Graph distance distribution for social network mining

Plan of the talk

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- *Computing distances* in large graphs (using HyperANF)

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- Running HyperANF on *Facebook* (the largest Milgram-like experiment ever performed)
- Other uses of distances (in particular: *robustness*)

Prelude

Milgram's experiment is 45

Where it all started...

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- A. Rapoport, W.J. Horvath: *A study of a large sociogram*. (Behav.Sci. 1961)
- S. Milgram, *An experimental study of the small world problem.* (Sociometry, 1969)

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  - 100 were random Nebraska inhabitants (group C)

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    - parcels could be directly sent *only* to someone the sender knows personally
    - 453 intermediaries happened to be involved in the experiments (besides the starting population and the target)

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  - What is the distribution of the number of hops required to reach the target? **Avg. was 5.2**
  - Is this distribution different for the three starting subpopulations? **Yes: avg. for groups A/B/C was 4.6/5.4/5.7, respectively**

Chain lengths

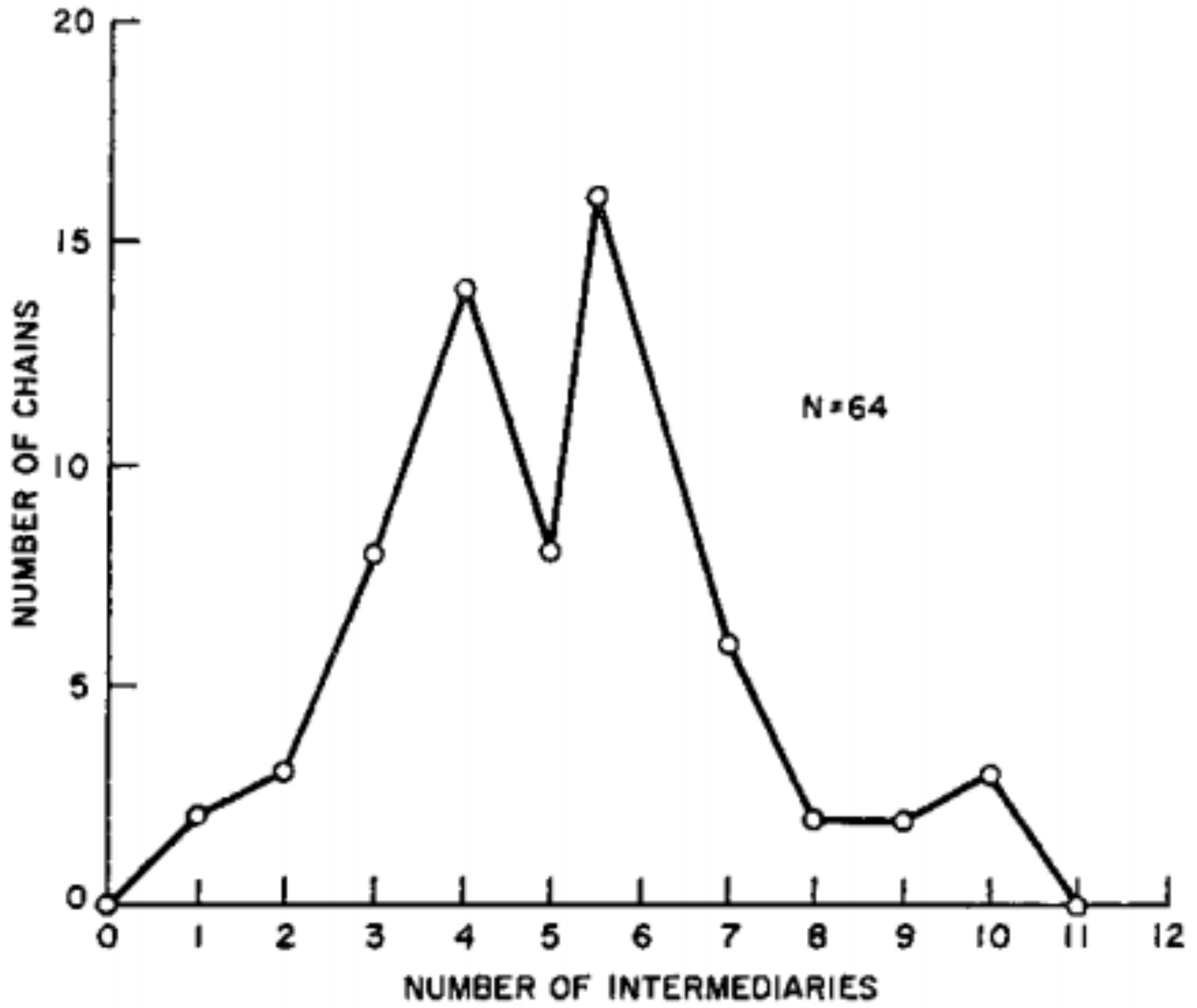

FIGURE 1

Milgram’s popularity

Milgram's popularity

- *Six degrees of separation* slipped away from the scientific niche to enter the world of popular immagination:

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  - ...a song sung by dolls in their national costume at Disneyland in a heart-warming exhibition celebrating the connectedness of people all

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  - The vast majority of chains were never completed
  - Extremely difficult to reproduce

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- But what did Milgram's experiment reveal, after all?

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Measuring what?

- But what did Milgram's experiment reveal, after all?

  i) That the world is small

  ii)That people are able to exploit this smallness

HyperANF

A tool to compute distances in large graphs

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- You want to obtain some information about its *global* structure (not simply triangle-counting/degree distribution/etc.)
- A natural candidate: *distance distribution*

Graph distances and distribution

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- For *undirected* graphs, $d(x,y)=d(y,x)$
- For every t, count the number of pairs (x,y) such that $d(x,y)=t$
- The fraction of pairs at distance t is (the density function of) a distribution

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    - if we repeat it from every source: $O(nm)$

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- Compute $d(x,y)$ with a BFS from x
- (Possibly: reject the pair if $d(x,y)$ is infinite)

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- Takes a BFS for every pair $O(m)$

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- Compute a full BFS from x

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Cohen’s sampling

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- Edith Cohen [JCSS 1997] came out with a very general framework for size estimation: powerful, but doesn't scale well, it is not easily parallelizable, requires direct access

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- Moreover $B_{t+1}(x)=\bigcup_{x\to y}B_t(y)\cup\{x\}$
- So computing B_{t+1} starting from B_t one just need a single (sequential) scan of the graph

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- What about using *approximated sets*?
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- Very small!

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- With 40 bits you can count up to 4 billion with a standard deviation of 6%
- Remember: one set per node!

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- This implies a guarantee on the *summation* of the counters
- This gives in turn precision bounds on the estimated distribution with respect to the real one

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- We use *broadword programming* to compute efficiently unions
- *Systolic computation* for on-demand updates of counters
- Exploited *microparallelization* of multicore architectures

Real speed?

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Real speed?

- Small dimension: 1.8min vs. 4.6h on a graph with 7.4M nodes
- Large dimension: HADI [Kang et al., 2010] is a Hadoop-conscious implementation of ANF. Takes 30 minutes on a 200K-node graph (on one of the 50 world largest supercomputers). HyperANF does the same in 2.25min on our workstation (20 min on this laptop).

Running it on Facebook!

{with Lars Backstrom and Johan Ugander}

Facebook

Facebook

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Facebook

- Facebook opened up to non-college students on September 26, 2006
- So, between 1 Jan 2007 and 1 Jan 2008 the number of users exploded

Experiments (time)

- We ran our experiments on snapshots of facebook
  - Jan 1, 2007
  - Jan 1, 2008 ...
  - Jan 1, 2011
  - {current} May, 2011

Experiments (dataset)

- We considered:
  - fb: the whole facebook
  - it / se: only Italian / Swedish users
  - it+se: only Italian & Swedish users
  - us: only US users
- Based on users' *current* geo-IP location

Active users

- We only considered *active* users (users who have done some activity in the 28 days preceding 9 Jun 2011)
- So we are not considering "old" users that are not active any more
- For fb [current] we have about 750M nodes

Distance distribution (fb)

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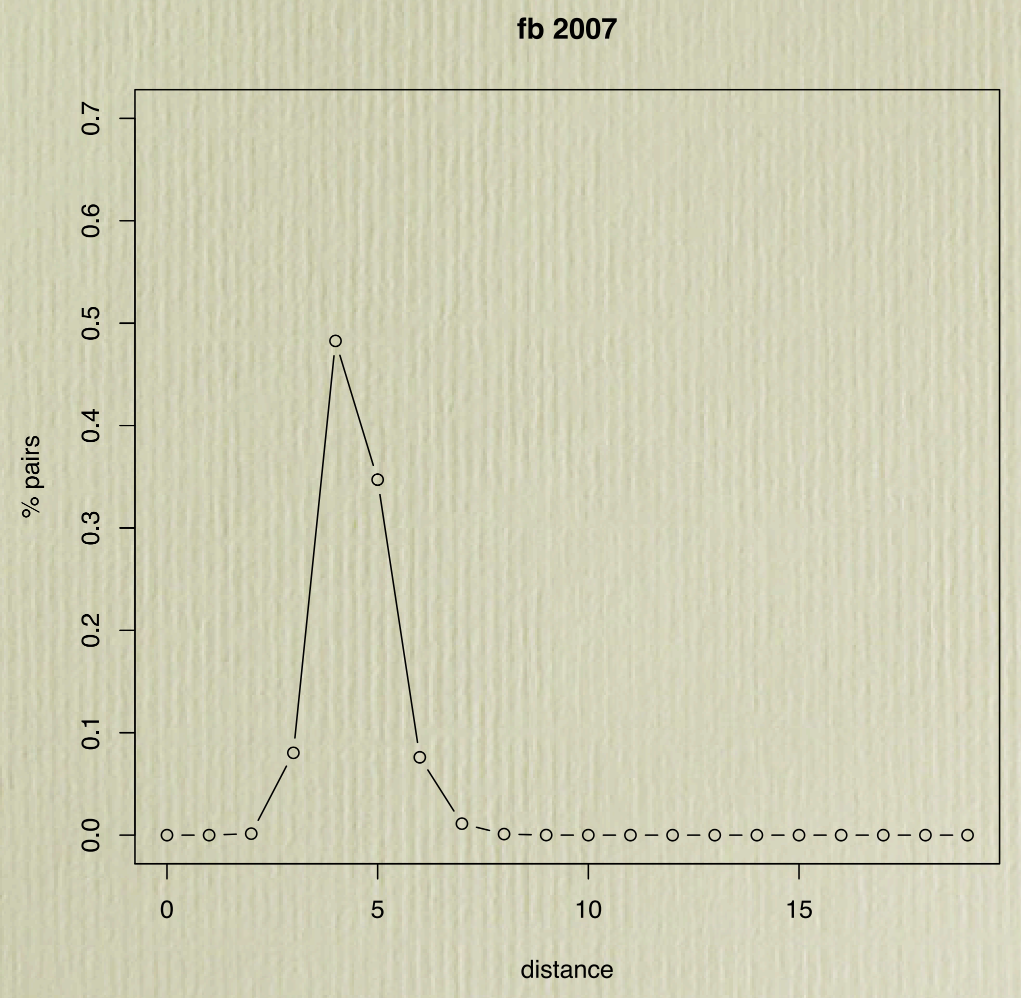

