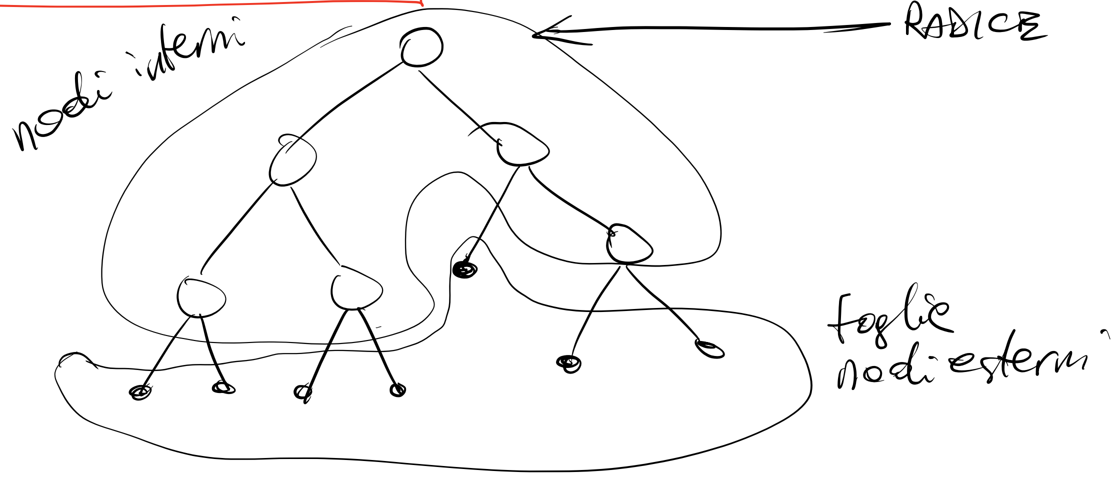
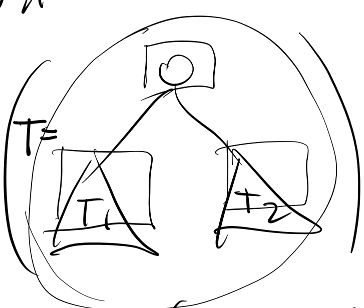
ALBERO BINARIO



Teorema: $\#\text{nodi esterni} = \#\text{nodi interni} + 1$

Dim: $\#\text{nodi esterni}(\bullet) = 1$

$\#\text{nodi interni}(\bullet) = 0$

$\#\text{nodi esterni}\Big(T=$  $\Big) = \#\text{nodi est}(T_1) + \#\text{nodi esterni}(T_2) =$

$$= 1 + \underbrace{\#\text{nodi interni}(T_1) + 1 + \#\text{nodi interni}(T_2)}_{} =$$

$$= 1 + \#\text{nodi interni}(T) \quad \square$$

Teorema: Ci sono

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$ [n-esimo n° di Catalano]

alberi binari con n nodi interni.

$$\boxed{x! \sim \sqrt{2\pi x}\left(\frac{x}{e}\right)^x}$$ Formula di Stirling

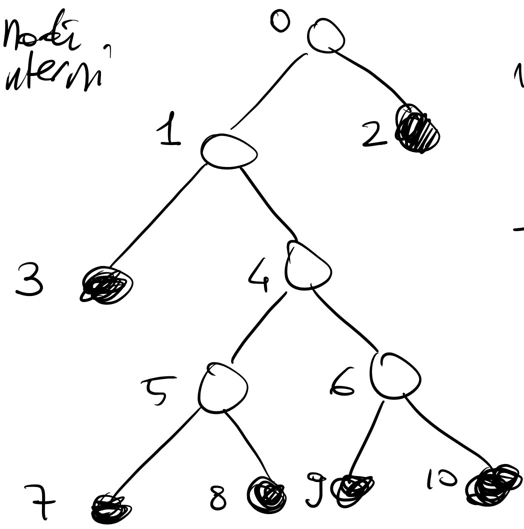
$$C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{1}{n+1}\frac{(2n)!}{n!(2n-n)!} =$$

$$= \frac{(2n)!}{(n+1)(n!)^2} = \frac{\sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n}}{(n+1)\,2\pi n\left(\frac{n}{e}\right)^{2n}} =$$

$$= \frac{1}{n+1}\frac{1}{\sqrt{\pi n}}\,2^{2n} \approx \frac{4^n}{\sqrt{\pi n^3}}$$

$$\log C_n = 2n - O(\log n)$$

Teorema: Per rappresentare un albero binario con n nodi interni servono $Z_n = 2n - O(\log n)$ bit n nodi interni



n = # nodi int. $= n$ "1"

$n+1$ = # nodi est. $= (n+1)$ "0"

