PAOLO BOLDI

ALGORITMI & COMPLESSITÀ

PROBLEMA: Π

1) ins. input $I_\Pi \subseteq 2^*$
2) ins. output $O_\Pi \subseteq 2^*$
3) $Sol_\Pi : I_\Pi \longrightarrow 2^{O_\Pi} \setminus \{\emptyset\}$

- Decidere se un numero è primo
- Dato l'elenco dei nomi e cognomi degli studenti presenti in aula, stampa il primo studente per cognome in ordine alfabetico

$$x \in I_\pi \longrightarrow \boxed{A} \longrightarrow y \in O_\pi$$

$y \in Sol_\pi(x)$

INTERLUDIO : NOTAZIONI

$\mathbb{N} \quad \mathbb{Z} \quad \mathbb{Q} \quad \mathbb{R}$

$\mathbb{N}^+ \quad \mathbb{Z}^+ \quad \mathbb{Q}^+ \quad \mathbb{R}^+$

INS. NUMERICI

MONOIDE LIBERO

Σ^* insieme delle stringhe sull'alfabeto Σ

$\Sigma = \{a, b, c\}$

$\Sigma^* = \{\varepsilon$

a b c

aa ab ac ba bb bc

ca cb cc

.... $\}$ MONOIDE LIBERO SU Σ

$w \in \Sigma^*$

$w = w_0 w_1 \dots w_{|w|-1}$

Es. $w = abacca$

$|w| = 6$

$w_0 = a \quad w_1 = b \quad w_2 = a$

$w_3 = c \quad w_4 = c \quad w_5 = a$

$\boxed{\text{POTENZE DI INSIEMI}}$

A, B insiemi

$$B^A = \{ f \mid f : A \to B \}$$

$A = \{a, b, c\}$ $\quad B = \{0, 1\}$

a	0
b	0
c	0

a	1
b	1
c	1

a	0
b	1
c	1

...

$$2^3 = |B|^{|A|}$$

NOTAZIONE

$k \in \mathbb{N}$

$$k = \{0, 1, \dots, k-1\}$$

0	$\emptyset$
1	$\{0\}$
2	$\{0, 1\}$

2^* $\qquad A = \{a, b, c\}$

$$A^2 = \{ f \mid f : 2 \to A \}$$

$$\begin{array}{c|c} 0 & a \\ 1 & b \end{array} \qquad \begin{array}{c|c} 0 & a \\ 1 & a \end{array} \qquad \begin{array}{c|c} 0 & c \\ 1 & b \end{array}$$

...