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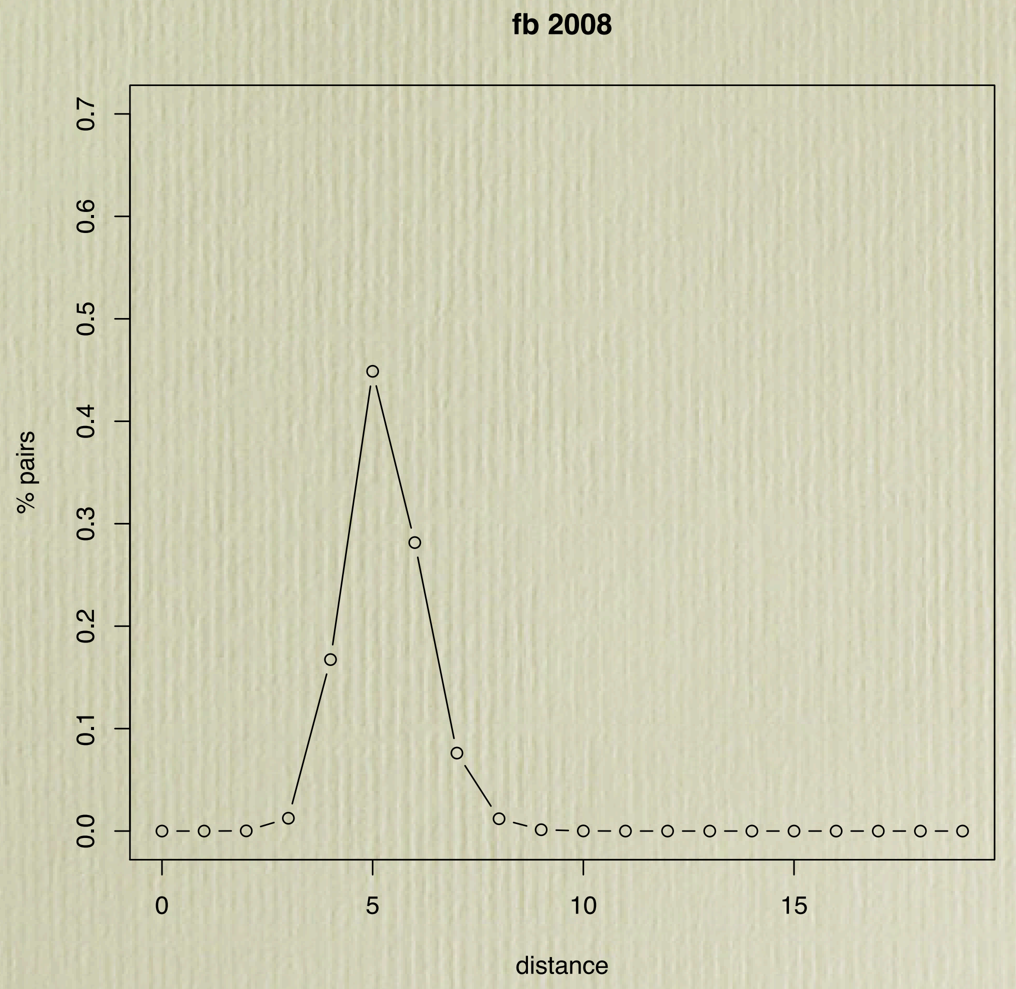

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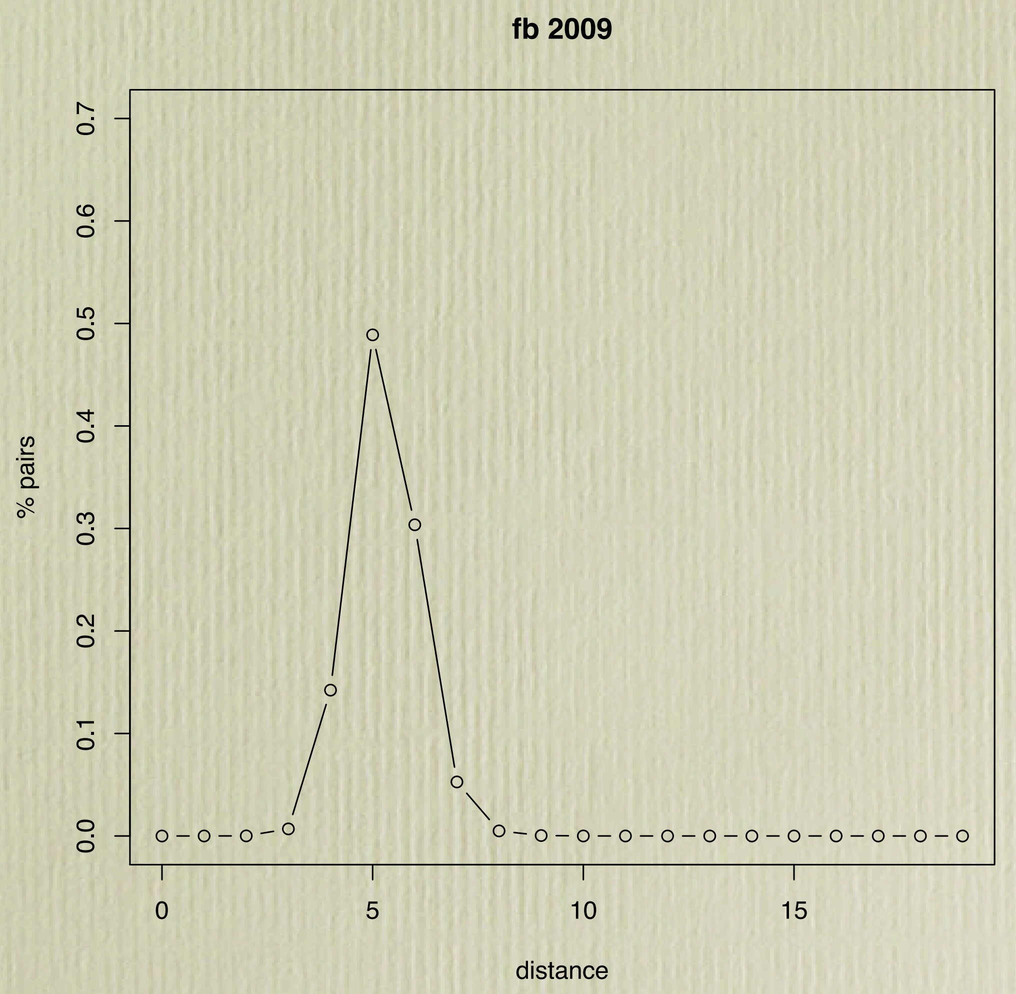

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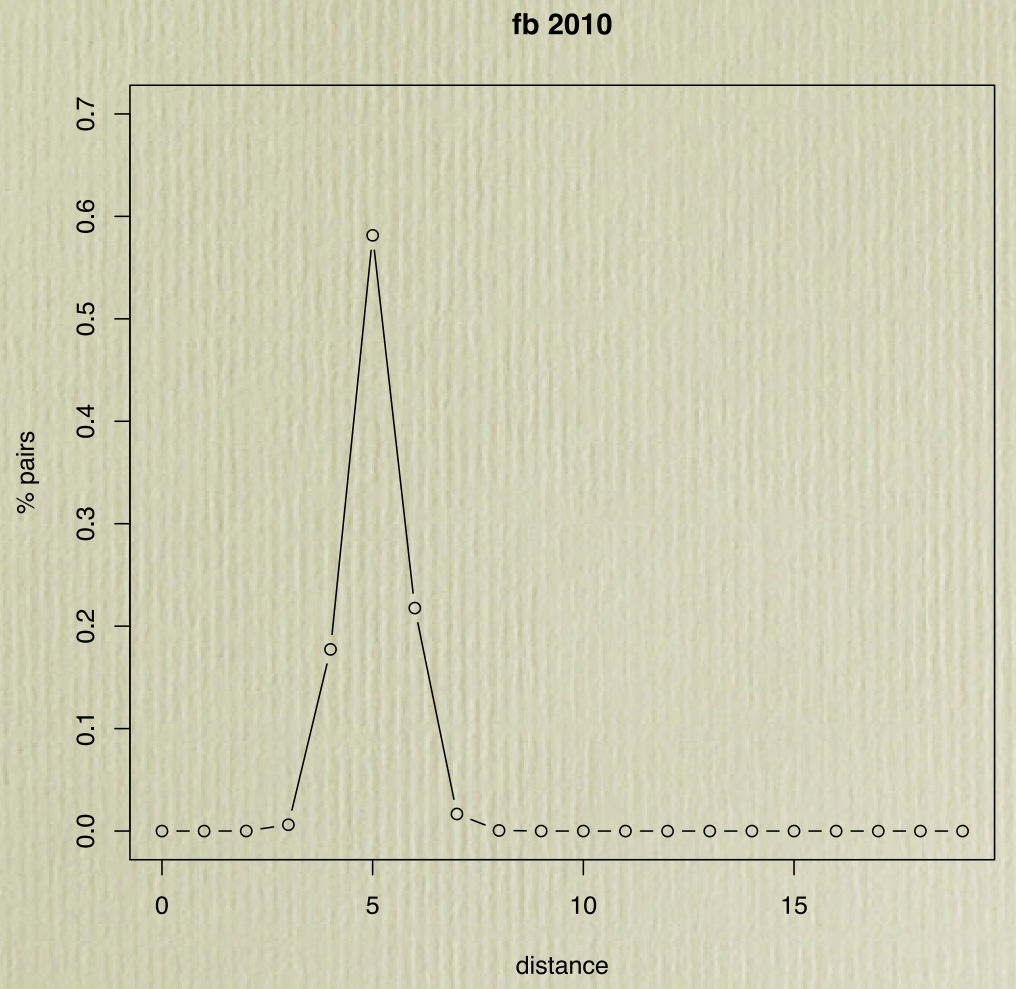

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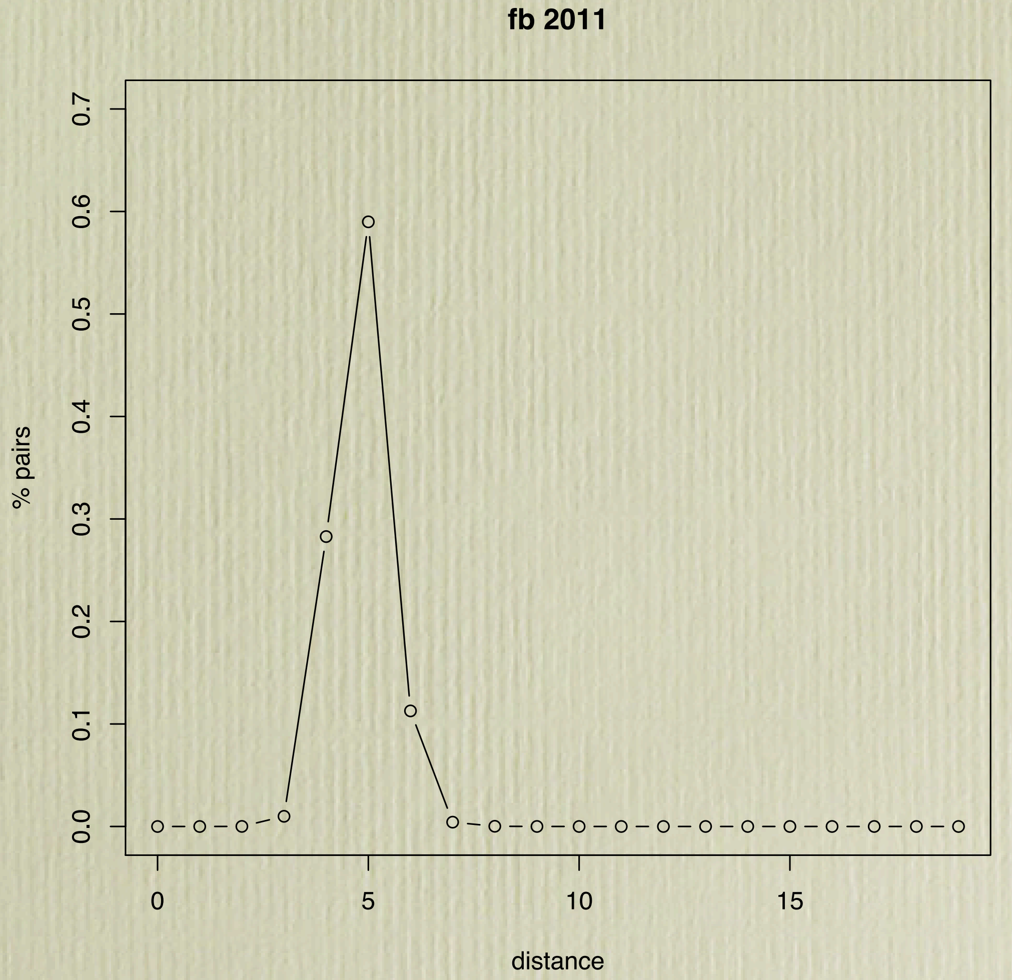

