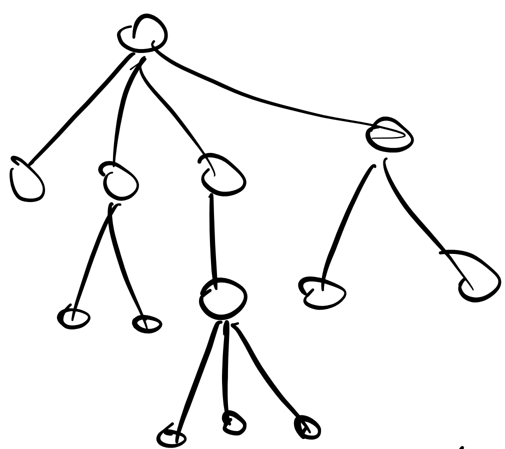
STRUTTURE SUCCINTE PER ALBERI BINARI

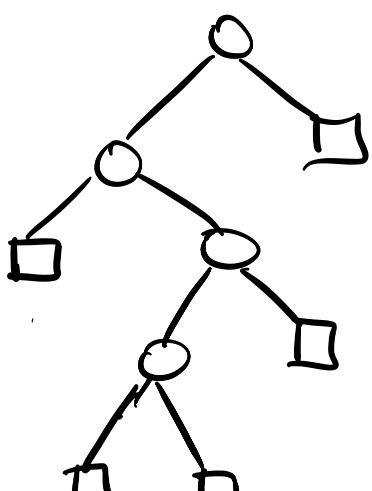
- Teoria dei grafi: albero = grafo non orientato connesso aciclo
- Informatica



ALBERI RADICATI E ORDINATI

ALBERI GENERALI

- ALBERI BINARI



○ nodi interni

□ nodi esterni (foglie)

Thm: # nodi esterni = # nodi interni + 1

$$|E| = |I| + 1$$

Thm: Numero di alberi con n nodi interni è

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$ [NUMERO DI CATALANO]

Approssimazione di Stirling:

$$x! \approx \sqrt{2\pi x}\left(\frac{x}{e}\right)^x$$

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \frac{1}{n+1}\frac{(2n)!}{n!\,n!} \approx$$

$$\approx \frac{1}{n+1}\frac{\sqrt{4\pi n}\left(\frac{2n}{e}\right)^{2n}}{2\pi n\left(\frac{n}{e}\right)^{2n}} =$$

$$= \frac{1}{n+1} \frac{1}{\sqrt{\pi n}} 2^{2n} \approx \frac{4^n}{\sqrt{\pi n^3}}$$

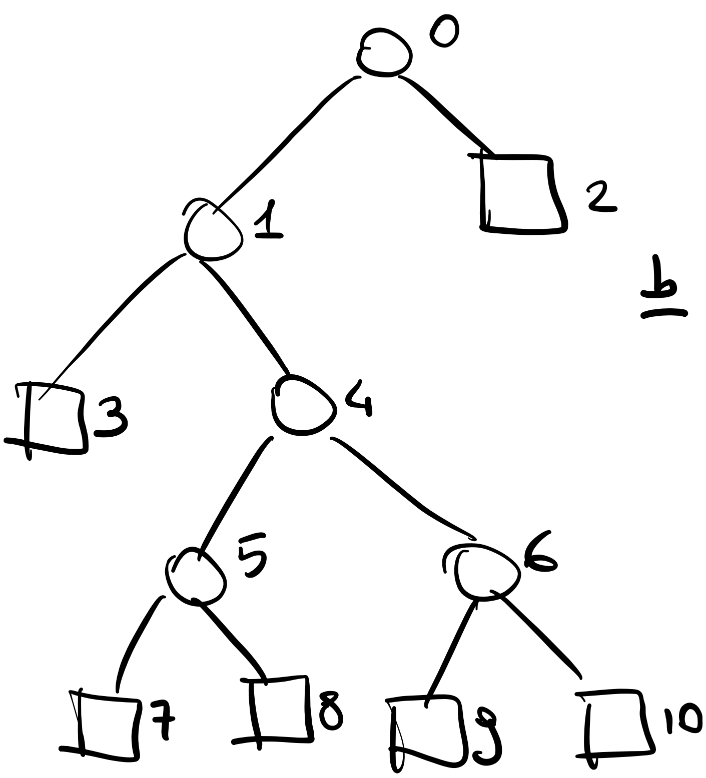
$$\lg C_n \approx \boxed{2n} - O(\lg n)$$

