$\boxed{\text{PROBLEMA}}$ Π

1) insieme di input I_π
2) insieme di output O_π
3) $sol_\pi : I_\pi \longrightarrow 2^{O_\pi} \setminus \{\emptyset\}$

NOTAZIONI

$\mathbb{N}$ $\mathbb{Z}$ $\mathbb{Q}$ $\mathbb{R}$

$\mathbb{N}^+$ $\mathbb{Q}^+$ $\mathbb{R}^+$

MONOIDE LIBERO

Σ insieme finito non-vuoto (alfabeto)

$$(\Sigma^*, \cdot, \varepsilon)$$

$$(\mathbb{N}, +, 0)$$

$\Sigma = \{a, b, c\}$

$\Sigma^* = \{\varepsilon, a, b, c, aa, ab, ac,$
$ba, bb, bc, ca, cb, cc,$
$\ldots, abaccacca, \ldots\}$

$w \in \Sigma^*$ $|w|$

$w = w_0 w_1 \ldots w_{|w|-1}$

NOTAZIONE $\quad B^A$

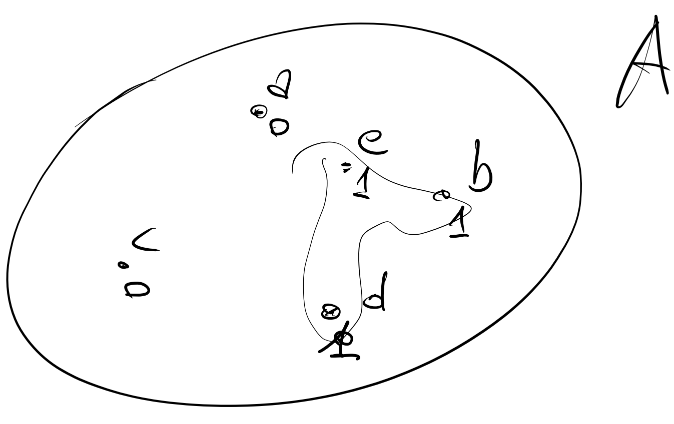
$$B^A = \{ f \mid f : A \to B \}$$

NOTAZIONE $\quad k \in \mathbb{N}$

$$k = \{0, 1, \dots, k-1\}$$

$$\begin{aligned} 0 &= \emptyset \\ 1 &= \{0\} \\ 2 &= \{0, 1\} \\ 3 &= \{0, 1, 2\} \\ &\dots \end{aligned}$$

$$2^A = \{ f \mid f : A \to 2 \} = \{ f \mid f : A \to \{0,1\} \} \simeq \{ X \mid X \subseteq A \} = \mathcal{P}(A)$$



2^{Σ^*}

2^{2^*}

$\{0, 00, 10, 001\} \in 2^{2^*}$

$A^2 = \{f \mid f : 2 \to A\} =$

$= \{f \mid f : \{0,1\} \to A\} \cong A \times A$

PROBLEMA Π

1) insieme di input $I_\pi \subseteq 2^*$
2) insieme di output $O_\pi \subseteq 2^*$
3) $sol_\pi : I_\pi \longrightarrow 2^{O_\pi} \setminus \{\emptyset\}$

ESEMPI

① Decidere se un numero naturale positivo è primo

$I_\pi = 1\{0,1\}^*$

$O_\pi = \{0,1\}$

② Calcolare il MCD fra due interi positivi

$$\begin{array}{cc} x & y \\ 3 & 5 \\ 11 & 101 \end{array}$$

11 11 10 1100 11

$2\lceil \log_2 x \rceil + 2 + 2\lceil \log_2 y \rceil$

$5 \qquad x$

$$\underbrace{1110}\,,\underbrace{101}$$

$$\lceil \log_2 x \rceil + 1 + \lceil \log_2 x \rceil$$

3) SAT

$$\overbrace{(x_1 \vee \underbrace{\neg x_2}_{\text{letterale}})}^{\text{clausola}} \wedge \overbrace{(x_1 \vee \neg x_3 \vee x_7)}^{\text{clausola}} \wedge \overbrace{(\neg x_2 \vee x_5)}^{\text{clausola}}$$