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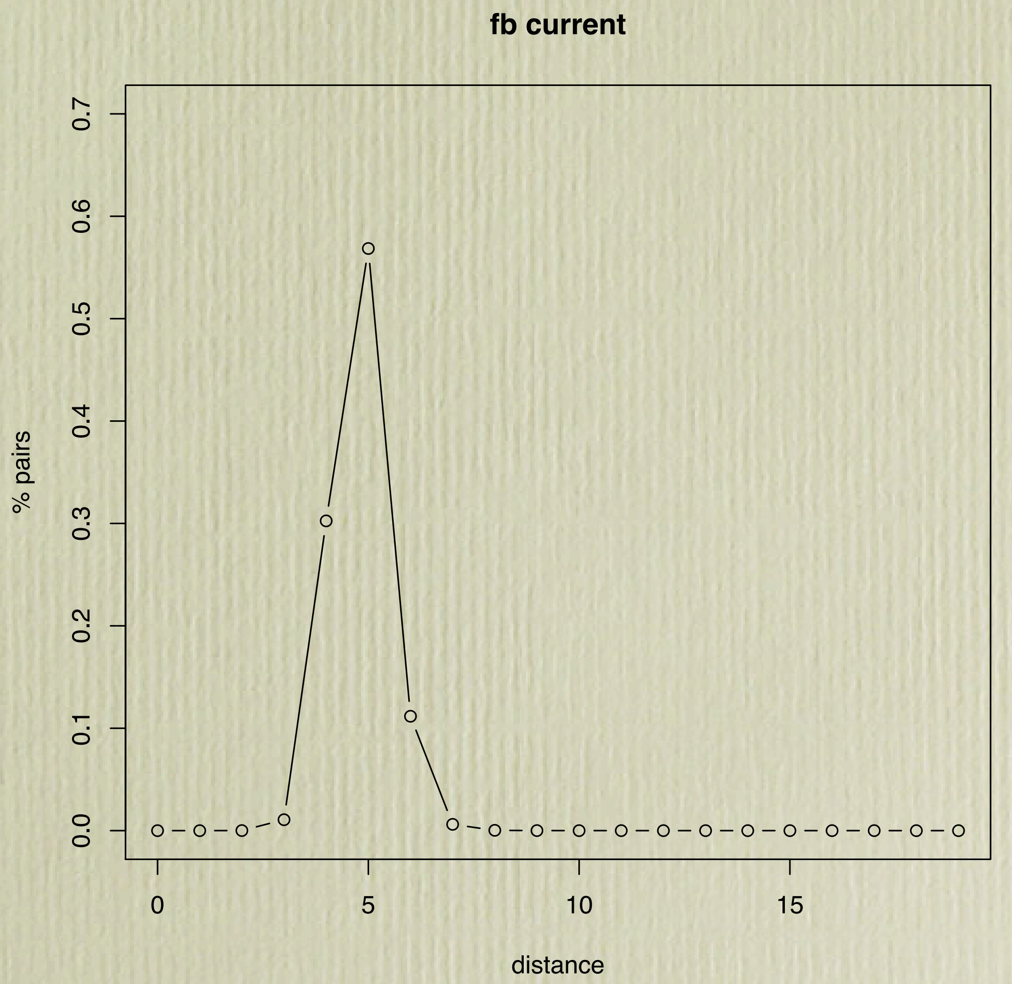

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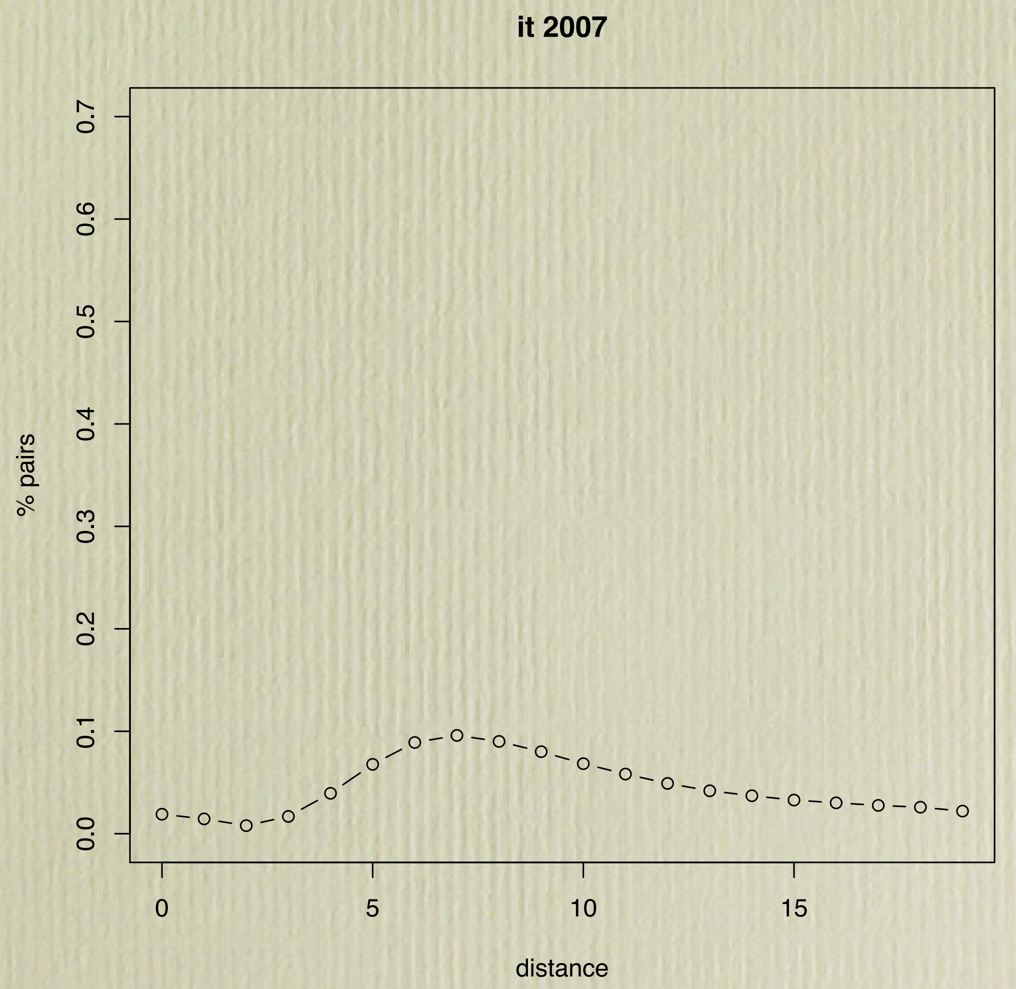

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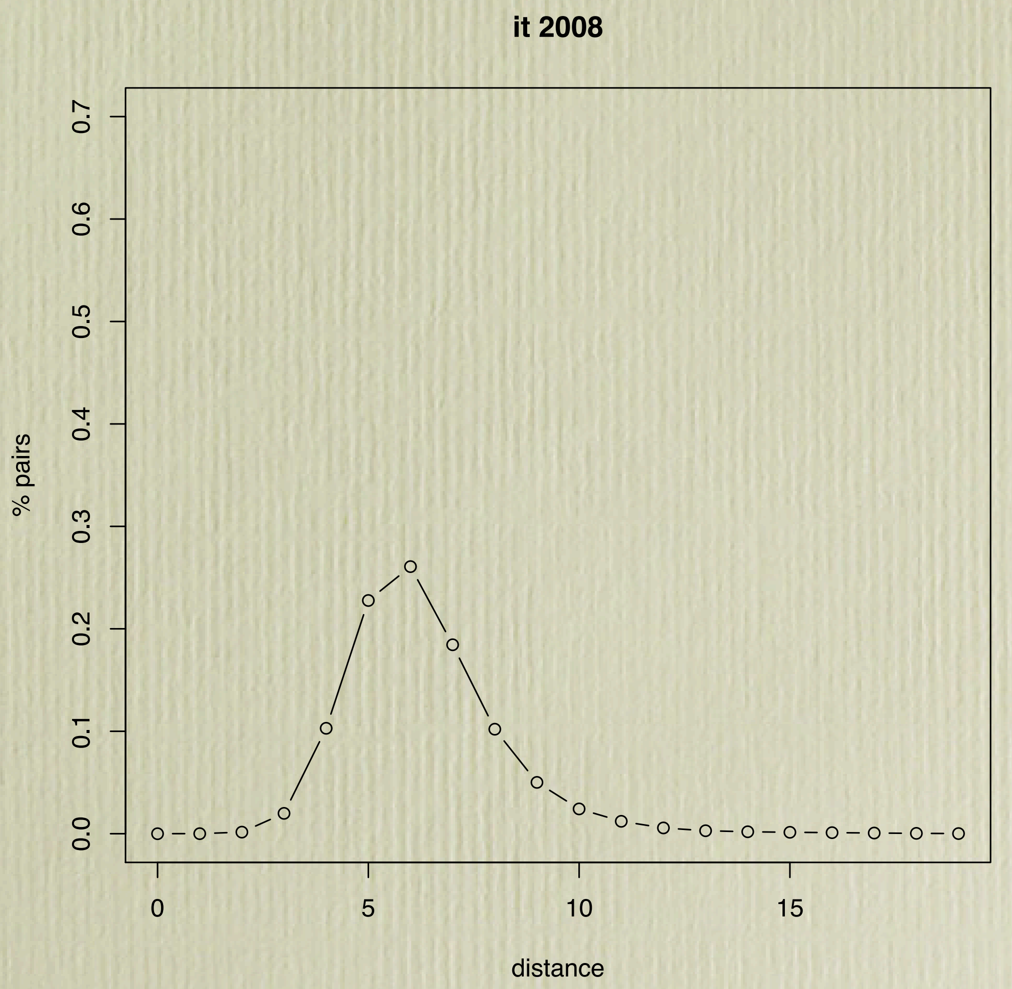

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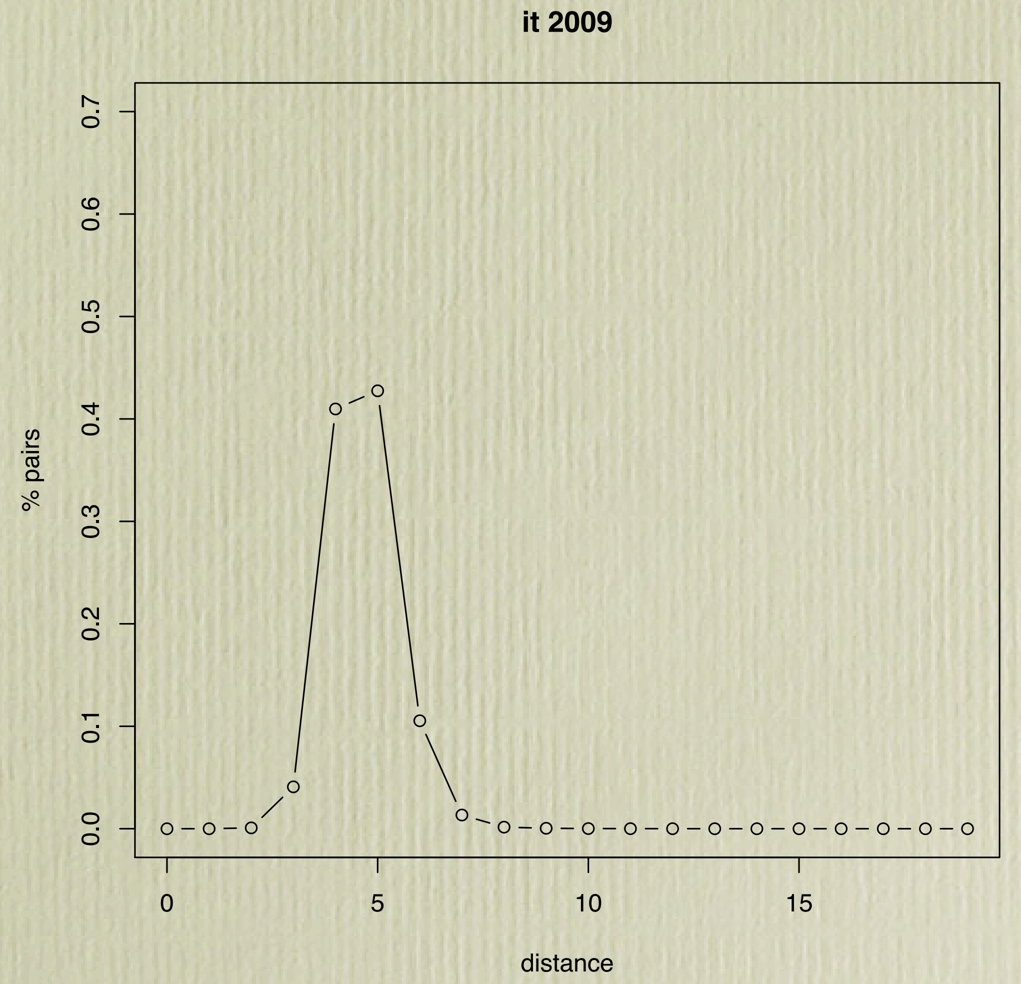

