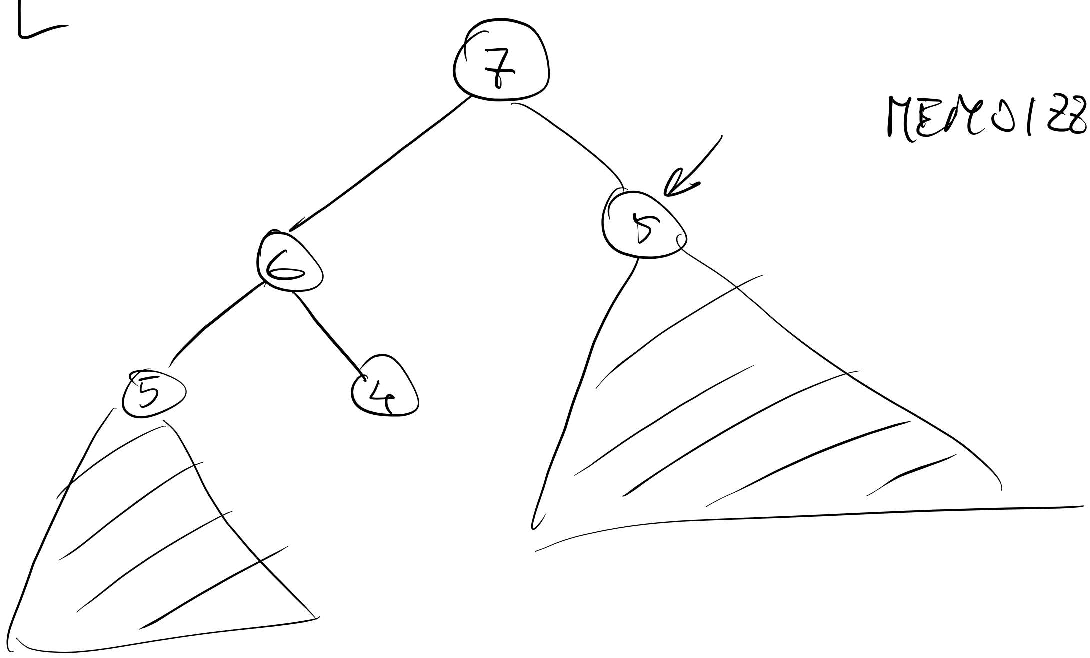
PROGRAMMAZIONE DINAMICA

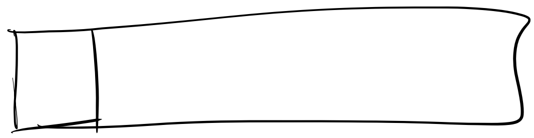
$$\Phi_0 = \Phi_1 = 1$$

$$\Phi_n = \Phi_{n-1} + \Phi_{n-2} \qquad \forall n > 1$$

```
def fib(n):
    if n < 2:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```



MEMOIZZAZIONE

f 

```
f[0] = f[1] = 1
for i = 2 ; i <= n ; i++ {
        f[i] = f[i-1] + f[i-2]
}
```

KNAPSACK

INPUT: $v_0, \dots, v_{n-1} \in \mathbb{N}^+$

$w_0, \dots, w_{n-1} \in \mathbb{N}^+$

$W \in \mathbb{N}^+$

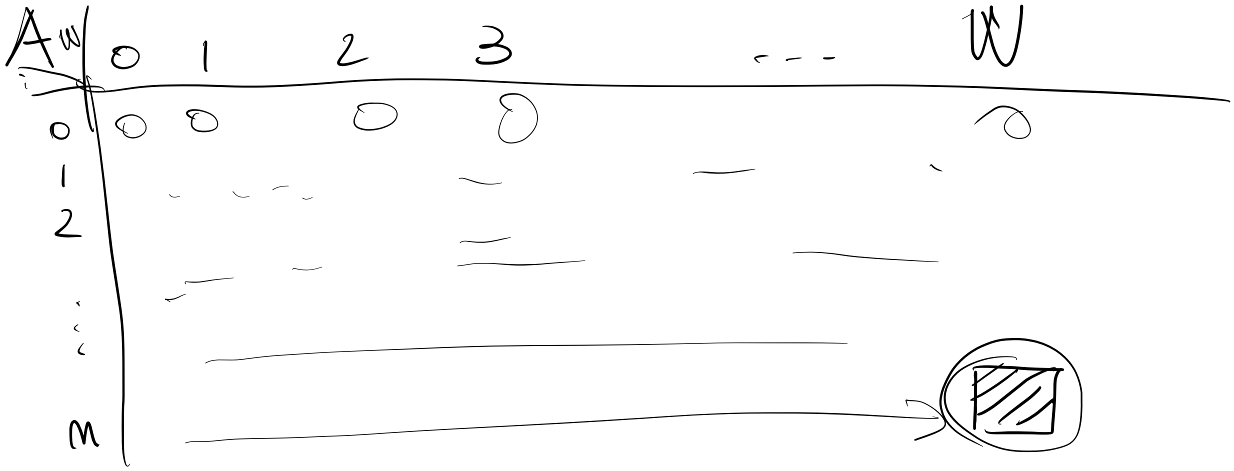
SOL. AMM.: $I \subseteq \{0, \dots, n-1\}$ t.c. $\sum_{i \in I} w_i \leq W$

FUNZ. OB.: $\sum_{i \in I} v_i$

TIPO: MAX

SOL. ESATTA TRAMITE PROGRAMM. DINAMICA (1)

$A[i, w]$ = massimo valore che riesco a ottenere con i primi i oggetti se lo zaino ha capacità w



$\forall w \quad A[0, w] = 0$

$$A[i+1, w] = \begin{cases} A[i, w] & w_i > w \\ \max\left(A[i, w],\ v_i + A[i, w - w_i]\right) & w_i \le w \end{cases}$$

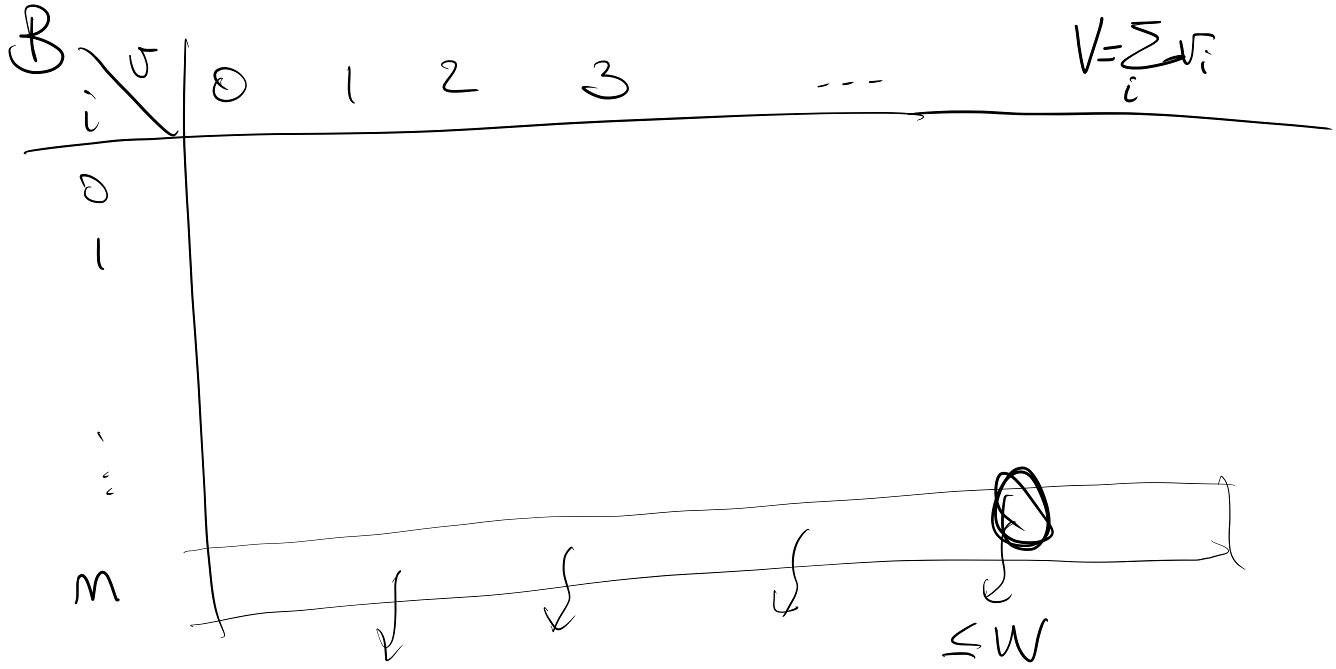
SOL. ESATTA TRAMITE PROGRAMM. DINAMICA (2)

$B[i, v]$ = minimo peso che mi devo sobbarcare per portare a casa $\geq v$ usando solo i primi i oggetti

$B[0,0] = 0$

$\forall v > 0 \quad B[0,v] = +\infty$