RAPPRESENTAZIONE SUCCINTA

$n = 5$

$\#\text{nodi} = 2n+1 = 11$

T



$\underline{b}$

0	1	2	3	4	5	6	7	8	9	10
1	1	0	0	1	1	1	0	0	0	0

+ RANK/SELECT

$$2n+1 + o(2n+1)$$

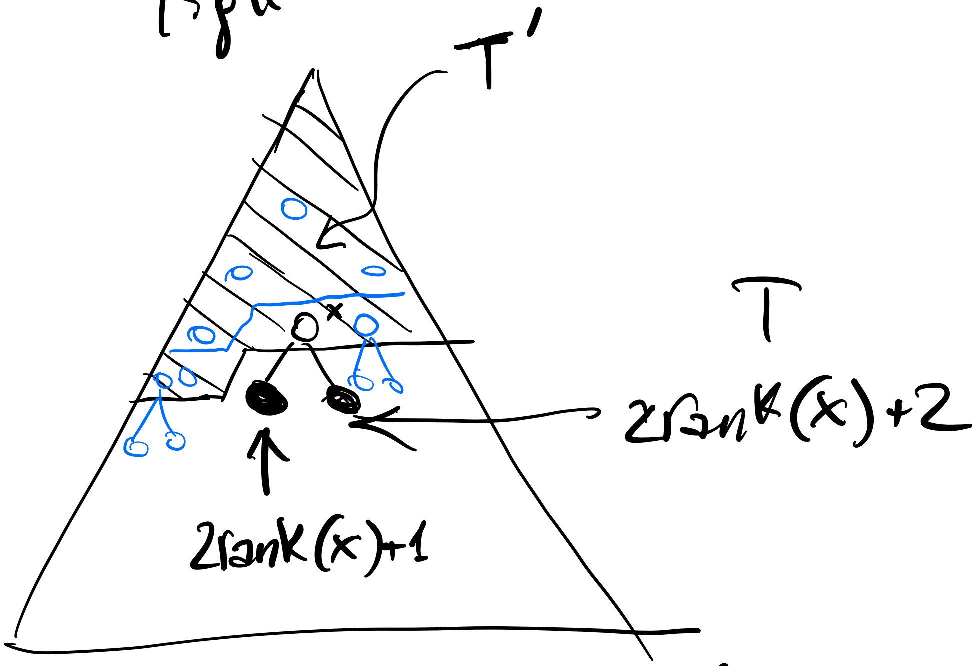
$$= \boxed{2n + o(n)}$$

NAVIGARE NELL'ALBERO

Se x è l'indice di un nodo

1) come capire se è una foglia

2) Quali sono gli indici dei figli



ACCESSO AI FIGLI

$\#$ nodi interni di T' =

= $\#$ nodi interni di T con indici $< x$ =

= $\#$ uni in $\underline{b}$ di indici $< x$ =

= $\mathrm{rank}_{\underline{b}}(x)$

$\#$ nodi $T' = 2\,\#$ nodi interni di $T' + 1 =$

$= 2\,\mathrm{rank}_{\underline{b}}(x) + 1$

ACCESSO AL GENITORE

Il genitore di x è un nodo p t.c.

$$2\,\mathrm{rank}(p)+1=x$$

opp. $$2\,\mathrm{rank}(p)+2=x$$

$$\left.\begin{aligned} &\mathrm{rank}(p)+\frac{1}{2}=\frac{x}{2} \\ \text{opp.}\quad &\mathrm{rank}(p)+1=\frac{x}{2} \end{aligned}\right\}$$

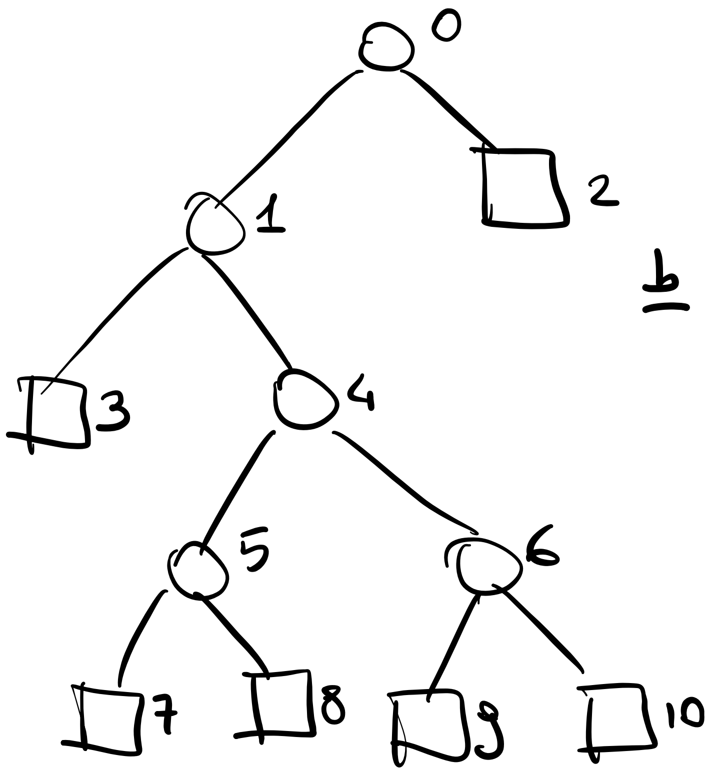
$$\mathrm{rank}(p)=\left\lfloor \frac{x}{2}-\frac{1}{2} \right\rfloor$$

$$\mathrm{select}(\mathrm{rank}(p))=\mathrm{select}\left(\left\lfloor \frac{x}{2}-\frac{1}{2} \right\rfloor\right)$$

$$p=\mathrm{select}\left(\left\lfloor \frac{x}{2}-\frac{1}{2} \right\rfloor\right)$$