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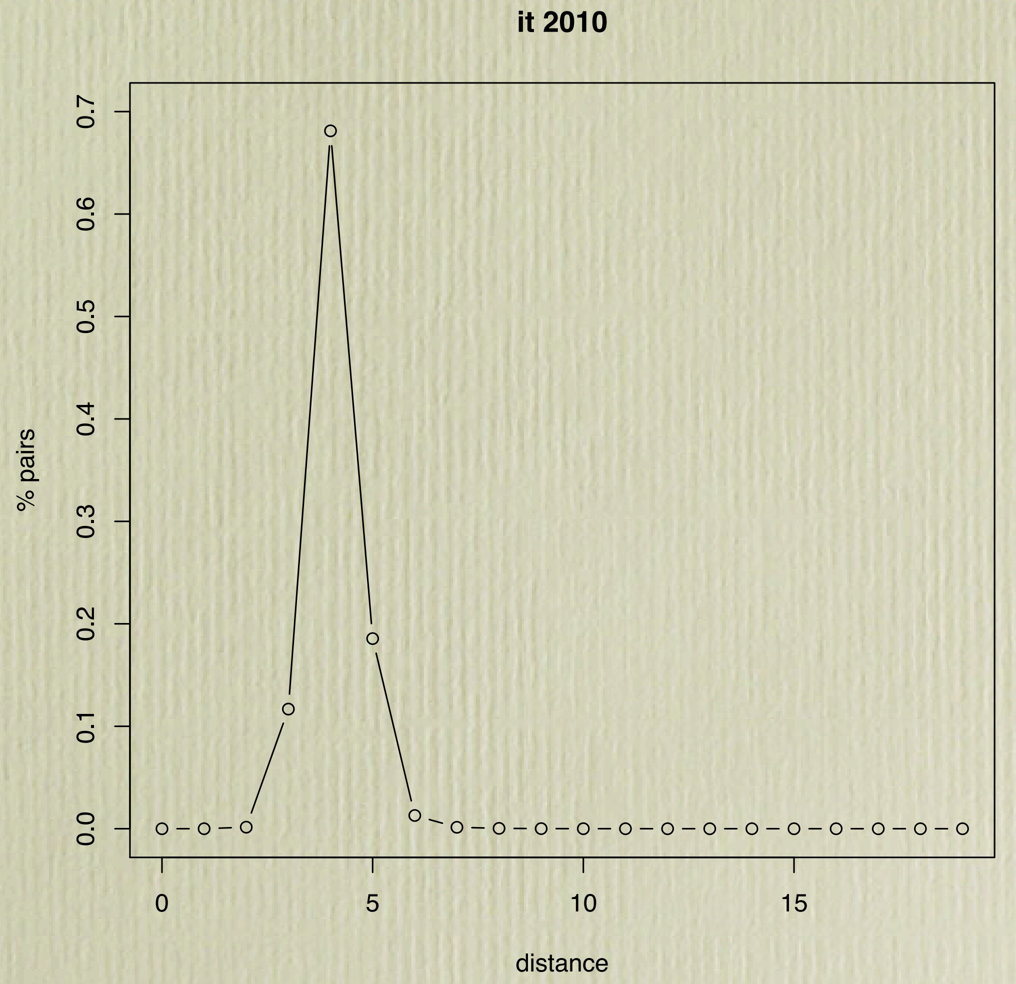

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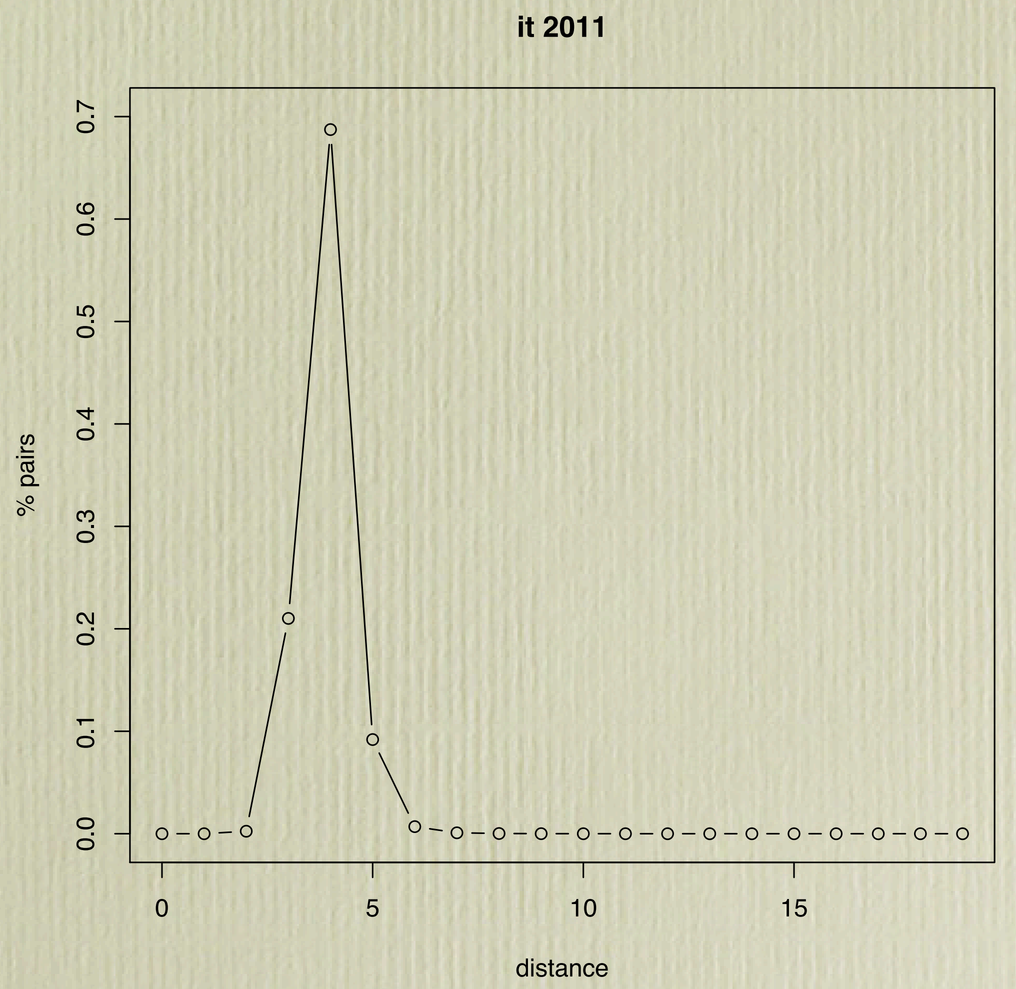

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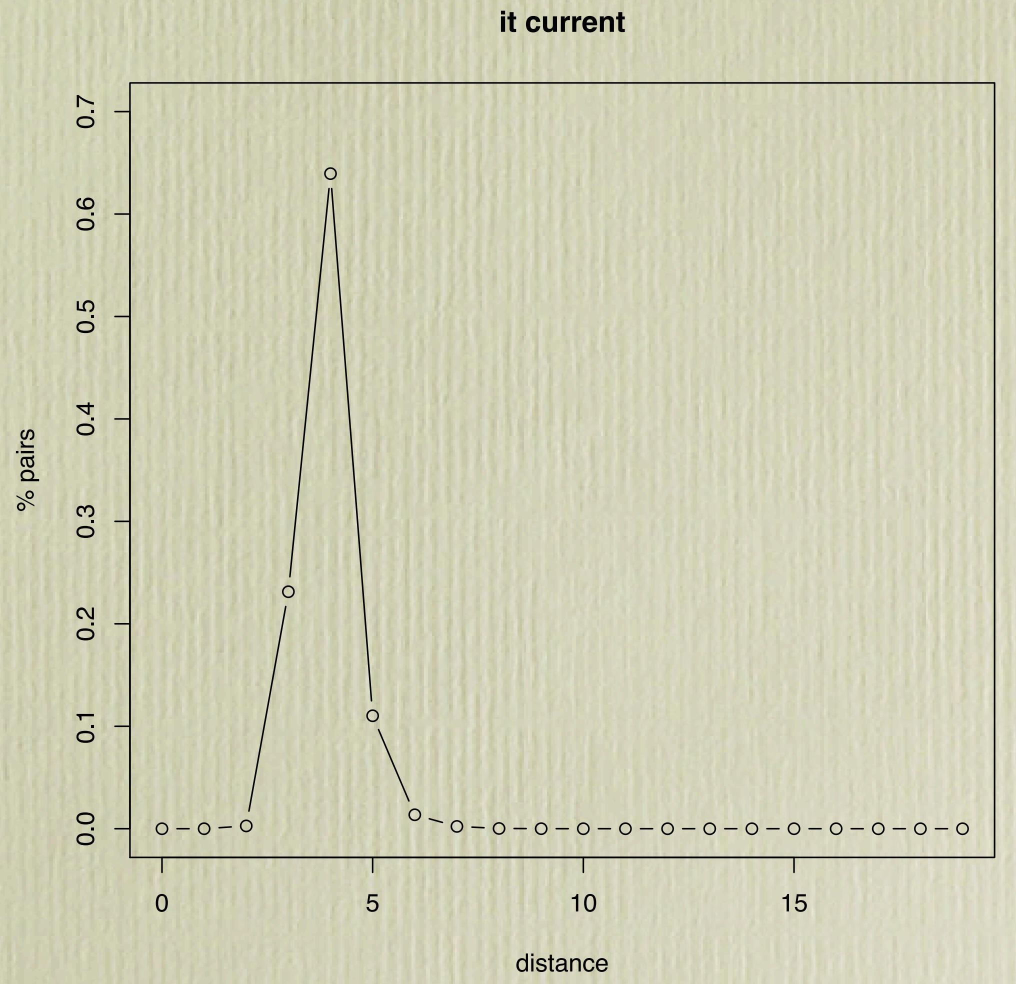

