

ALGORITHM

CENTER SELECTION LUS

INPUT: $S \subseteq \Omega$ ($|S| \leq n > 0$), $k > 0$,
 $\pi \in \mathbb{R}^{>0}$

$C \leftarrow \{\bar{s}\}$ for some $\bar{s} \in S$
while true

if $\exists \bar{s} \cdot d(\bar{s}, C) > 2\pi$
 $C \leftarrow C \cup \{\bar{s}\}$
else break

if $|C| \leq k$
output C

else
output "Impossible"

Teorema: GREEDY CENTER SELECTION

è una 2-approximazione
per CENTER SELECTION

Dim: Per assurdo, supponiamo che
GREEDY CENTER SELECTION esista
una soluzione con $p > 2p^*$.
Vul dire $\exists \hat{S} \in S$. $d(\hat{S}, C) > 2p^*$.
Sia $\bar{s}_i$ l'i-esimo centro aggiunto
e sia $\bar{C}_i$ l'insieme dei centri
in quel momento.

$$d(\bar{s}_i, \bar{C}_i) \geq d(\hat{S}, \bar{C}_i) \geq d(\hat{S}, C) > 2p^*$$

Ma allora l'esecuzione
una delle esecuzioni possibili
di CENTER SELECTION PLUS quando
 $r = p^*$.

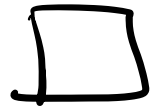
Ma CENTER SELECTION PLUS con $r = p^*$
produce un output
 $\Rightarrow$ termina entro k iterazioni
f

$\Rightarrow$ tutti gli $s \in S$ sono i.c.

$$d(s, C) \leq 2p^*$$

Ma non è vero: $d(\bar{s}, C) > 2p^*$

Assurdo.



Teorema: Se $P \neq NP$, non esiste
un algoritmo polinomiale
che α -approssimi
per qualche $\alpha < 2$.
CENTER SELECTION

Def:

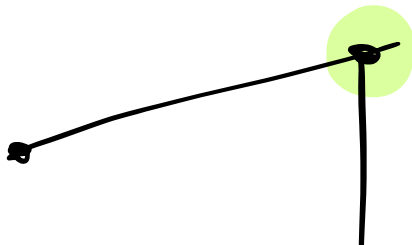
DOMINATING SET

INPUT: $G=(V, E)$, $k \geq 0$

OUTPUT: $\exists D \subseteq V$ t.c. $|D| \leq k$
e $\forall x \in V \exists y \in D$ $xy \in E$

DOMINATING SET $\in NP_c$

$k=2$



DATA G, k (input DOMINATING SET)

$$\Omega = V = S$$

$$d(x, y) = \begin{cases} 1 & \text{se } xy \in E \\ 0 & \text{se } x = y \\ 2 & \text{se } xy \notin E \end{cases}$$

$$\underbrace{d(x, y)}_{1, 2} \leq \underbrace{\underbrace{d(x, z)}_{1, 2} + \underbrace{d(z, y)}_{1, 2}}_{2, 3, 4}$$

CENTER SELECTION (S, k)

$$p^*(S, k) \in \{1, 2\}$$

$$p^*(S, k) = 1 \quad \text{se} \quad \exists C^* \subseteq S \quad |C^*| \leq k$$

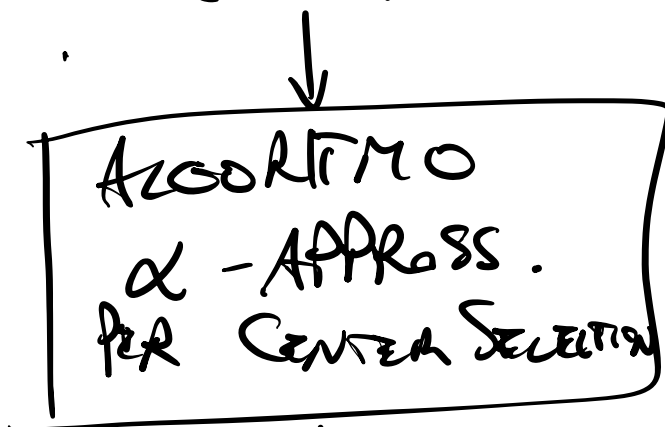
centri t.c. $\forall s \in S, d(s, c) \leq 1$

$$\text{sse} \quad \exists C^* \subseteq S \quad |C^*| \leq k \\ \text{t.c.} \quad \forall s \quad \min_{c \in C^*} d(s, c) = 1$$

$$\text{sse} \quad \exists C^* \subseteq S \quad |C^*| \leq k \\ \text{t.c.} \quad \forall s \quad \exists c \in C^* \quad s, c \in E$$

