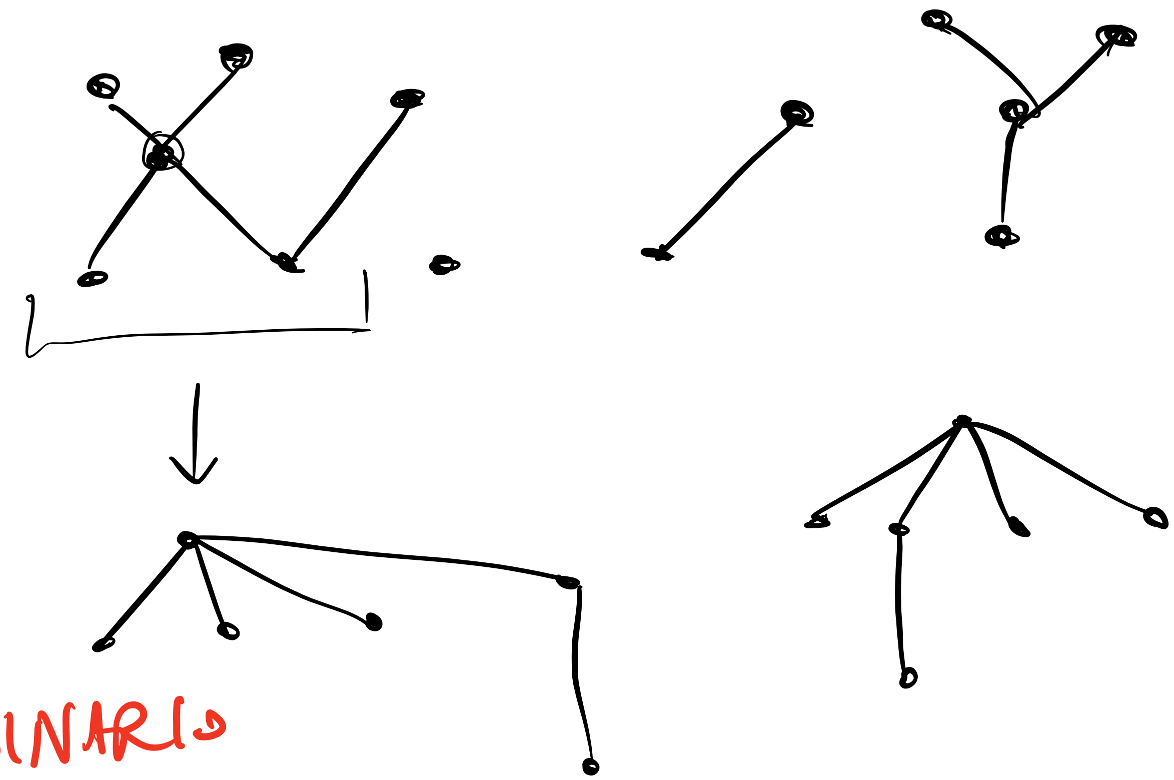
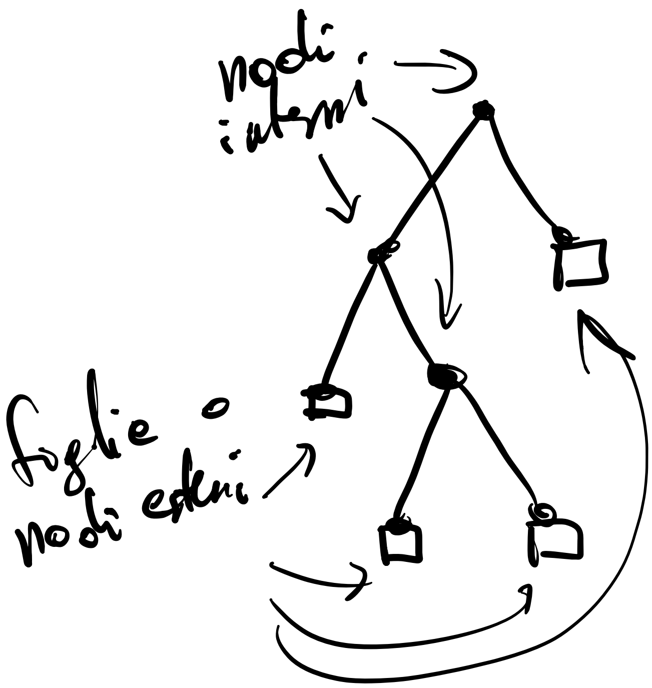
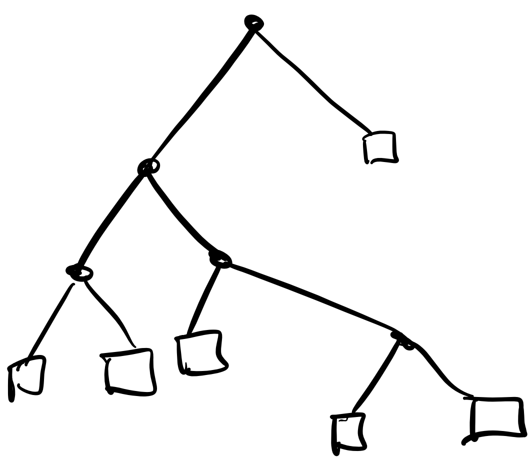
ALBERI