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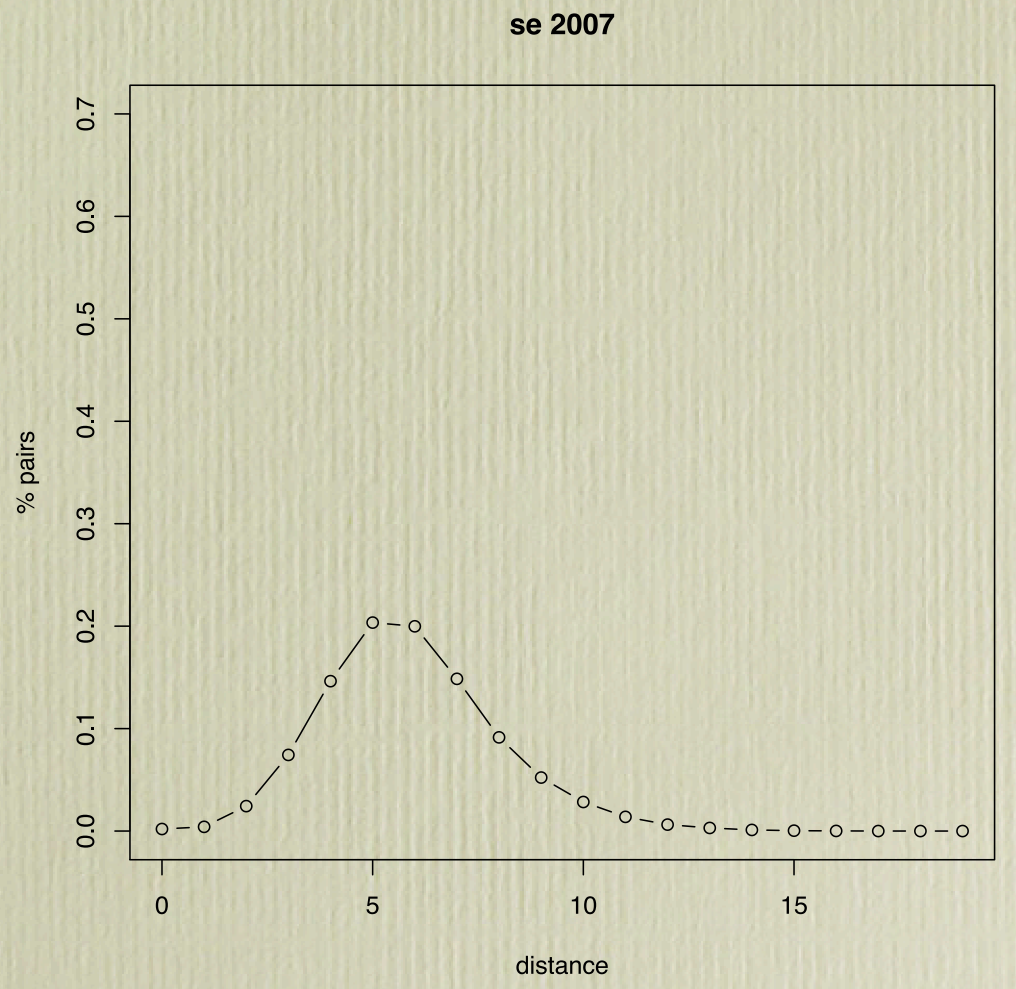

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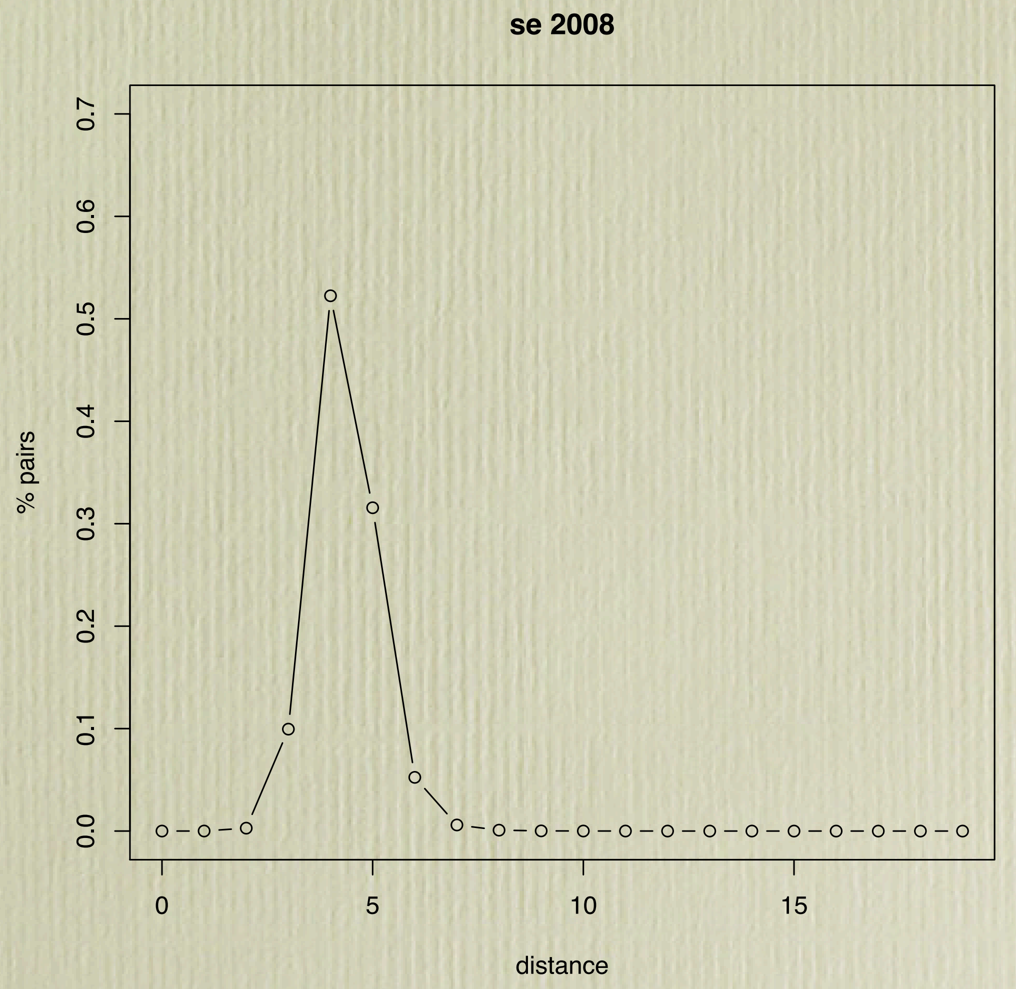

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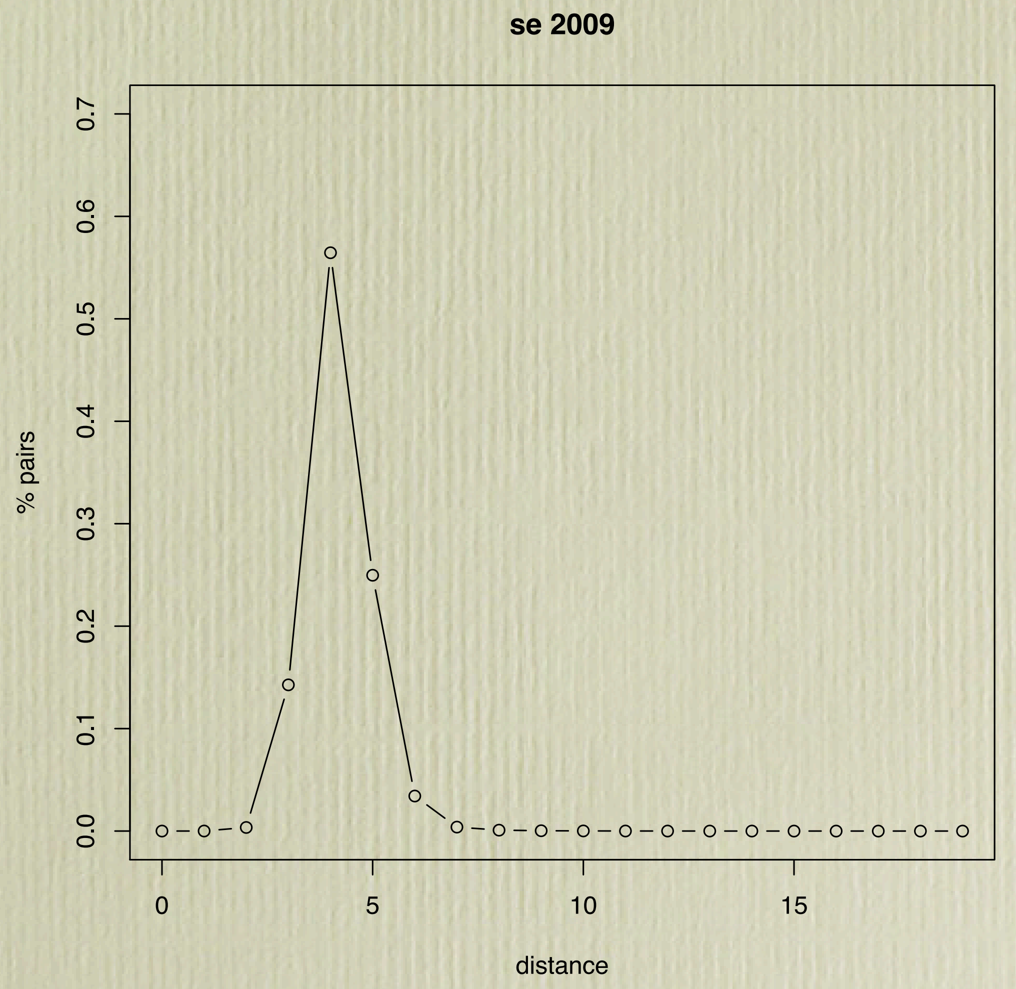

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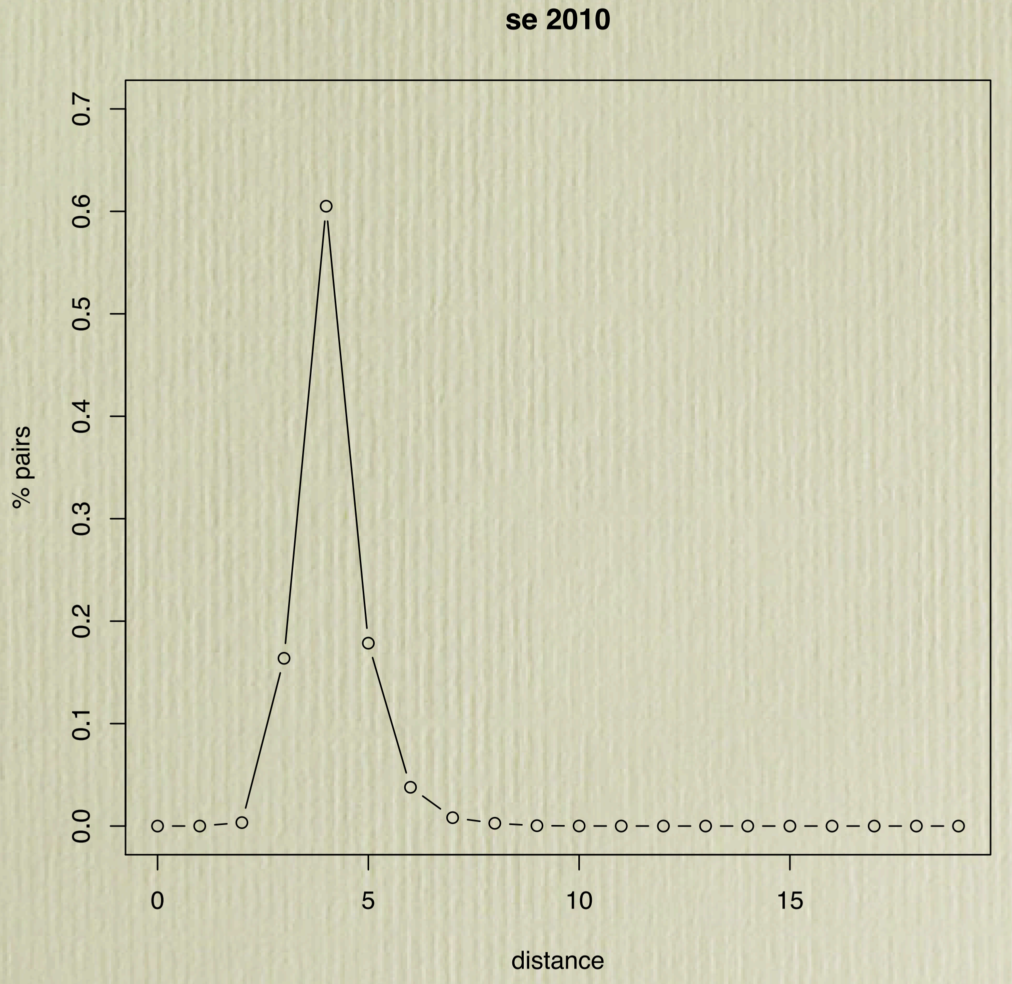