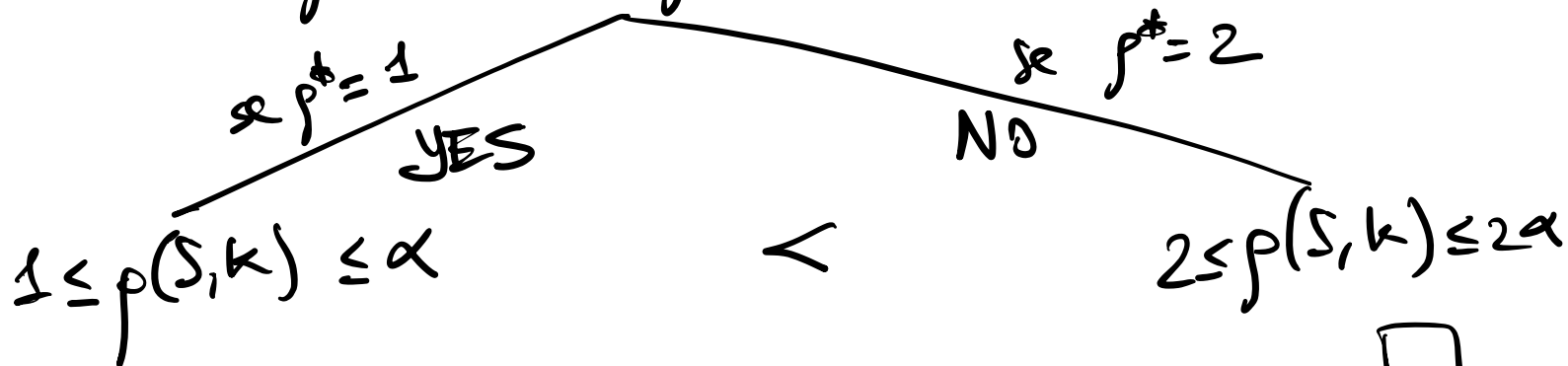
sse C^* è un dominating set

(S, k)



$\alpha < 2$

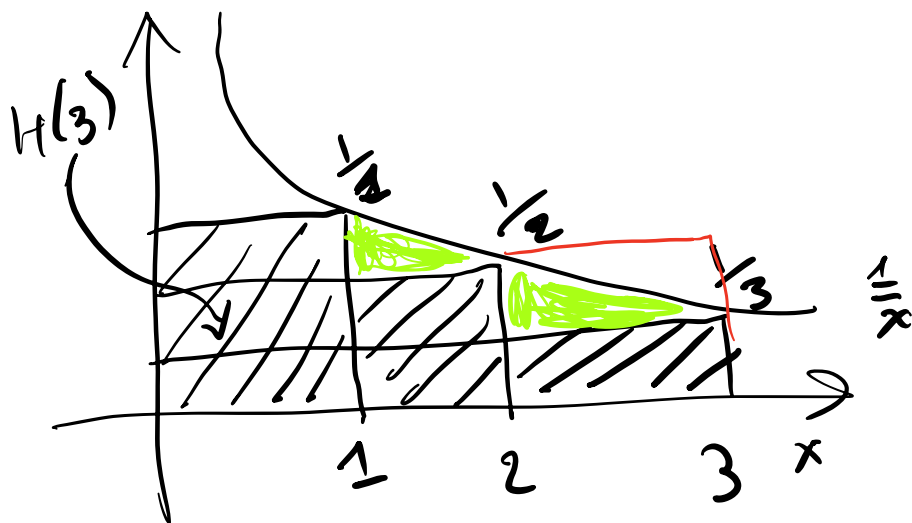
$$p^*(S, k) \leq p(S, k) \leq \alpha p^*(S, k)$$