DATI ANCILLARI

T



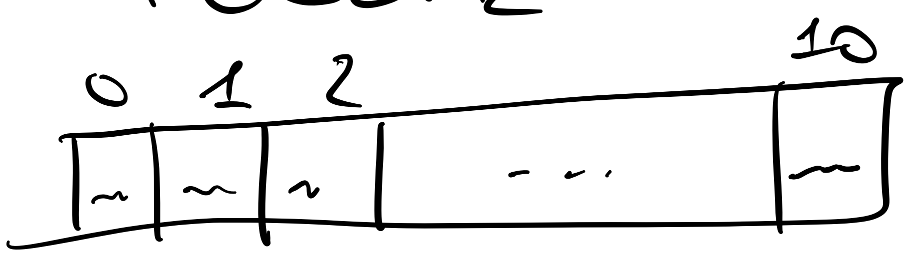
b

0	1	2	3	4	5	6	7	8	9	10
1	1	0	0	1	1	1	0	0	0	0

+ RANK/SELECT

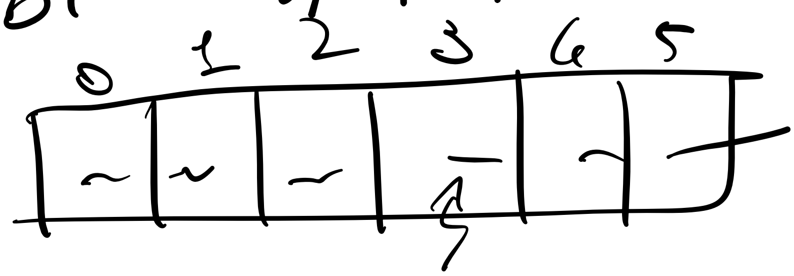
DATI ANCILLARI ANCHE SULLE FOGLIE

A



DATI ANCILLARI SOLO SU NODI INTERNI

B

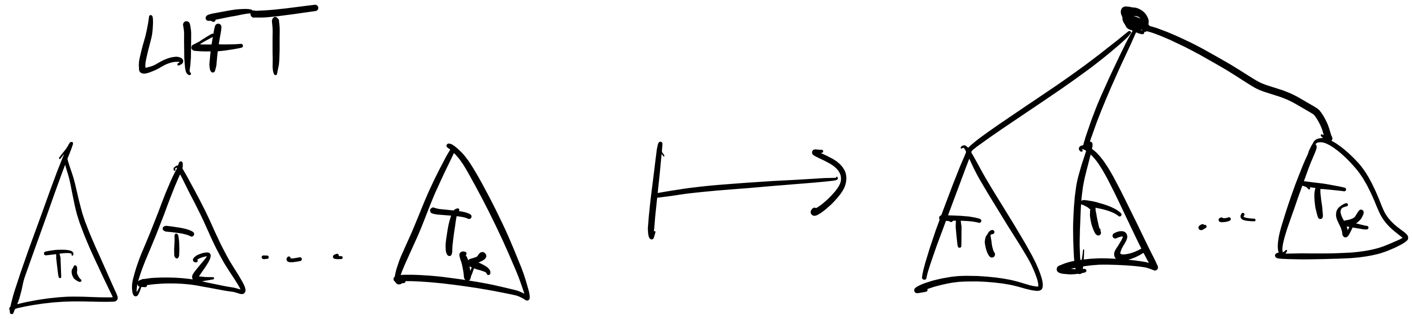


BIIEZIONI FRA ALBERI BINARI, ALBERI GENERALI E FORESTE

B_n	insieme alberi binari con n nodi interni
F_n	insieme delle foreste ordinate con n nodi
T_n	insieme degli alberi generali con n nodi