ALBERO BINARIO



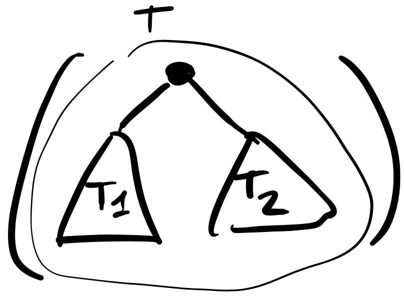
ALBERO RADICATO ORDINATO IN CUI OGNI NODO HA {0, 2} FIGLI

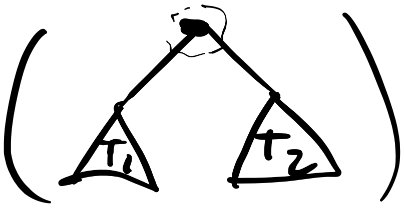


Teorema: $\#\text{nodi esterni} = \#\text{nodi interni} + 1$

<u>Dim</u>: • $E(\square) = 1 \qquad I(\square) = 0$

$$1 = 0 + 1$$

• $E\Big($  $\Big) = E(T_1) + E(T_2)$

$\rightarrow I\Big($  $\Big) = I(T_1) + I(T_2) + 1$

$$\left.\begin{array}{l} E(T_1) = I(T_1) + 1 \\ E(T_2) = I(T_2) + 1 \end{array}\right] (*)$$

$$E(T) = E(T_1) + E(T_2) =$$

$$= I(T_1) + 1 + I(T_2) + 1 =$$

$$= \underbrace{I(T_1) + I(T_2) + 1}_{} + 1 =$$

$$= I(T) + 1 \qquad \square$$

Teorema: Ci sono $C_n = \dfrac{1}{n+1}\dbinom{2n}{n}$ [numero di Catalano] alberi binari con n nodi

Catalan.

Approssimazione di Stirling

$$x! \approx \sqrt{2\pi x}\left(\frac{x}{e}\right)^x$$

$$C_n = \frac{1}{n+1}\binom{2n}{n} =$$

$$= \frac{1}{n+1}\frac{(2n)!}{n!\,(2n-n)!} = \frac{1}{n+1}\frac{(2n)!}{(n!)^2} \approx$$

$$\approx \frac{1}{n+1}\frac{\sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n}}{\left(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n\right)^2} =$$

$$= \frac{1}{n+1}\frac{\cancel{2}\sqrt{\pi n}}{\cancel{2}\pi n}\left(\frac{2n}{\cancel{e}}\right)^{2n}\left(\frac{\cancel{e}}{n}\right)^{2n} =$$

$$= \frac{1}{n+1}\frac{1}{\sqrt{\pi n}}\,2^{2n}\frac{\cancel{n^{2n}}}{\cancel{n^{2n}}} = \frac{4^n}{(n+1)\sqrt{\pi n}} \approx$$