$$x \in I_\pi \longrightarrow \boxed{A} \longrightarrow y \in O_\pi$$

t.c. $y \in sol_\pi(x)$

Complessità
- algoritmica
- strutturale

$$T_A : I_\pi \to \mathbb{N}$$

$\{$ worst-case

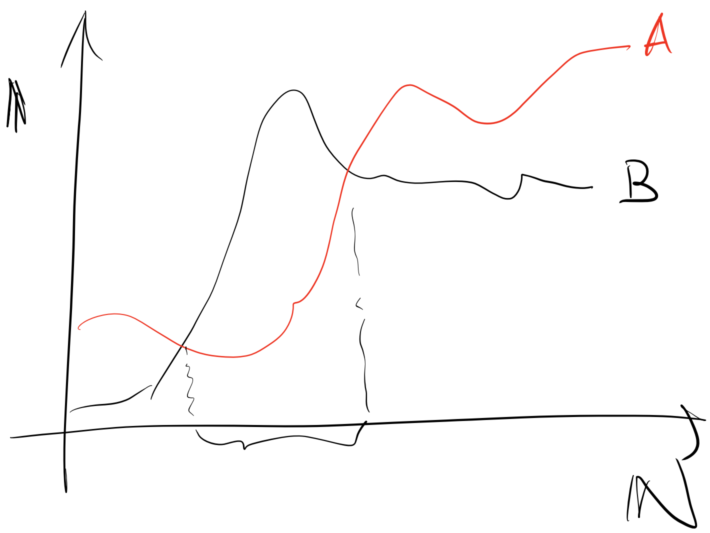
$$t_A : \mathbb{N} \to \mathbb{N}$$

$$t_A(n) = \max_{\substack{x \in I_\pi \\ |x| = n}} T_A(x)$$

$$t_A(17) = 3500$$

$$t_A(n) = O(n^2 \log n)$$

$$t_B(n) = O(n^2)$$