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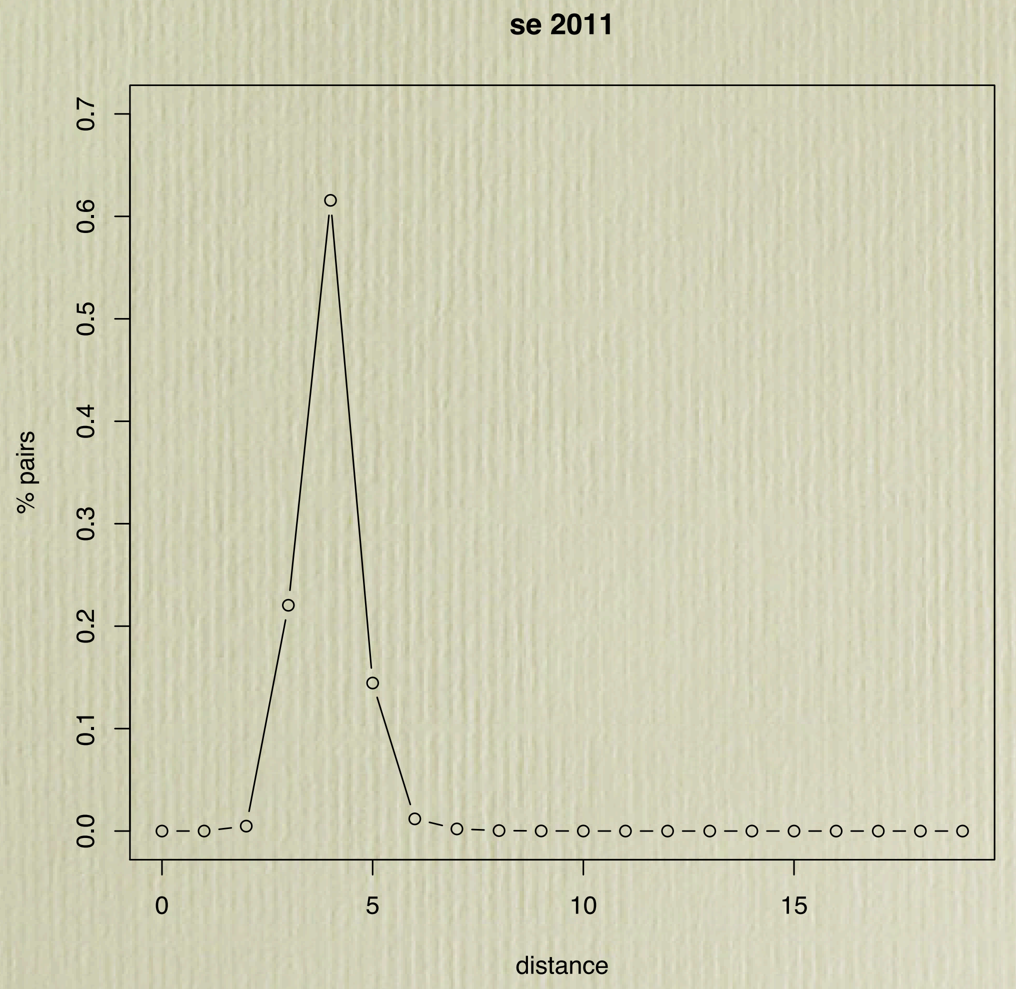

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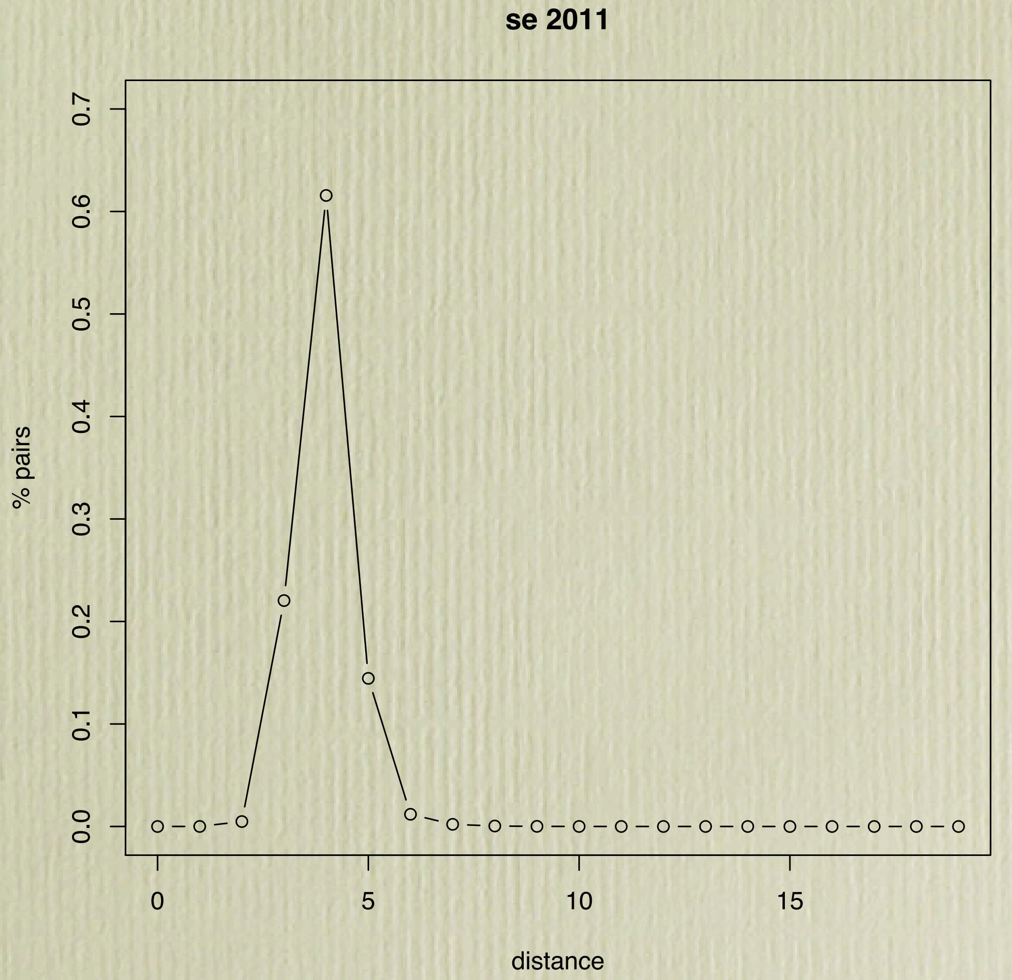