$$B[i+1, v] = \min\left(B[i, v], w_i + B[i, \max(v - v_i, 0)]\right)$$



$$\Pi = (v_i,\ w_i,\ W)$$ istanza di Knapsack v^* I^*

$\varepsilon \in (0,1]$

$$\vartheta = \frac{\varepsilon v_{MAX}}{2n}$$

$$\overline{\Pi} = \left(\overline{v}_i = \left\lceil \frac{v_i}{\vartheta} \right\rceil \vartheta,\ w_i,\ W\right) \qquad \overline{v}^*\ \ \overline{I}^*$$

$$\hat{\Pi} = \left(\hat{v}_i = \left\lceil \frac{v_i}{\vartheta} \right\rceil,\ w_i,\ W\right) \qquad \hat{v}^*\ \ \hat{I}^*$$

Lemma 1: $\hat{I}^* = I^*$, $\overline{v}^* = \hat{v}^* \vartheta$

Lemma 2: Sia I una sol. ammissibile di Π, allora

$$(1+\varepsilon)\sum_{i\in\hat{I}^*} v_i \geq \sum_{i\in I} v_i .$$

Dim:

$$\sum_{i\in I} v_i \leq \sum_{i\in I} \bar{v}_i$$

(ARROT. PER ECCESSO)

I^* è ottimo per $\bar{\Pi}$

$$\leq \sum_{i\in \bar{I}^*} \bar{v}_i$$

$\bar{I}^* = \hat{I}^*$ per lemma 1

$$\circledast \quad = \sum_{i\in\hat{I}^*} \bar{v}_i$$

$\bar{v}_i = \left\lceil \frac{v_i}{\vartheta} \right\rceil \vartheta \leq$
$\leq v_i + \vartheta$

$$\leq \sum_{i\in\hat{I}^*} (v_i + \vartheta) \leq$$

$$\leq \sum_{i\in\hat{I}^*} v_i + n\vartheta =$$