$$\varphi : F_n \xrightarrow{\cong} T_{n+1}$$

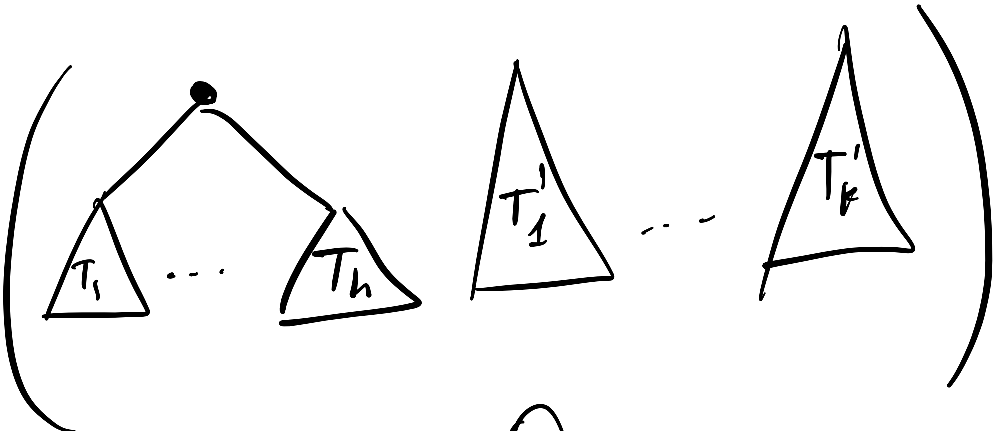
LIFT

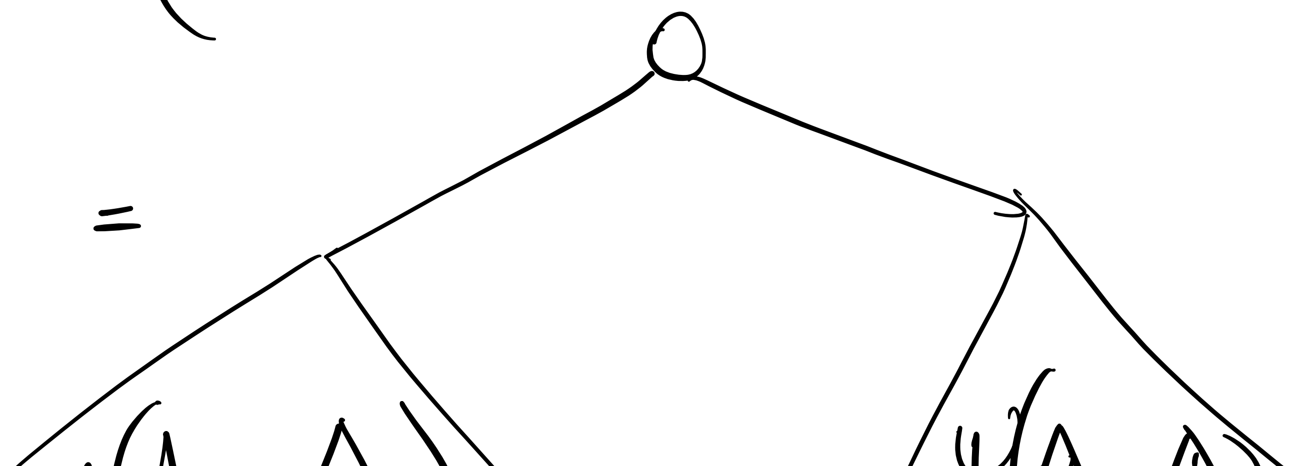


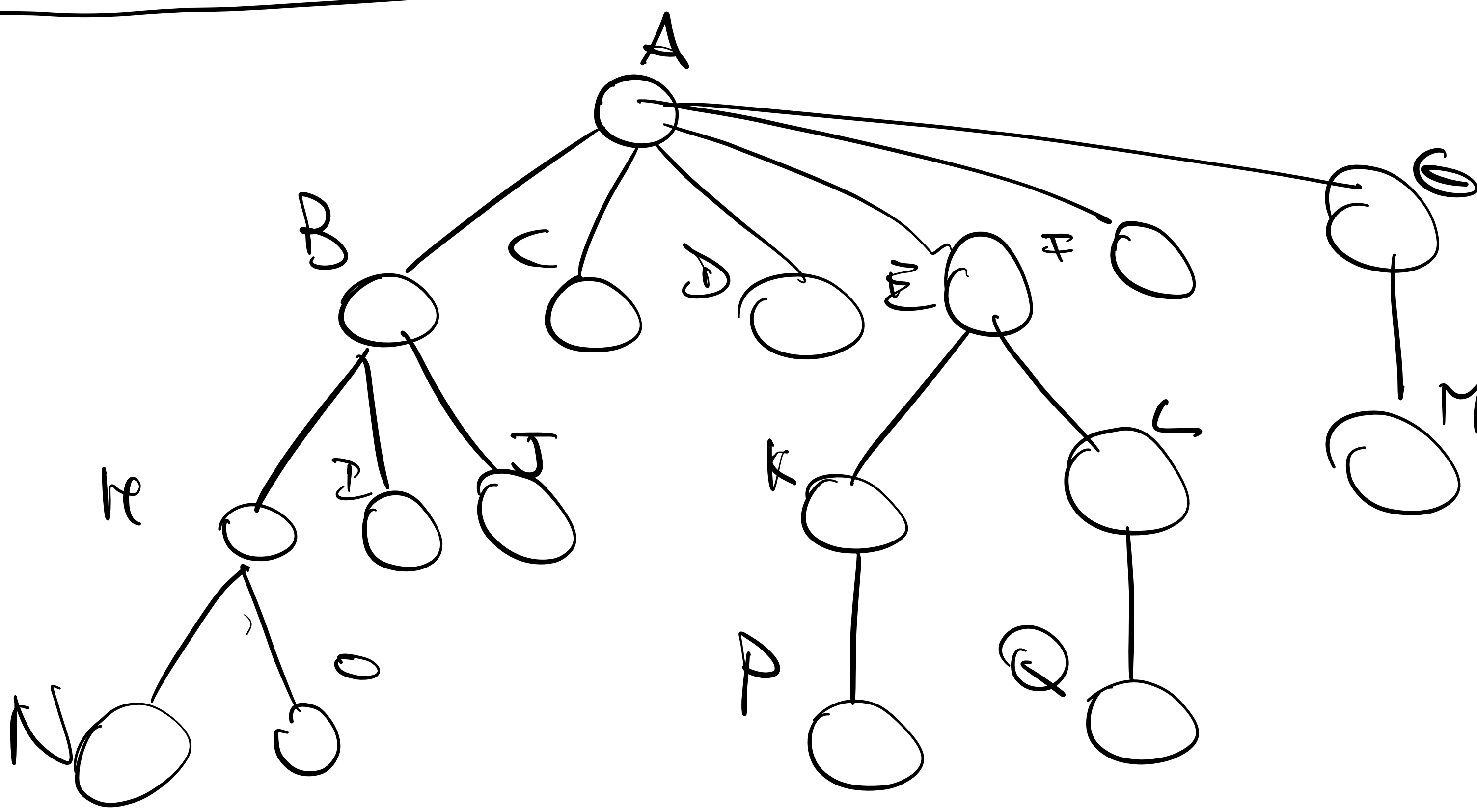
$$\psi : F_n \xrightarrow{\cong} B_n$$

FCNS (FIRST-CHILD NEXT-SIBLING)