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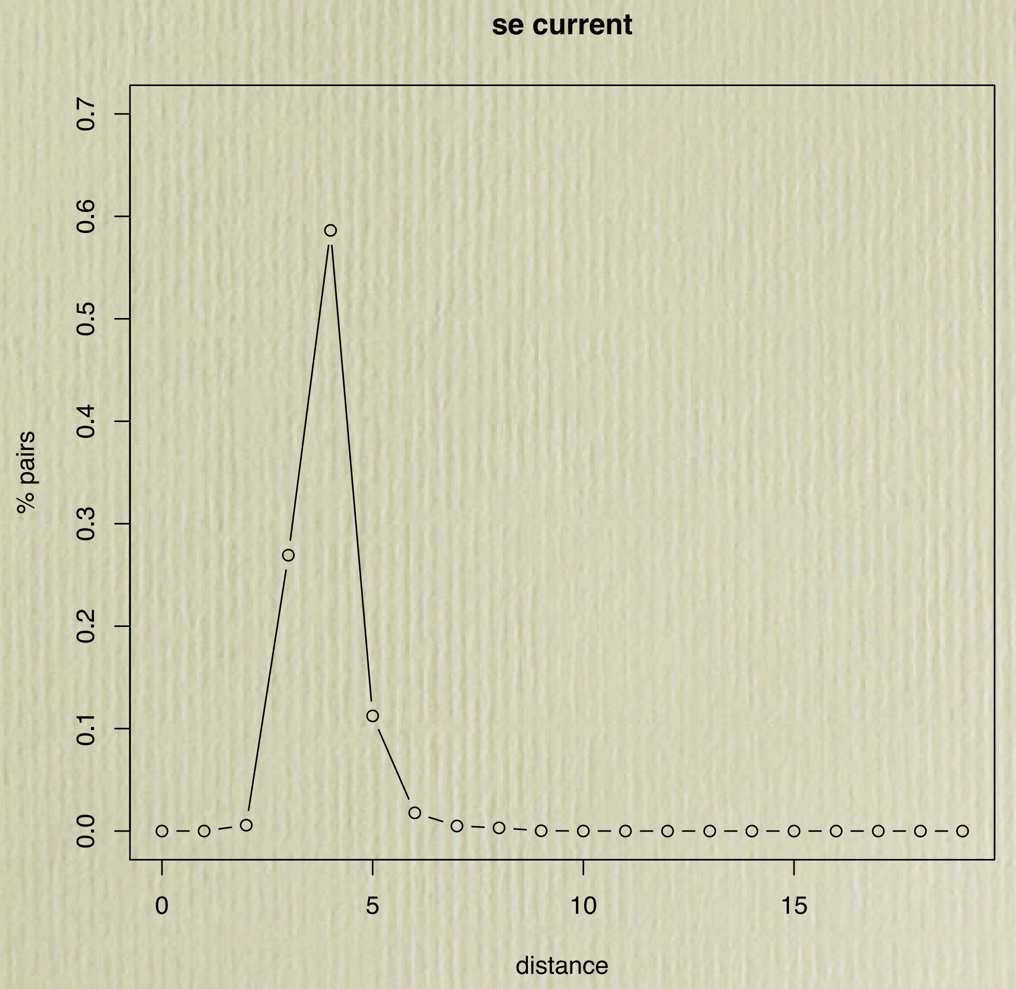
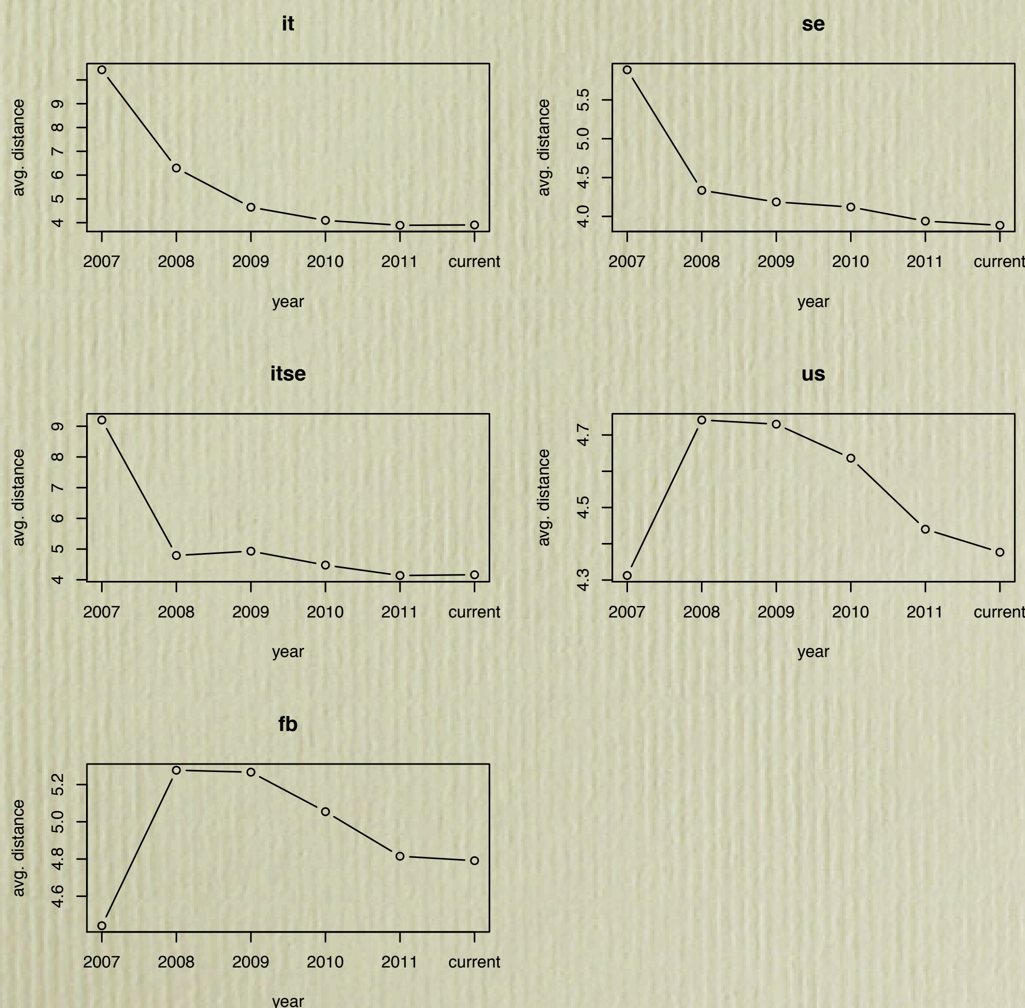
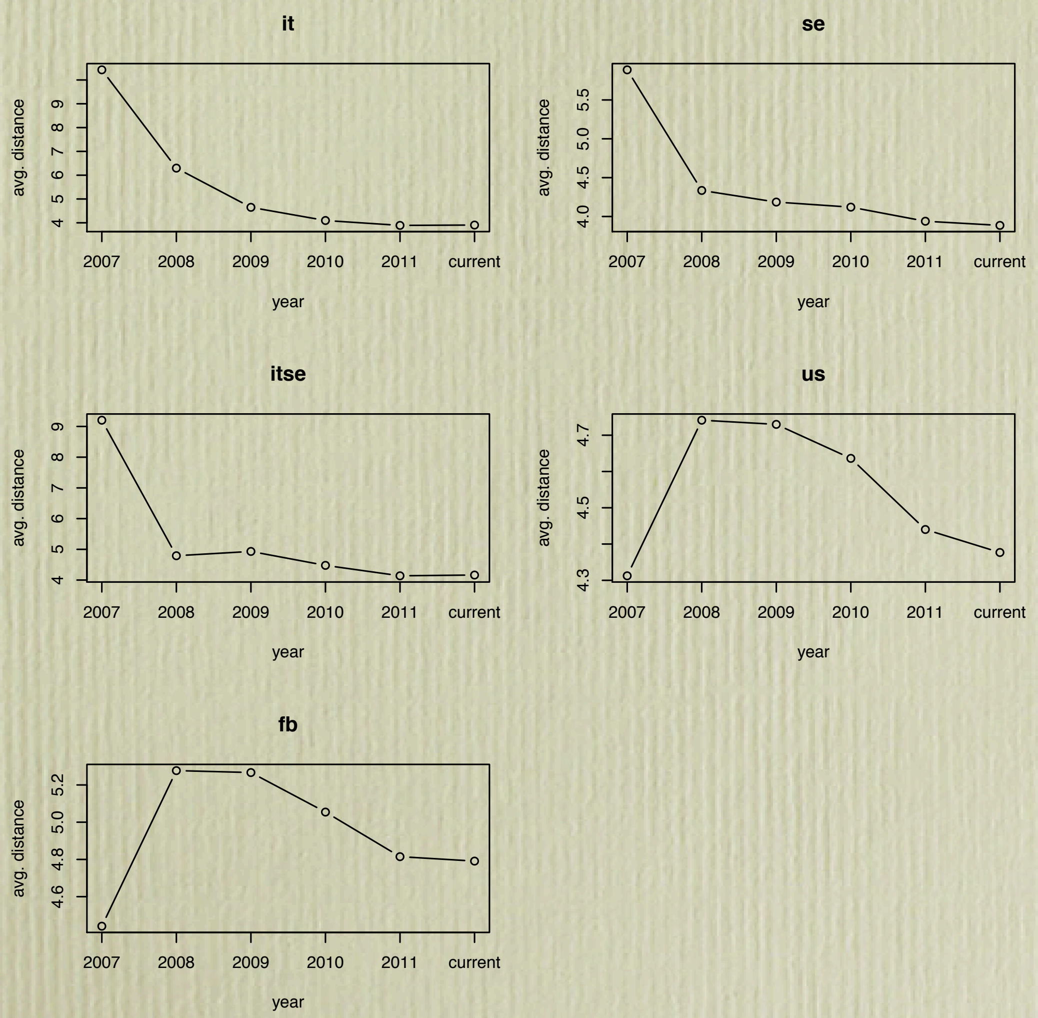


Average distance



	2008	*curr*
it	6.58	3.90
se	4.33	3.89
it+se	4.9	4.16
us	4.74	4.32
fb	5.28	4.74

Average distance



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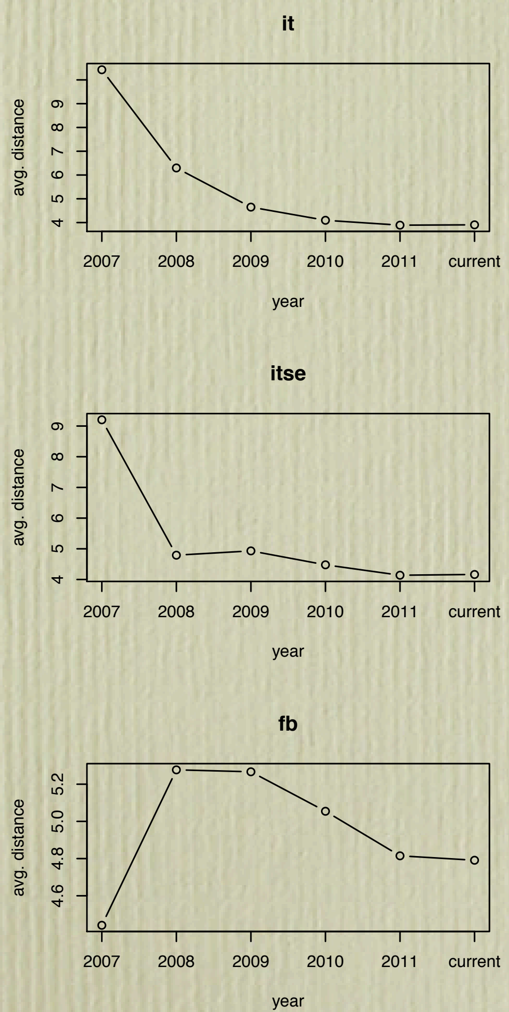

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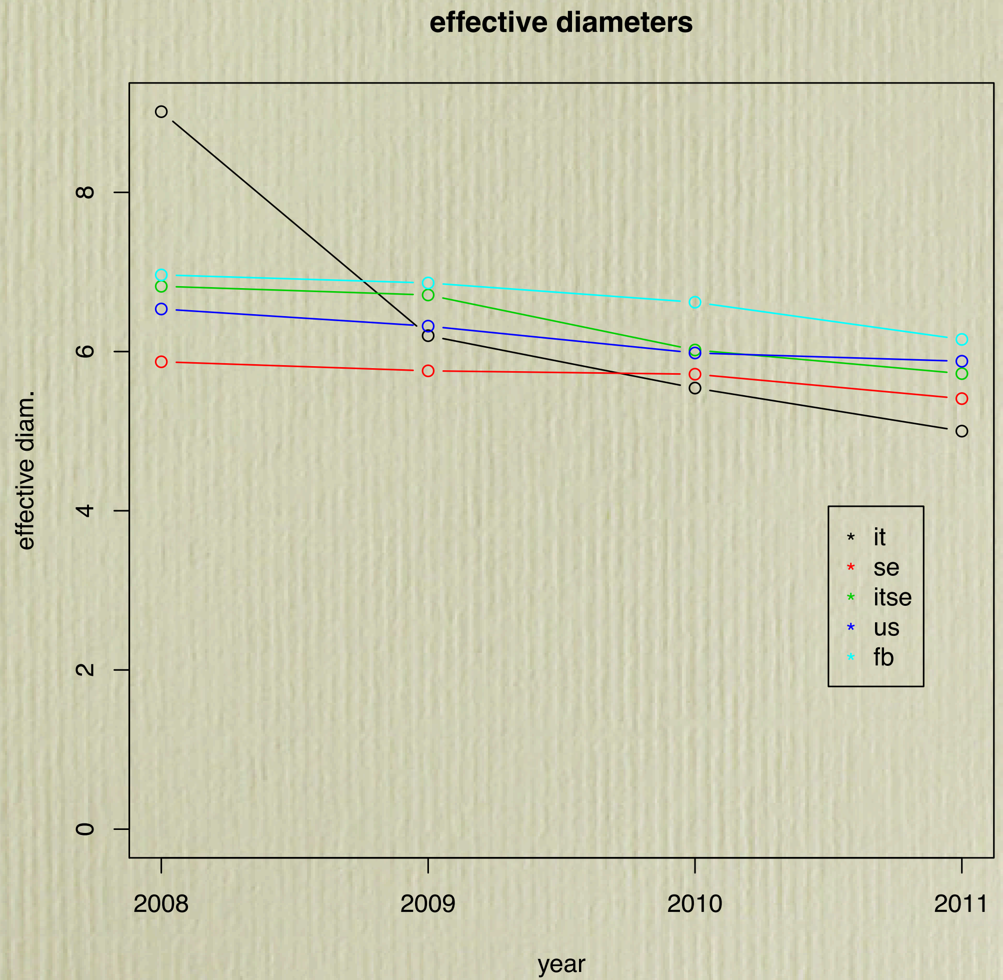
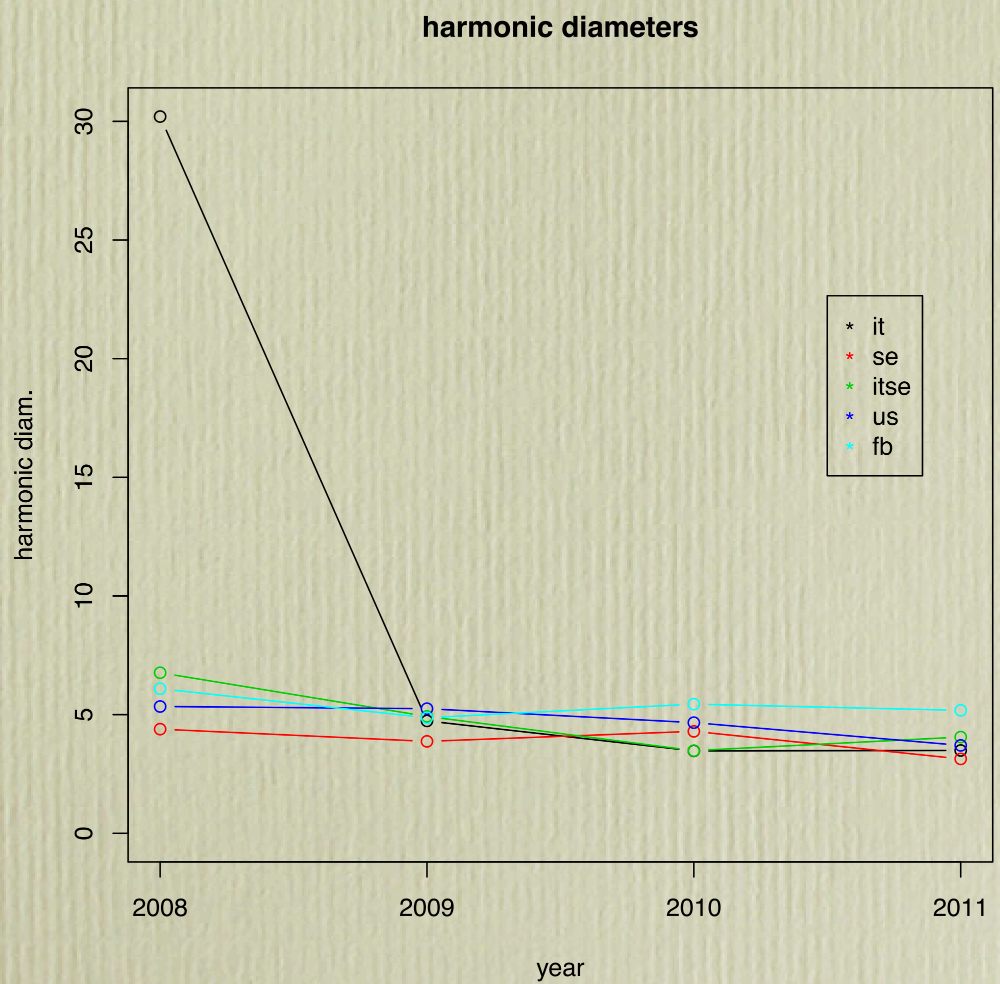
fb (current): 92% pairs are reachable!