$$2^A = \{ f \mid f : A \to 2 \}$$

$$\cong \mathcal{P}(A)$$

$$2^{2^*} = \{ X \mid X \text{ è un insieme di stringhe binarie} \}$$

$A = \emptyset$

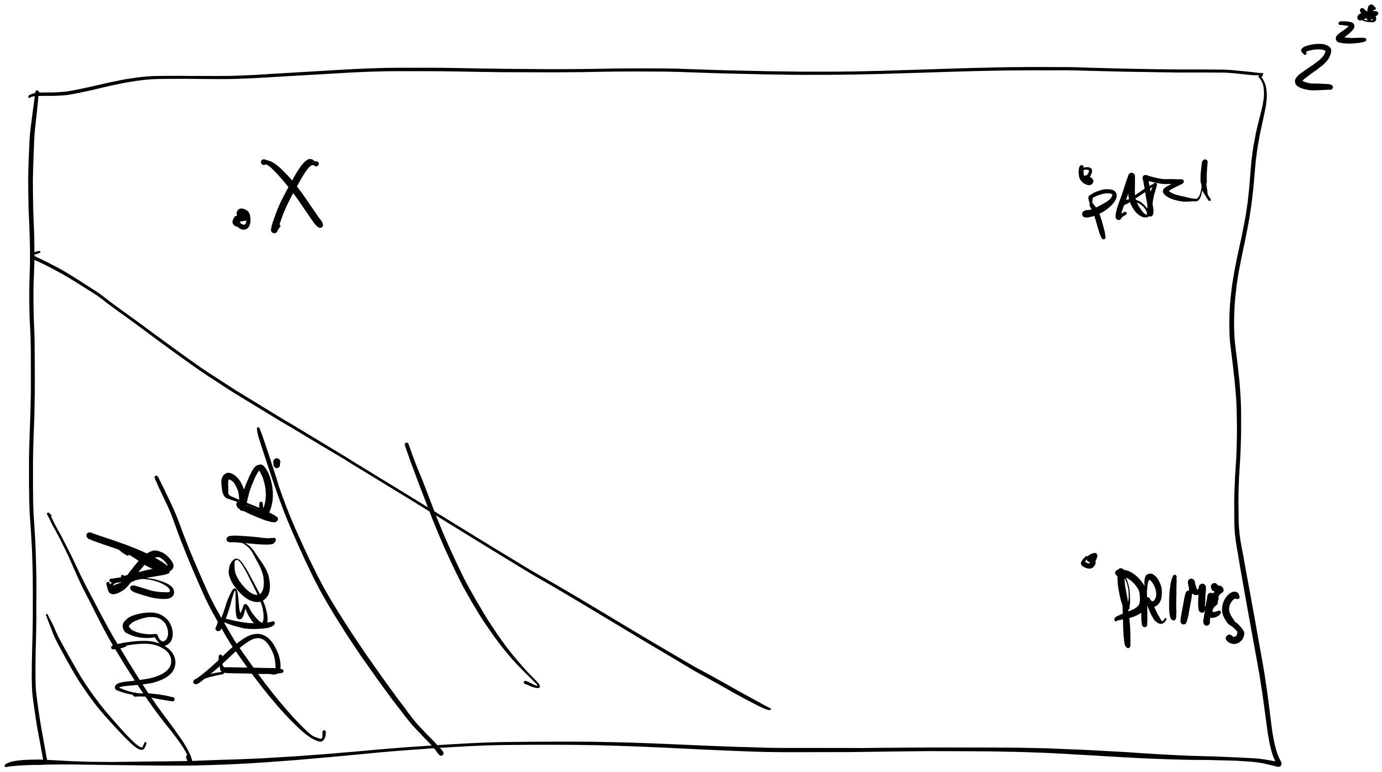
$A, B, C, D \in 2^{2^*}$

$B = 2^*$

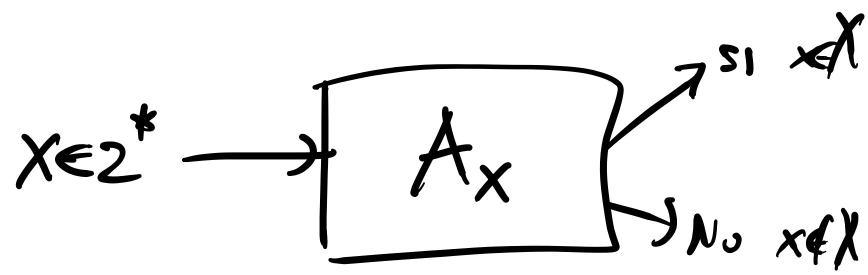
$A, B, C, D \subseteq 2^*$

$C = \{0, 10, 1010, \dots, x0\}$

$PRIMES = \{ x \mid x$ è la rappresentazione binaria di un numero primo $\}$



101



$\boxed{\text{ESEMPIO NON SEMPLICE}}$

- Dati due interi positivi x e y, calcolare $MCD(x, y)$

$x = 12$

$y = 7$

1100 111

$\underbrace{1100}111$

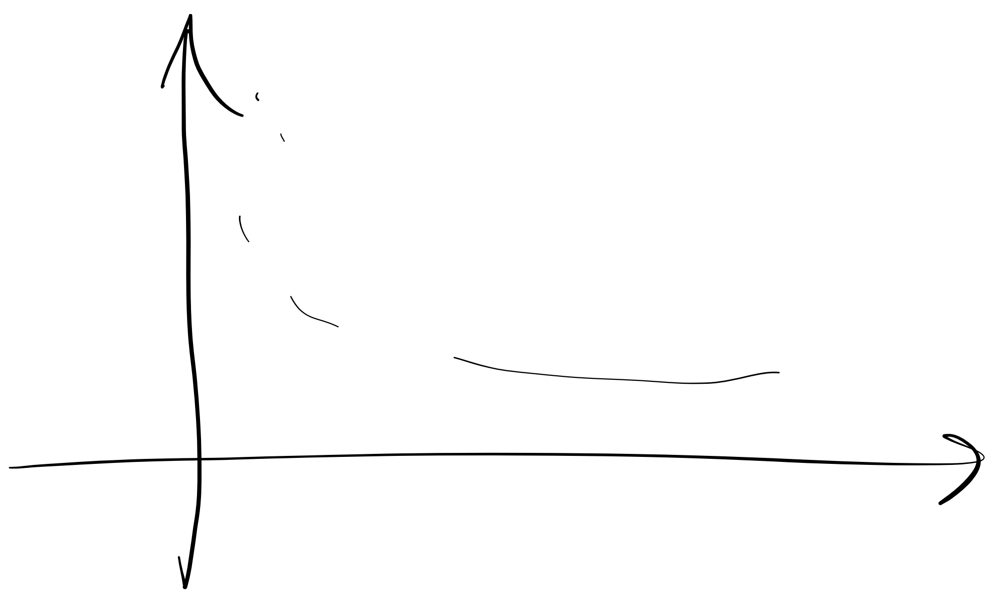
$\underbrace{\underbrace{00001}\,1100}\ \underbrace{\underbrace{0001}\ 111}$

Elias' γ

$\boxed{\text{Matteo}}$ x , y

<u>Bit</u> $8\lceil \log_{10} x \rceil + 8 + 8 \lceil \log_{10} y \rceil$

$\boxed{\text{PAOLO}}$ $2\lceil \log_2 x \rceil + 2 \lceil \log_2 y \rceil$



$\boxed{\text{Il modo brutto}}$ x, y

$12 \quad 7$

$$\underbrace{000000000000}_{12}1\underbrace{0000000}_{7}1$$