$$\psi(\emptyset) = \square$$

ψ  =

= 


A
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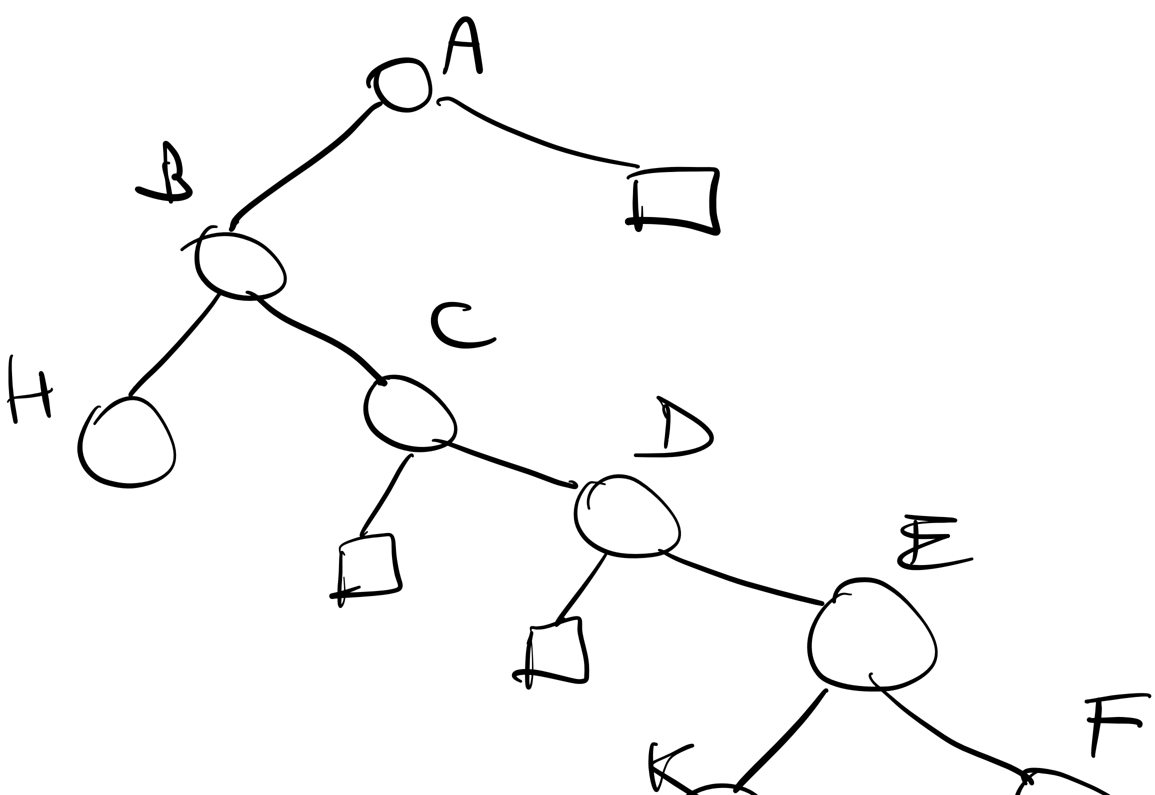
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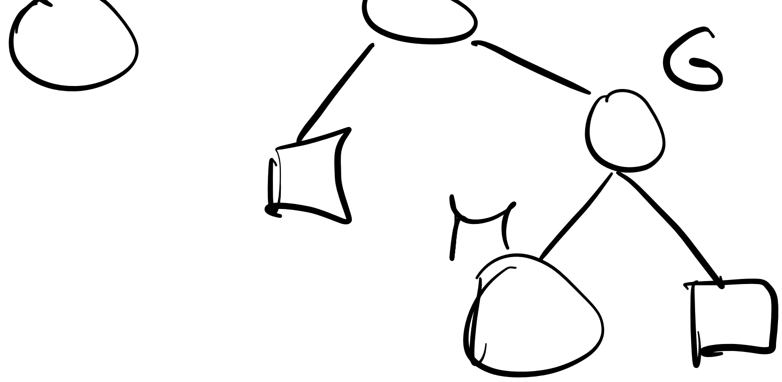