$\underline{b}$

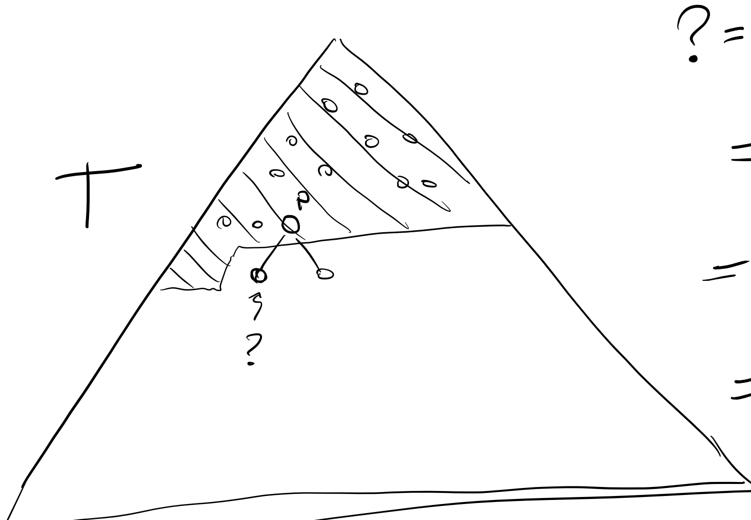
0	1	2	3	4	5	6	7	8	9	10
1	1	0	0	1	1	1	0	0	0	0

$2n+1$

left-child(p)

right-child(p)

parent(p)



? = # nodi di $=$

$= 2$ # nodi interni $+1 =$

$= 2\left(\#\text{ nodi interni di } T < p\right) + 1 =$

$= 2\,\text{rank}_{\underline{b}}(p) + 1$

$$\boxed{\text{left-child}(p) = 2\,\text{rank}_{\underline{b}}(p) + 1}$$

$$\boxed{\text{right-child}(p) = 2\,\text{rank}_{\underline{b}}(p) + 2}$$

Se $x = \text{left-child}(p)$, chi è p?
Se $x = \text{right-child}(p)$, chi è p?

$$\begin{cases} 2\,\text{rank}(p) + 1 = x \\ 2\,\text{rank}(p) + 2 = x \end{cases}$$

$$\begin{cases} \text{rank}(p) = \frac{x}{2} - \frac{1}{2} & \text{con } x \text{ disp.} \\ \text{rank}(p) = \frac{x}{2} - 1 & \text{con } x \text{ pari} \end{cases}$$

poiché $b_p = 1$

$$\text{rank}(p) = \left\lfloor \frac{x}{2} - \frac{1}{2} \right\rfloor$$

$$\text{select}(\text{rank}(p)) = \text{select}\left(\left\lfloor \frac{x}{2} - \frac{1}{2} \right\rfloor\right)$$

$$p = \text{select}\left(\left\lfloor \frac{x}{2} - \frac{1}{2} \right\rfloor\right)$$

$$\boxed{\text{parent}(p) = \text{select}\left(\left\lfloor \frac{p}{2} - \frac{1}{2} \right\rfloor\right)}$$

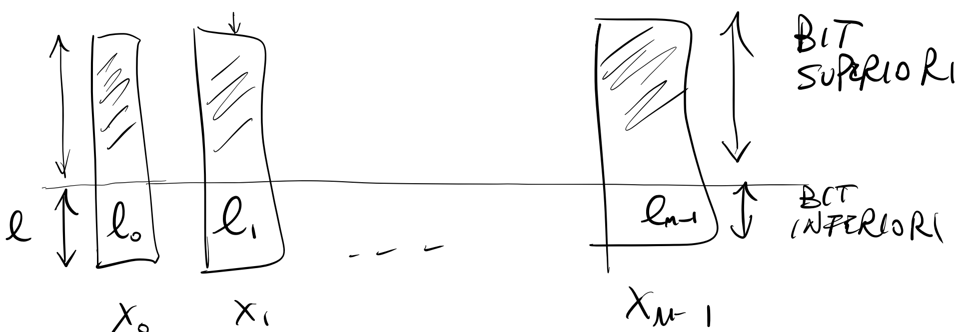
$$\left.\begin{aligned} D_n &= 2n + 1 + o(n) \\ Z_n &= 2n - O(\lg n) \end{aligned}\right] \text{succinta}$$