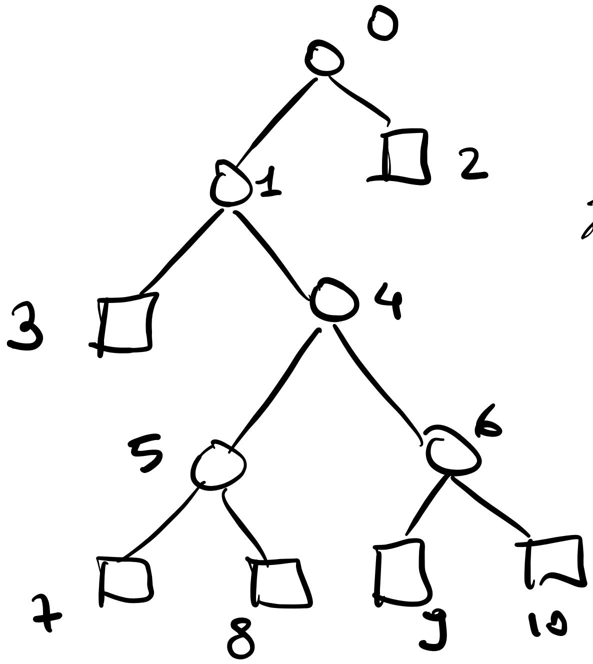
$$\approx \frac{4^n}{\sqrt{\pi n^3}}$$

$$Z_n = \log C_n \approx \underbrace{\log 4^n}_{} - \log\sqrt{\pi n^3} =$$

$$= 2n - O(\log n)$$

INF. TH. LOWER BOUND

RAPPRESENTAZIONE SUCCINTA

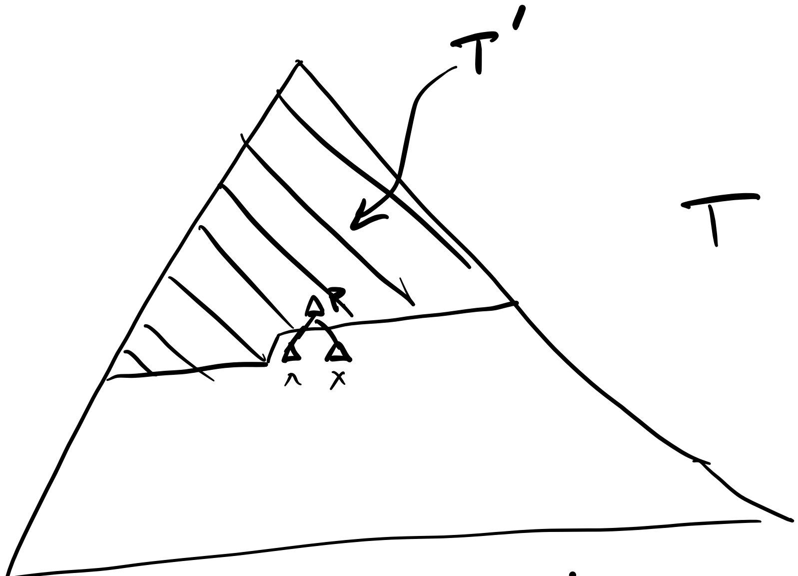


$n = 5$ nodi interni

0	1	2	3	4	5	6	7	8	9	10
1	1	0	0	1	1	1	0	0	0	0

1 = nodo interno

0 = nodo esterno



$$\text{figlisx}(p) = \#\text{nodi di } T' = 2\,\#\text{nodi interni di } T' + 1$$

$$= 2\,\text{rank}_{\underline{b}}(p) + 1$$

$$\text{figliodx}(p) = 2\,\text{rank}_b(p) + 2$$

$$\text{isleaf}(p) = \quad b_p == 0$$

Chi è p' t.c. p' è il genitore di p?