$$|T_{n+1}| = |F_n| = |B_n| = C_n$$

PAROLE DI DYCK

$$\Sigma = \{ \underset{0}{(} , \underset{1}{)} \}$$

$w \in \Sigma^*$ è una parola di Dyck

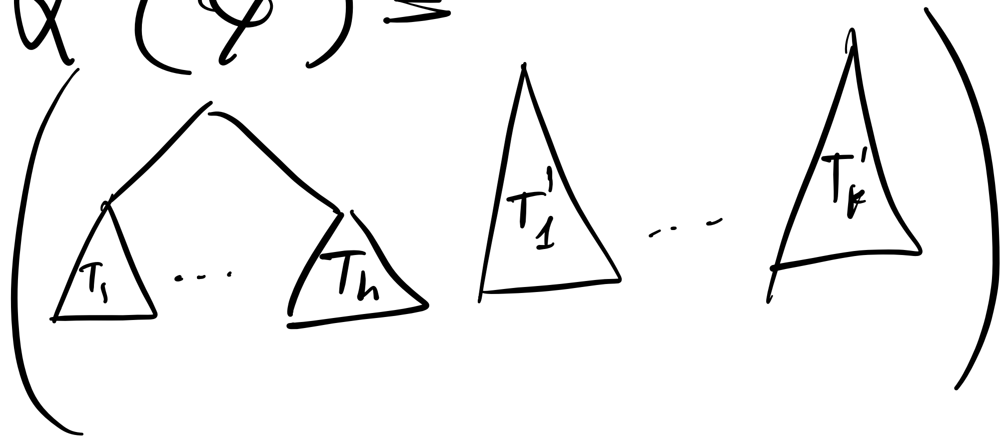
1) $\#_(\, w = \#_) \, w$

2) $\forall v$ prefisso di w

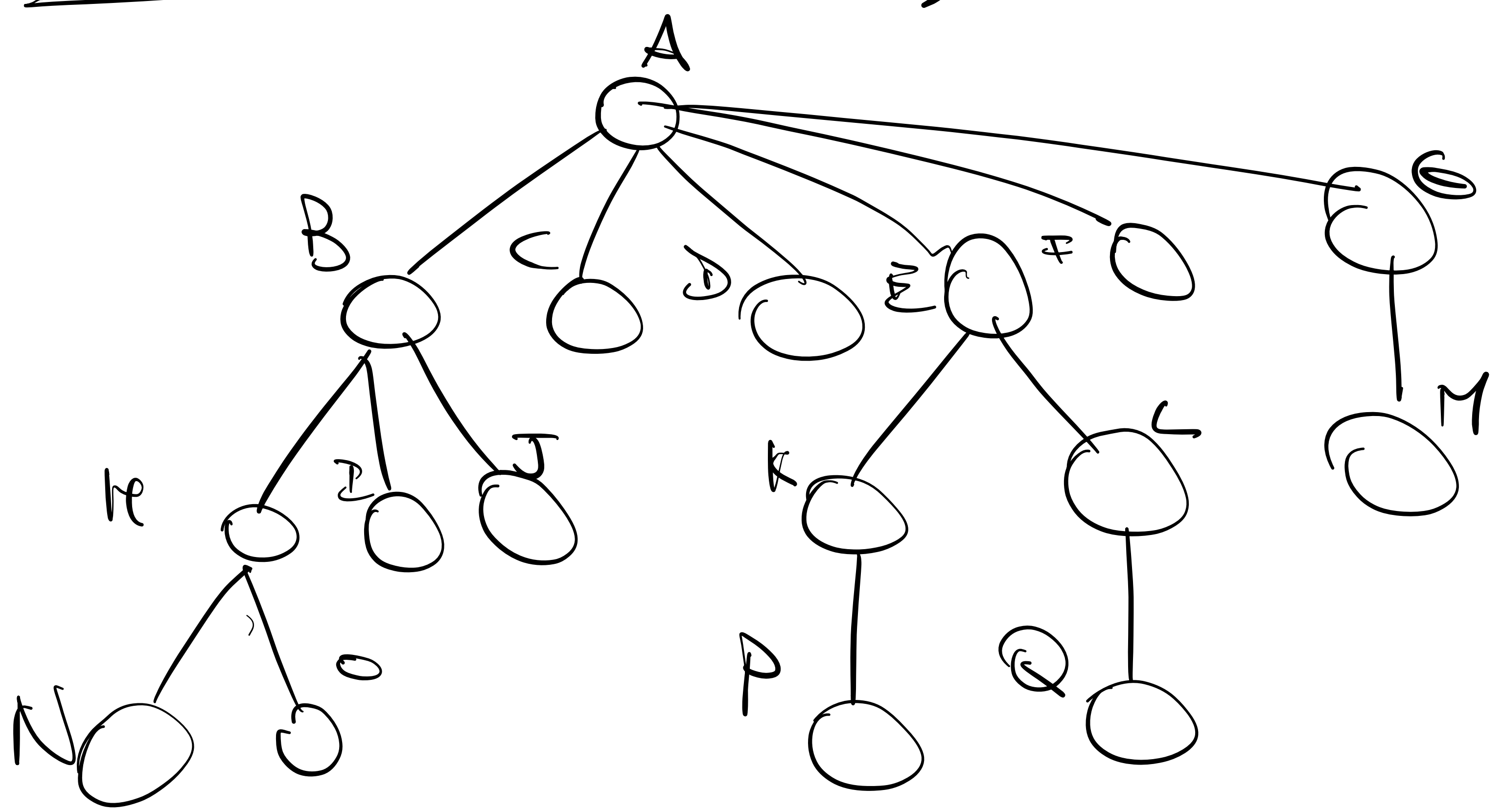
$\#_(v \geq \#_) v$ Parole con n aperte

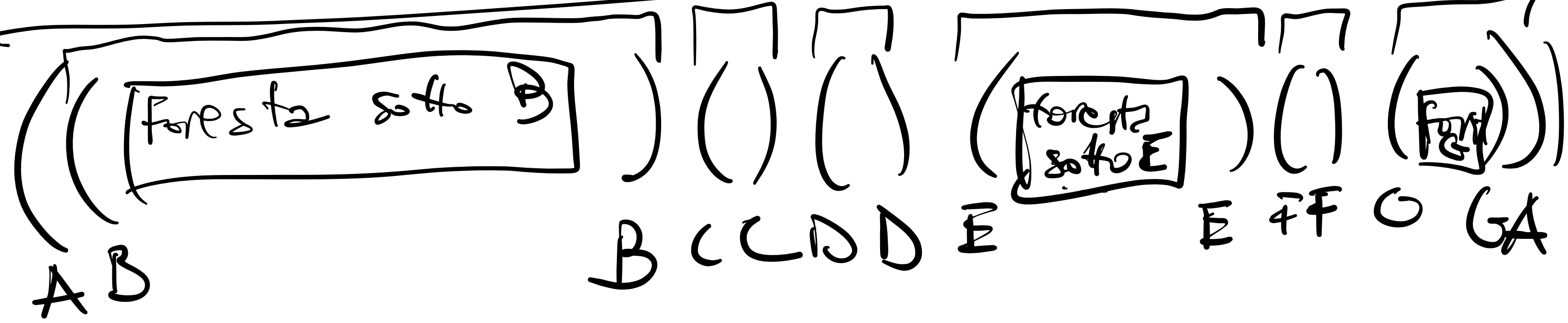
$$\alpha : F_n \longrightarrow D_n$$

$$\alpha(\emptyset) = \varepsilon$$

α  $=$

$$\left(\alpha\left(T_1 \ldots T_h\right)\right)\alpha(T_1')\ldots\alpha(T_k')$$





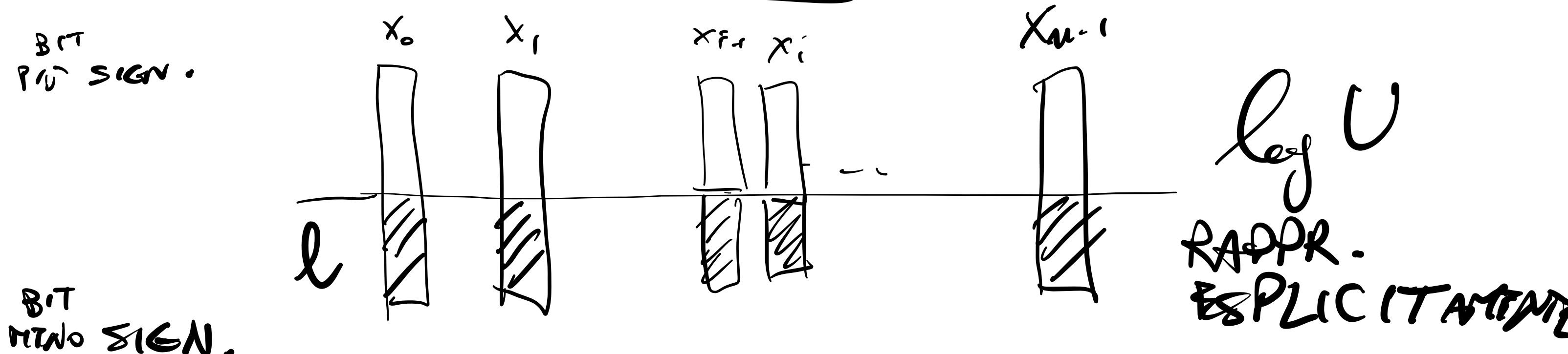
$$|D_n| = |F_n| = C_n$$

UNA RAPPRESENTAZIONE SUCCINTA PER SEQUENZE MONOTONE

$$\boxed{0 \le x_0 \le x_1 \le x_2 \le \dots \le x_{n-1} < U}$$

$$i \mapsto x_i$$

RAPPRESENTAZIONE DI ELIAS-FANO



$$\ell = \max\left(0, \left\lfloor \log \frac{U}{n} \right\rfloor\right)$$

$$\left.\begin{aligned} \ell_0 &= x_0 \bmod 2^\ell \\ \ell_1 &= x_1 \bmod 2^\ell \\ &\vdots \\ \ell_{n-1} &= x_{n-1} \bmod 2^\ell \end{aligned}\right] \text{LOWER PARTS}$$

$x_{-1} = 0$