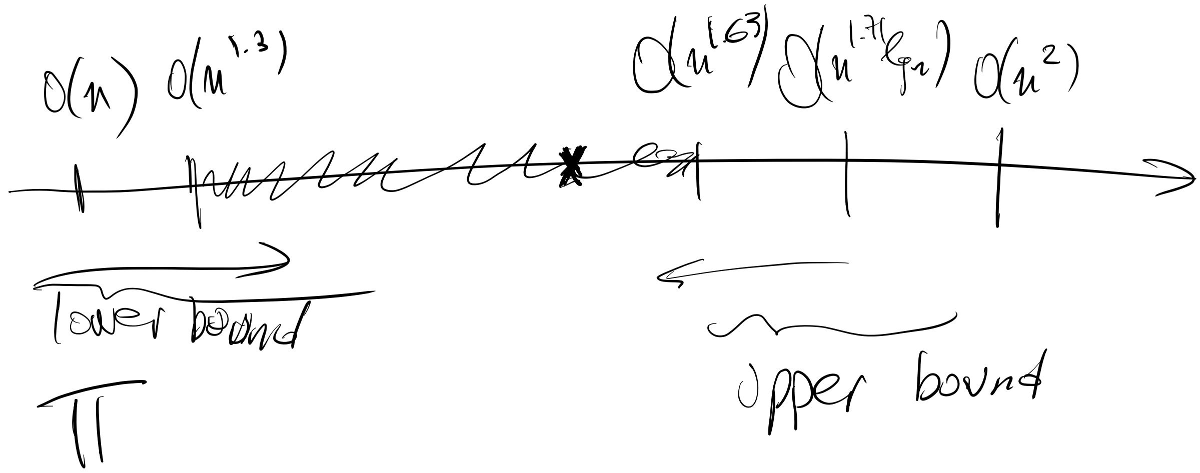


$$t_A(n) \le_{\text{def.}} K\, n^2 \log n$$

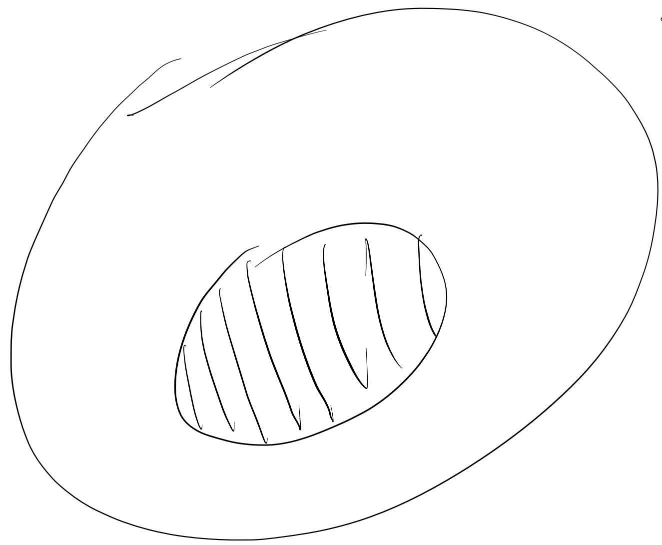
$$t_B(n) \le_{\text{def.}} K'\, n^2$$

$O(n^{1.71} \lg n)$ ← upper bound

$\boxed{O(n^{1.63} \lg n)}$ ←



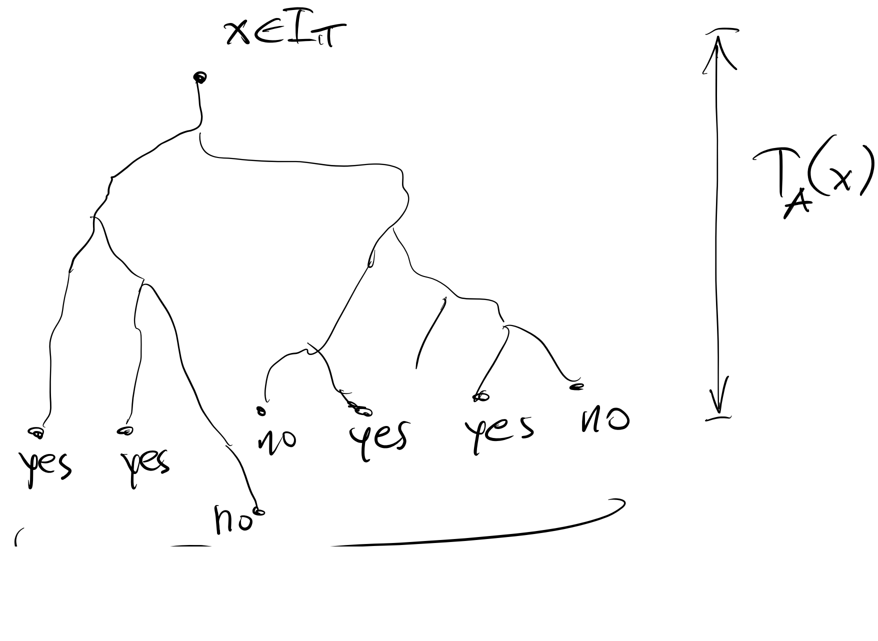
$P = \{ \Pi \mid \Pi$ problema di dec. t.c. $\exists A$ ris. per Π $t_A(n)$ è dominata da un polinomio $\}$