$2\,\mathrm{rank}(p') + 1 = p$ oppure

$2\,\mathrm{rank}(p') + 2 = p$

$\Downarrow$

$\mathrm{rank}(p') = \left\lfloor \frac{p}{2} - \frac{1}{2} \right\rfloor$

$p' = \mathrm{select}\left(\left\lfloor \frac{p}{2} - \frac{1}{2} \right\rfloor\right)$

$\mathrm{parent}(p) \triangleq \mathrm{select}_{\underline{b}}\left(\left\lfloor \frac{p}{2} - \frac{1}{2} \right\rfloor\right)$

$\boxed{\text{OCCUPAZIONE DI MEMORIA}}$

$D_n = \underbrace{2n + 1}_{\underline{b}} + o(n)$

$Z_n = 2n - O(\log n)$

succinta

CODIFICA DI ELIAS-FANO PER SEQUENZE MONOTONE

35	12
76	12

17 1543 3700

1940 3000

$$0 \leq x_0 \leq x_1 \leq x_2 \leq x_3 \quad \dots \leq x_{n-1} < u$$

$$\ell = \max\left\{0, \left\lfloor \log \frac{u}{n} \right\rfloor \right\}$$

(A) GLI ℓ BIT INFERIORI

ℓn BIT

$$\begin{cases} \ell_0 = x_0 \bmod 2^\ell \\ \ell_1 = x_1 \bmod 2^\ell \\ \vdots \\ \ell_{n-1} = x_{n-1} \bmod 2^\ell \end{cases}$$

x_i $\underbrace{\quad\dots\quad}_{\ell}$

ESPLICITAMENTE

(B) I BIT SUPERIORI

$$\left\lfloor \frac{x_0}{2^\ell} \right\rfloor, \left\lfloor \frac{x_1}{2^\ell} \right\rfloor, \dots, \left\lfloor \frac{x_{n-1}}{2^\ell} \right\rfloor$$

$u_0, u_1, u_2, \dots$

$$u_i = \left\lfloor \frac{x_i}{2^l} \right\rfloor - \left\lfloor \frac{x_{i-1}}{2^l} \right\rfloor \qquad (x_{-1} = 0)$$

$$u_0 = \left\lfloor \frac{x_0}{2^l} \right\rfloor$$

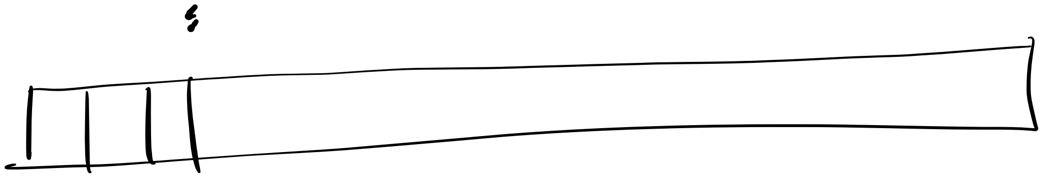
$$u_1 = \left\lfloor \frac{x_1}{2^l} \right\rfloor - \left\lfloor \frac{x_0}{2^l} \right\rfloor$$

$$u_2 = \left\lfloor \frac{x_2}{2^l} \right\rfloor - \left\lfloor \frac{x_1}{2^l} \right\rfloor$$

$$\vdots$$

MEMORIZZATI IN UNARIO IN UN UNICO VETTORE

3 → $\overline{0001}$



$$B = \sum_{i=0}^{n-1} u_i + 1 =$$

$$= \sum_{i=0}^{n-1} \left(\left\lfloor \frac{x_i}{2^l} \right\rfloor - \left\lfloor \frac{x_{i-1}}{2^l} \right\rfloor + 1 \right) =$$

$$= n + \sum_{i=0}^{n-1} \left(\left\lfloor \frac{x_i}{2^l} \right\rfloor - \left\lfloor \frac{x_{i-1}}{2^l} \right\rfloor \right) =$$