$$u_i = \left\lfloor \frac{x_i}{2^\ell} \right\rfloor - \left\lfloor \frac{x_{i-1}}{2^\ell} \right\rfloor \leftarrow \text{IN UNARIO}$$

RANK-SELECT $\leftarrow$ $\underline{b} = \to \boxed{\underbrace{0000}_{u_0} 1 \underbrace{0000}_{u_1} 1 \underbrace{000}_{u_2} 1}$

SPAZIO OCCUPATO

LOWER BIT $\quad \ell n$ BIT

UPPER BIT

$$\leq \sum_{i=0}^{n-1} (u_i + 1) =$$

$$= \sum_{i=0}^{n-1} \left(\left\lfloor \frac{x_i}{2^\ell} \right\rfloor - \left\lfloor \frac{x_{i-1}}{2^\ell} \right\rfloor + 1 \right) =$$

$n-1 \quad \left(\left\lfloor x_i \right\rfloor \quad \left\lceil x_{i-1} \right\rceil \right)$

$$= n + \sum_{i=0} \left(\left\lfloor \frac{x_i}{2^\ell} \right\rfloor - \left\lfloor \frac{x_{i-1}}{2^\ell} \right\rfloor \right)$$

$$= n + \left\lfloor \frac{x_{n-1}}{2^\ell} \right\rfloor \leq n + \left\lfloor \frac{U}{2^\ell} \right\rfloor \leq$$

$$\leq n + \frac{U}{2^\ell} \leq n + \frac{U}{2^{\lfloor \lg \frac{U}{n} \rfloor}} =$$

$$\leq n + \frac{U}{2^{\lg \frac{U}{n} - 1}} = n + \frac{2U}{U/n} =$$

$$= 3n$$

$$D_n = \ell n + 3n \qquad \text{BIT}$$