P

$\boxed{NP}$

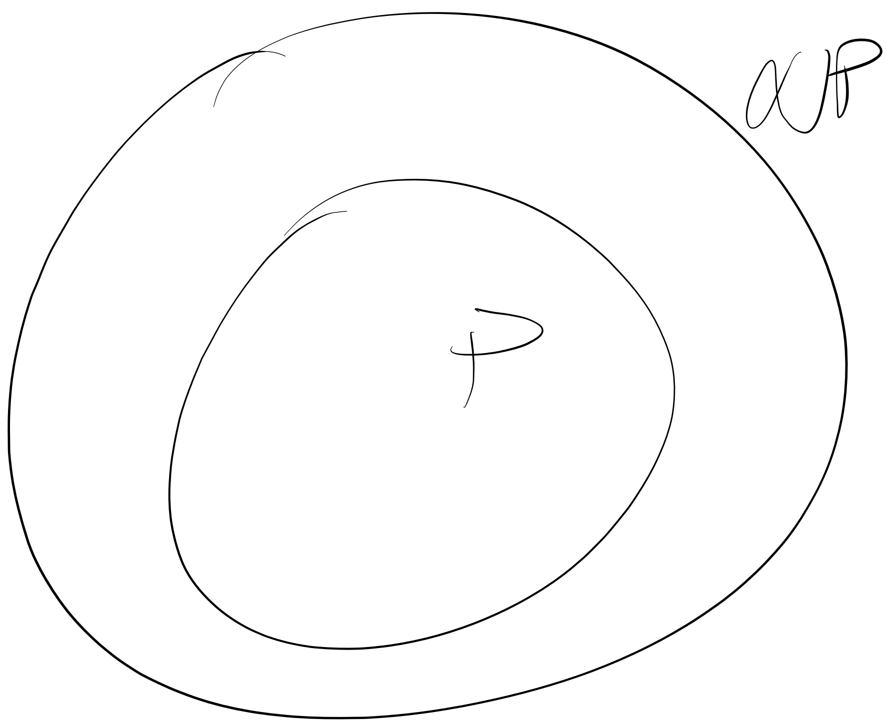
$X = ?$ ⟵ magic instruction



$\underbrace{\qquad\qquad\qquad}_{OR}$

$NP = \{ \Pi \mid \Pi$ problema di dec. t.c. $\exists A$ n.d. per Π t.c. $t^{\text{non det.}}_A(n)$ è dominata da un polinomio$\}$

$$P \subseteq NP$$



RIDUZIONE POLINOMIALE

Π_1, Π_2 probl. decisione

$\Pi_1 \leq_p \Pi_2$ sse

$\exists f : 2^* \longrightarrow 2^*$

1) f è calcolabile in tempo pol.

2) $\forall x \in I_{\Pi_1}$ t.c. $f(x) \in I_{\Pi_2}$

$$Sol_{\Pi_1}(x) = Sol_{\Pi_2}(f(x))$$

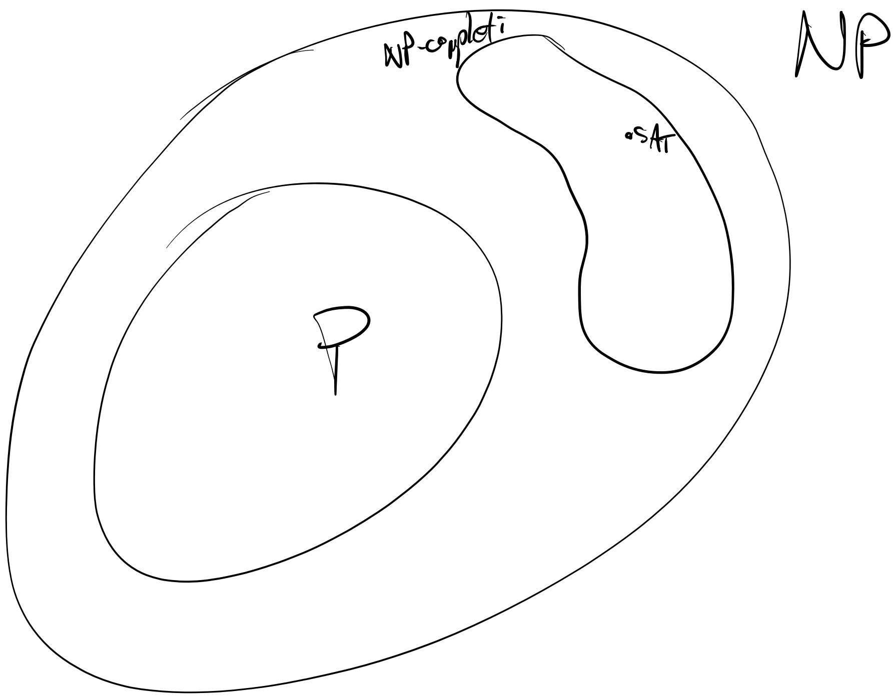
Π è NP-completo

1) $\Pi \in NP$

2) $\forall \Pi' \in NP$ $\quad \Pi' \leq_p \Pi$

Teorema di Cook:

SAT è NP-completo


NP-completi
SAT
P
NP