$$= n + \frac{x_{n-1}}{2^l} - \frac{x_{-1}}{2^l} =$$

$$= n + \frac{x_{n-1}}{2^l} \le n + \frac{u}{2^l} =$$

$$= n + \frac{u}{2^{\lfloor \lg \frac{u}{n} \rfloor}} \le n + \frac{u}{2^{\lg \frac{u}{n} - 1}} =$$

$$2^{\lfloor \log n \rfloor} \qquad 2$$

$$= n + \frac{2u}{u/n} = \underline{3n}$$

$$D_n = A + B = ln + 3n = (l+3)n =$$

$$= \left(\left\lfloor \log \frac{u}{n} \right\rfloor + 3 \right) n =$$

$$= 2n + n \left\lceil \log \frac{u}{n} \right\rceil + o(n)$$

$$\underline{u} = \underbrace{0001}_{u_0}\ \underbrace{00001}_{u_1}\ \underbrace{001}_{u_2}\ \underbrace{0000001}_{u_3} \ldots$$

$$x_i = \ ?$$

$$\text{select}_{\underline{u}}(i) = i + u_0 + u_1 + \ldots + u_i =$$

$$= i + \left\lfloor \cancel{\frac{x_0}{2^l}} \right\rfloor + \left\lfloor \frac{x_1}{2^l} \right\rfloor - \left\lfloor \cancel{\frac{x_0}{2^l}} \right\rfloor + \ldots =$$

$$= i + \left\lfloor \frac{x_i}{2^l} \right\rfloor$$

$\boxed{\text{INFORMATION THEORETICAL LOWER BOUND}}$

Fissati n e u, quante sono le sequenze

$$0 \le x_0 \le x_1 \le \dots \le x_{n-1} < u \quad ?$$

$$\{0, 1, \dots, u-1\} \leftarrow$$ contare quanti multi insiemi di cardinalità n ci sono

$$\underbrace{c_0 + c_1 + \dots + c_{u-1} = n}$$

quante soluzioni in $\mathbb{N}^u$ ha questa equazione?

STARS & STRIPES COUNTING

$$\overbrace{**}^{c_0}\;\overbrace{|}^{c_1}\,|\;\overbrace{*}^{c_2}\;||||\;*\;|$$

n $\qquad$ $u-1$

Quante sequenze di n stelle e $u-1$ barre esistono?

$$\binom{u+n-1}{u-1} = \frac{(u+n-1)!}{(u-1)!\,(u+n-1-u+1)!}$$

$$= \frac{(u+n-1)!}{n!\,(u-1)!}$$

$$Z_n = \log \binom{u+n-1}{u-1} = \log \binom{u+n-1}{n} =$$

$$\boxed{\log\binom{A}{B} \sim B\log\frac{A}{B} + (A-B)\log\frac{A}{A-B}}$$

$A = U+M-1$
$B = M$

$$Z_M \sim M\log\frac{U+M-1}{M} + \underbrace{(U-1)\log\left(\frac{U+M-1}{U-1}\right)}_{\text{TRASC. se } M \ll U} =$$

≈ 1

$$= M\log\frac{U}{M}\left(1+\frac{M}{U}-\frac{1}{U}\right) =$$

$$= M\log\frac{U}{M} + M\underbrace{\log\left(1+\frac{M}{U}-\frac{1}{U}\right)}$$

$$\boxed{x \approx \log(1+x)}$$

$$Z_M \approx M\log\frac{U}{M} + M\left(\frac{M}{U}-\frac{1}{U}\right) \leq$$

$$\leq M\log\frac{U}{M} + \frac{M^2}{U} \qquad \text{se } M \ll U$$

$$D_M = 2M + M\left\lceil\log\frac{U}{M}\right\rceil + o(M) =$$

$$= M\left(2+\left\lceil\log\frac{U}{M}\right\rceil\right) + o(M)$$

$$D_M = O(Z_M)$$

$\Rightarrow$ COMPATTA CON $n \leq \sqrt{U}$

SUCCINTA