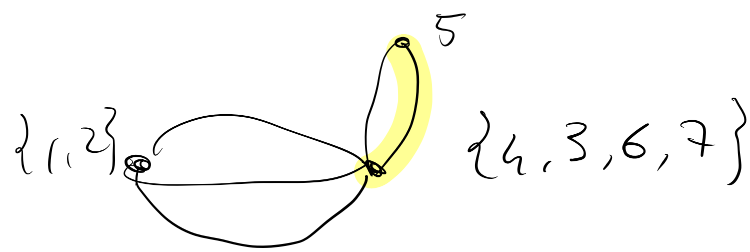

{1,2}
{3,6,7}
4
5



$S^* \subseteq V$ sia il taglio minimo e $k^* = |E_{S^*}|$.

Sia $G = G_1, G_2, \dots$ la sequenza di grafi di Karger

1) # vertici di G_i = $m-i+1$

lati di $G_i \leq m-i+1$

2) Ogni taglio in G_i è un taglio in G dello stesso costo

3) Il taglio minimo in G_i è $\geq k^*$

(altrimenti G avrebbe un taglio $< k^*$)

$$2(n-i+1) \geq 2\#\text{lati di } G_i = \sum_{v \in V_{G_i}} d_{G_i}(v) \geq k^*(n-i+1)$$

①

$$\Rightarrow \underbrace{n-i+1}_{\#\text{lati in } G_i} \geq \underbrace{\frac{(n-i+1)k^*}{2}}$$

$\mathcal{E}_i =$ "al passo i-esimo abbiamo contratto un lato $e \notin E_{S^*}$"

Lemma: $\forall i$

$$P[\mathcal{E}_i \mid \mathcal{E}_1, \dots, \mathcal{E}_{i-1}] \geq \frac{n-i-1}{n-i+1}$$

Dim: $P[\mathcal{E}_i \mid \mathcal{E}_1, \dots, \mathcal{E}_{i-1}] =$

$$= 1 - P[\neg\mathcal{E}_i \mid \mathcal{E}_1, \dots, \mathcal{E}_{i-1}] \geq$$

$$\geq 1 - \frac{k^* \quad 2}{(n-i+1)k^*} = 1 - \frac{2}{n-i+1} =$$

$$= \frac{n-i+1-2}{n-i+1} = \frac{n-i-1}{n-i+1} \quad \square$$

$$P[\mathcal{E}_1 \wedge \mathcal{E}_2 \wedge \dots \wedge \mathcal{E}_{n-2}] =$$

$$= P[\mathcal{E}_1] \cdot P[\mathcal{E}_2 \mid \mathcal{E}_1] \cdot P[\mathcal{E}_3 \mid \mathcal{E}_1, \mathcal{E}_2]$$

$$\dots =$$