$\boxed{\text{PROBLEMA}} \longleftrightarrow \boxed{\text{ALGORITMO}}$

$= \text{MdT}$

calcolabilità

complessità $\longleftrightarrow$ TEMPO

$\Pi \longleftarrow A$ algoritmo per Π

$\forall x \in I_\Pi$

$T_A(x)$ ⇜ tempo richiesto da A su x

$$\boxed{\text{WORST-CASE ASSUMPTION}}$$

$t_A : \mathbb{N} \longrightarrow \mathbb{N}$

$$t_A(n) = \max_{\substack{x \in I_\pi \\ |x| = n}} T_A(x)$$

A n 1500 ⇜ x 1000000

y
 z
 w ≤ 100

$$\boxed{\text{ASYMPTOTIC ASSUMPTION}}$$

$t_A = O(n^{3.71})$

$$\boxed{t_{A'} = O(n^2 \lg n + n^{1.75})}$$

$t_{A''} = O(n^{2.15})$

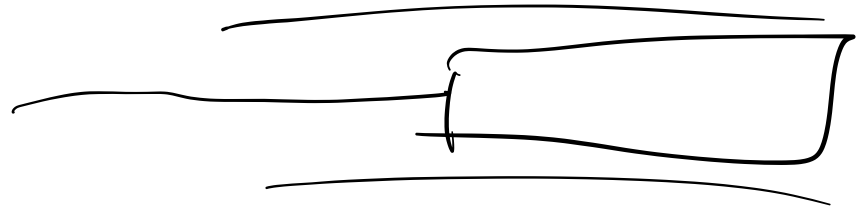
complessità
- algoritmica
- strutturale

Π problema da risolvere

upper bound ↓

$O(n^{2.71})$

$O(n^{1.81} \lg n)$



lower bound ↑

$\Omega(n^{1.5} \lg n)$

$\Omega(n \lg n)$

Algoritmi
- polinomiali ok
- super-polinomiali NON ok

Upper bound esponenziale $O(2^n)$

Lower bound polinomiale

SAT

$$(x_1 \lor x_2) \land (\neg x_1 \lor x_2 \lor x_3) \land (x_1 \lor x_i) \land (\neg x_1 \lor x_7)$$