$$= \sum_{i\in\hat{I}^*} v_i + n\frac{\varepsilon v_{MAX}}{2n} = \sum_{i\in\hat{I}^*} v_i + \frac{\varepsilon v_{MAX}}{2}$$

Per qualunque I ammissibile.

$I = \{MAX\}$

$$v_{MAX} \leq \sum_{i\in\hat{I}^*} v_i + \frac{\varepsilon v_{MAX}}{2} \leq$$

$\varepsilon \leq 1$

$$\leq \sum_{i\in\hat{I}^*} v_i + \frac{v_{MAX}}{2}$$

$\circledast\circledast$

$$\Rightarrow \quad \frac{v_{MAX}}{2} \leq \sum_{i\in\hat{I}^*} v_i$$

Riprendendo (*)

$$\sum_{i\in I} v_i \overset{(*)}{\le} \sum_{i\in \hat{I}^*} v_i + \frac{\varepsilon v_{MAX}}{2} \overset{(**)}{\le} (1+\varepsilon)\sum_{i\in \hat{I}^*} v_i$$

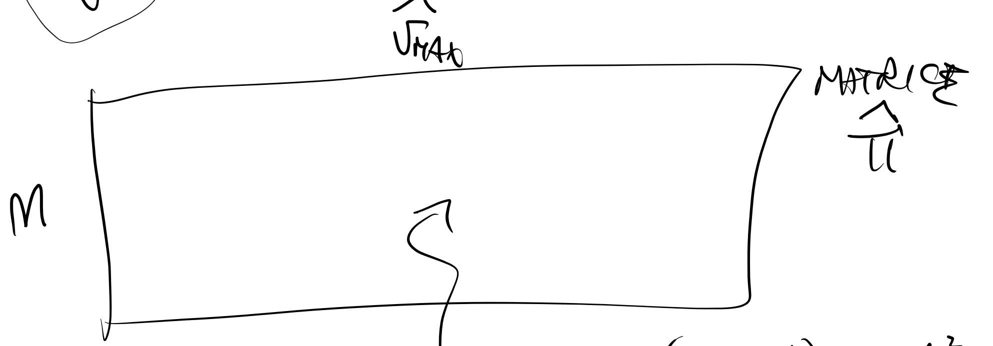
□

<u>Corollario</u>: $(1+\varepsilon)\hat{v}^* \ge v^*$.

<u>Dim</u>: Applica Lemma 2 a $I = I^*$. □

$$\hat{v}_{MAX} = \left\lceil \frac{v_{MAX}}{\vartheta} \right\rceil = \left\lceil \frac{2\cancel{v_{MAX}}\, n}{\varepsilon \cancel{v_{MAX}}} \right\rceil = \left\lceil \frac{2n}{\varepsilon} \right\rceil \le \frac{2n}{\varepsilon} + 1$$

$$\frac{v^*}{\hat{v}^*} \le 1+\varepsilon$$



$$n\,\hat{v}_{MAX} \le n\left(\frac{2n}{\varepsilon} + 1\right) = \frac{2n^2}{\varepsilon} + 1$$

$O\left(\frac{n^2}{\varepsilon}\right)$ TEMPO e SPAZIO
pol. in n
e in $1/\varepsilon$

<u>Corollario</u>: KNAPSACK ∈ FPTAS.