$$= 3n + n \left\lfloor \lg \frac{U}{n} \right\rfloor =$$

$$= 2n + n \left\lceil \lg \frac{U}{n} \right\rceil$$

$$\mathrm{select}_{\underline{b}}(i) = i-1 + u_0 + u_1 + \dots + u_i =$$

$$u_i = \left\lfloor \frac{x_i}{2^\ell} \right\rfloor - \left\lfloor \frac{x_{i-1}}{2^\ell} \right\rfloor \leftarrow \text{IN UNARIO}$$

$$-\ \underline{b} = \rightarrow \underbrace{0000}_{u_0} 1 \underbrace{00000}_{u_1} 1 \underbrace{000}_{u_2} \overset{i}{\downarrow}{1} \dots$$

$$= i-1 + \left\lfloor \frac{x_0}{2^\ell} \right\rfloor + \left\lfloor \frac{x_1}{2^\ell} \right\rfloor - \left\lfloor \frac{x_0}{2^\ell} \right\rfloor + \left\lfloor \frac{x_2}{2^\ell} \right\rfloor - \left\lfloor \frac{x_1}{2^\ell} \right\rfloor + \dots + \left\lfloor \frac{x_i}{2^\ell} \right\rfloor - \left\lfloor \frac{x_{i-1}}{2^\ell} \right\rfloor = i-1+\left\lfloor \frac{x_i}{2^\ell} \right\rfloor$$

$$\mathrm{select}_b(i) = i-1+\left\lfloor \frac{x_i}{2^\ell} \right\rfloor$$

$$\Rightarrow \left\lfloor \frac{x_i}{2^\ell} \right\rfloor = \mathrm{select}_b(i) - i + 1$$

$\left[\,2^{t}\quad\right] \qquad b$