CODIFICA DI ELIAS-FANO PER SEQUENZE MONOTONE

DIM. UNIVERSO

$$\underbrace{0 \le x_0 \le x_1 \le \dots \le x_{n-1} < u}$$

$$get(i) = x_i$$



$$\ell = \max\left\{0, \left\lfloor \log \frac{u}{n} \right\rfloor\right\}$$

$$\boxed{\ell_i = x_i \bmod 2^\ell}$$

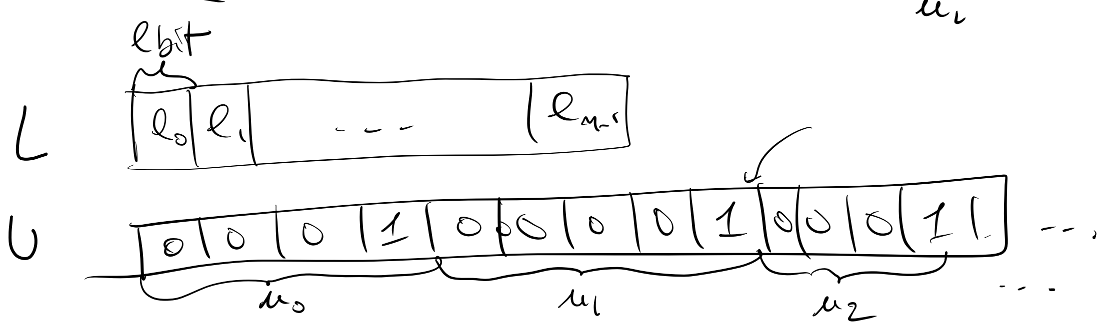
MEMORIZZATO CON ℓ bit

$x_{-1} = 0$

$$\boxed{u_i = \left\lfloor \frac{x_i}{2^\ell} \right\rfloor - \left\lfloor \frac{x_{i-1}}{2^\ell} \right\rfloor}$$

MEMORIZZATO IN UNARIO

$$\underbrace{00 \ldots 0}_{u_i} 1$$



$\boxed{\text{MEMORIA}}$

$L \longrightarrow \ell n$ bit

$U \longrightarrow \sum_{i=0}^{n-1} (u_i + 1) = \sum_{i=0}^{n-1} \left(\left\lfloor \frac{x_i}{2^\ell} \right\rfloor - \left\lfloor \frac{x_{i-1}}{2^\ell} \right\rfloor + 1 \right) \leq$

$\leq n + \left\lfloor \frac{x_{n-1}}{2^\ell} \right\rfloor \leq n + \frac{u}{2^\ell} \leq n + \frac{u}{2^{\lfloor \log \frac{u}{n} \rfloor}}$

Se $\frac{U}{n}$ è una pot. 2

$\longrightarrow \leq n + \frac{u}{u/n} = 2n$

Se $\frac{u}{n}$ non è una pot. 2

$\leq n + \frac{u}{2^{\log \frac{u}{n} - 1}} = n + \frac{2u}{u/n} =$

$= 3n$

$\boxed{\begin{cases} \ell n + \boxed{2n} \\ \ell n + \boxed{3n} \end{cases} \text{ bit in tutto}}$

$u \geq n$

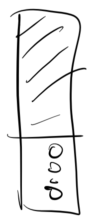
$\frac{D_n}{n} = \boxed{2 + \left\lceil \log \frac{u}{n} \right\rceil \text{ bit per elemento}}$

$\boxed{D_n = 2n + n \left\lceil \log \frac{u}{n} \right\rceil} \leftarrow$

$$\text{select}(i) - i = u_0 + u_1 + \dots + u_i =$$

$$= \sum_{j=0}^{i} \left\lfloor \frac{x_j}{2^\ell} \right\rfloor - \left\lfloor \frac{x_{j-1}}{2^\ell} \right\rfloor = \left\lfloor \frac{x_i}{2^\ell} \right\rfloor$$

$$\boxed{x_i = \underbrace{(\text{select}(i) - i)}_{\left\lfloor \frac{x_i}{2^\ell} \right\rfloor} 2^\ell + \ell_i}$$



INFORMATION-THEORETICAL LOWER BOUND

Quante sono le seq. per dati n e u

$$0 \le x_1 \le x_2 \dots \le x_n < u$$

$\updownarrow$

multisottoinsiemi di $\{0, 1, \dots, u-1\}$ aventi cardinalità n

$\updownarrow$

Quante sono le soluzioni $(c_0, \dots, c_{u-1}) \in \mathbb{N}^{u-1}$

$$c_0 + c_1 + \dots + c_{u-1} = n$$

$\updownarrow$

$$* * \,|\,|\,|\, * * * \,|\, * \; * \,|\,|\, * \,|$$

$U-1$ $\qquad$ n

$$\binom{U+n-1}{U-1} = \frac{(U+n-1)!}{n!\,(U-1)!} = \binom{U+n-1}{n}$$

$$\log\binom{A}{B} \approx B\log\frac{A}{B} + (A-B)\log\frac{A}{A-B}$$

$A = U+n-1$

$B = n$

$A-B = U-1$

$\frac{A}{A-B} = \frac{U+n-1}{U-1} \approx 1$

$\log\frac{A}{A-B} \Rightarrow 0$

$$Z_n = \log\binom{U+n-1}{n} \approx$$

$$\approx n\log\frac{U+n-1}{n} = \qquad \frac{U}{n}+1-\frac{1}{n}$$

$$= n\log\left(\frac{U}{n}\left(1+\frac{n}{U}-\frac{1}{U}\right)\right) =$$

$$= n\log\frac{U}{n} + n\log\left(1+\frac{n}{U}-\frac{1}{U}\right) \approx$$

$x \approx \log(1+x)$

$n \ll U$

$n \le \sqrt{U}$

$$\approx n\log\frac{u}{n} + \frac{n^2}{U}$$

$$\boxed{Z_n \approx n\log\frac{u}{n}}$$

$$D_n = 2n + n\left\lceil \log\frac{u}{n} \right\rceil$$

$$D_n = O(2^n)$$