FUNZIONE ARMONICA

$$H: \mathbb{N}^{\infty} \rightarrow \mathbb{R}$$

$$H(n) = \sum_{i=1}^n \frac{1}{i}$$



Proprietà 1: $H(n) \leq 1 + \int_1^n \frac{1}{x} dx$

$$H(n) \leq 1 + \left[\ln x \right]_1^n =$$

$$= 1 + \ln n - \ln 1 =$$

$$= 1 + \ln n$$

$$\boxed{H(n) \leq 1 + \ln n}$$

Proprietà 2:

$$(1 + \int_1^n \frac{1}{x} dx) -$$

$$\int_t^{t+1} \frac{1}{x} dx \leq \frac{1}{t}$$

$$= \frac{1}{t} \int_t^{t+1} 1 dx = \frac{1}{t}$$

$$H(n) = \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{n} \geq$$

$$\geq \int_1^2 \frac{1}{x} dx + \int_2^3 \frac{1}{x} dx + \dots + \int_n^{n+1} \frac{1}{x} dx =$$

$$= \int_1^{n+1} \frac{1}{x} dx = [\ln x]_1^{n+1} =$$

$$= \ln(n+1) - \ln 1 =$$

$$= \ln(n+1)$$

$$\boxed{H(n) \geq \ln(n+1)}$$

$$\ln(n+1) \leq H(n) \leq 1 + \ln(n)$$

PROBLEMA SET COVER

INPUT: $S_0, \dots, S_{m-1} \mid \bigcup_{i \in I} S_i = U$
 $W_0, \dots, W_{m-1}$ $m = |U|$

SOL. AMMISSIBILI: $I \subseteq m$ t. c.

$$\bigcup_{i \in I} S_i = U$$

FUNZ. OBIETTIVO:

$$W = \sum_{i \in I} W_i$$

TIPO: $NP(N)$

GREEDY SET COVER

$R \leftarrow U$
 $I \leftarrow \emptyset$
while $R \neq \emptyset$
 choose S_i minimizing $\frac{W_i}{|S_i \cap R|}$
 $I \leftarrow I \cup \{i\}$
 $R \leftarrow R \setminus S_i$

$\uparrow$
 $\forall s \in S_i \cap R$
 $c(s) := \frac{W_i}{|S_i \cap R|}$

output I

$|S_i \cap R|$

Lemma 1: Alla fine
$$W = \sum_{s \in U} c(s)$$

Dici:

$$W = \sum_{i \in I} w_i = \sum_{i \in I} \left[\sum_{s \in S_i \cap R} c(s) \right] = \sum_{s \in U} c(s) \cdot \square$$

$\downarrow$
 $c(s) := \frac{w_i}{|S_i \cap R|}$

Lemma 2: Per ogni k

$$\sum_{s \in S_k} c(s) \leq H(|S_k|) w_k$$

Dici: $S_k = \{s_1, \dots, s_d\}$

enumerati in ordine
di copertura

Consideriamo l'iterazione che
copre s_j (usando S_k)

$s_1, s_2, \dots, s_d$

$$K = \{s_j, s_{j+1}, \dots, s_d\}$$

$$\Rightarrow |S_k \cap R| \geq d - j + 1$$

$$\Rightarrow c(s_j) = \frac{w_k}{|S_k \cap R|} \underset{\min}{\leq} \frac{w_k}{|S_k \cap R|} \leq$$

$$\sum_{s \in S_k} c(s) = \sum_{j=1}^d c(s_j) \leq \sum_{j=1}^d \frac{w_k}{d-j+1} =$$

$$= \frac{w_k}{d} + \frac{w_k}{d-1} + \dots + \frac{w_k}{1} =$$

$$= w_k \left(\frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{d} \right) =$$

$$= w_k H(d) = w_k H(|S_k|) \quad \square$$