Effective diameter (@ 90%)



	2008	curr
it	9.0	5.2
se	5.9	5.3
it +se	6.8	5.8
us	6.5	5.8
fb	7.0	6.2

Harmonic diameter



	2008	curr
it	23.7	3.4
se	4.5	4.0
it +se	5.8	3.8
us	4.6	4.4
fb	5.7	4.6

Average degree vs. density (fb)

	Avg. degree	*Density*
2009	88.7	$6.4 * 10^{-7}$
2010	113.0	$3.4 * 10^{-7}$
2011	169.0	$3.0 * 10^{-7}$
curr	190.4	$2.6 * 10^{-7}$

Actual diameter

	2008	*curr*
it	>29	=25
se	>16	=25
it+se	>21	=27
us	>17	=30
fb	>16	>58

Actual diameter

Used the fringe/double-sweep technique for "="

	2008	curr
it	>29	=25
se	>16	=25
it+se	>21	=27
us	>17	=30
fb	>16	>58

Other applications

Spid, network robustness and more...

What are distances good for?

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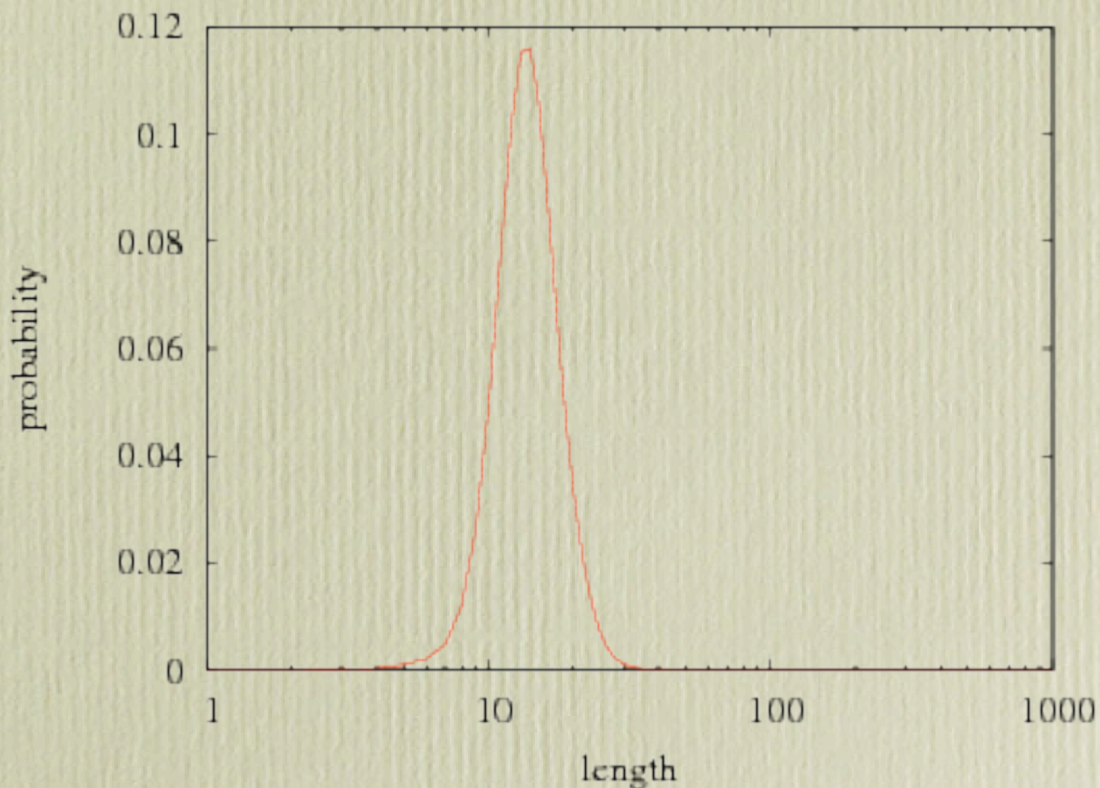
- Network models are usually studied on the base of the local statistics they produce
- Not difficult to obtain models that behave correctly locally (i.e., as far as degree distribution, assortativity, clustering coefficients... are concerned)

Global = more informative!



An application

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An application

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- Typical example: if I modify the graph with a certain criterion, how much does the distance distribution change?

Node elimination

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- Consider a certain *ordering of the vertices* of a graph

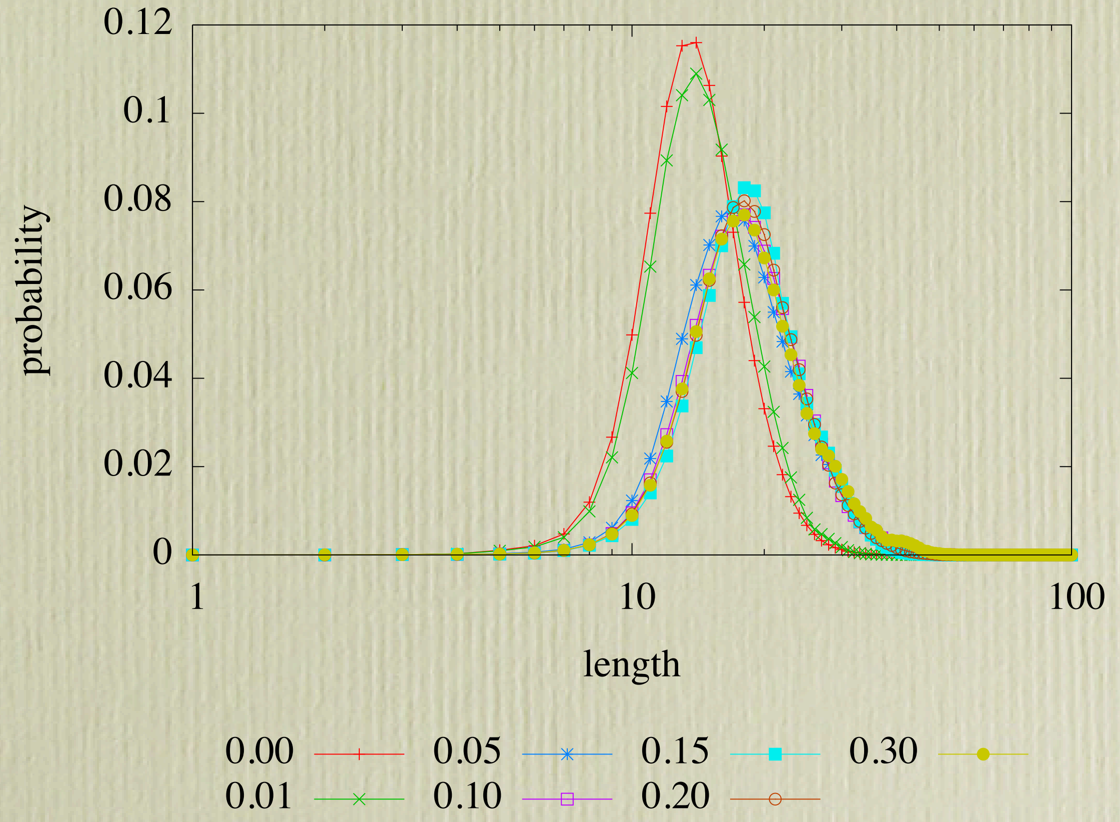
Node elimination

- Consider a certain *ordering of the vertices* of a graph
- Fix a threshold ϑ, delete all *vertices* (and all incident arcs) in the specified order, until ϑm arcs have been deleted

Node elimination

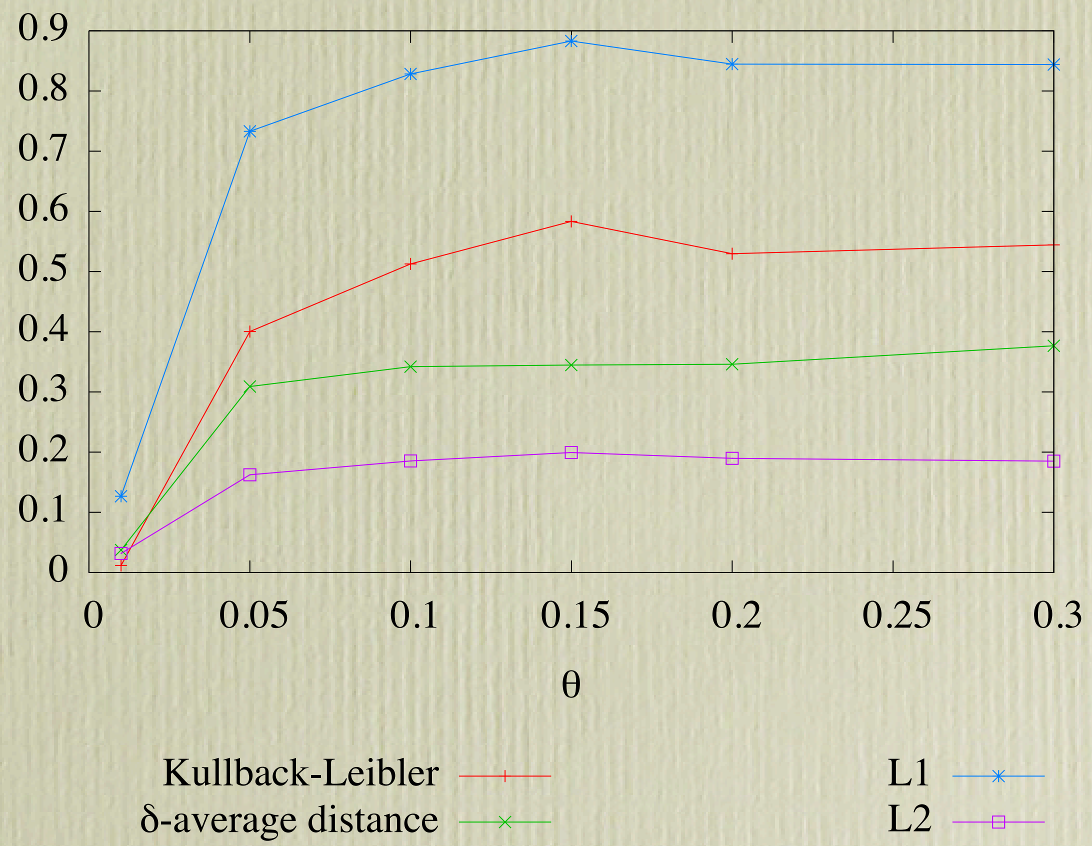
- Consider a certain *ordering of the vertices* of a graph
- Fix a threshold ϑ, delete all *vertices* (and all incident arcs) in the specified order, until ϑm arcs have been deleted
- Compute the “difference” between the graph you obtained and the original one

Experiment



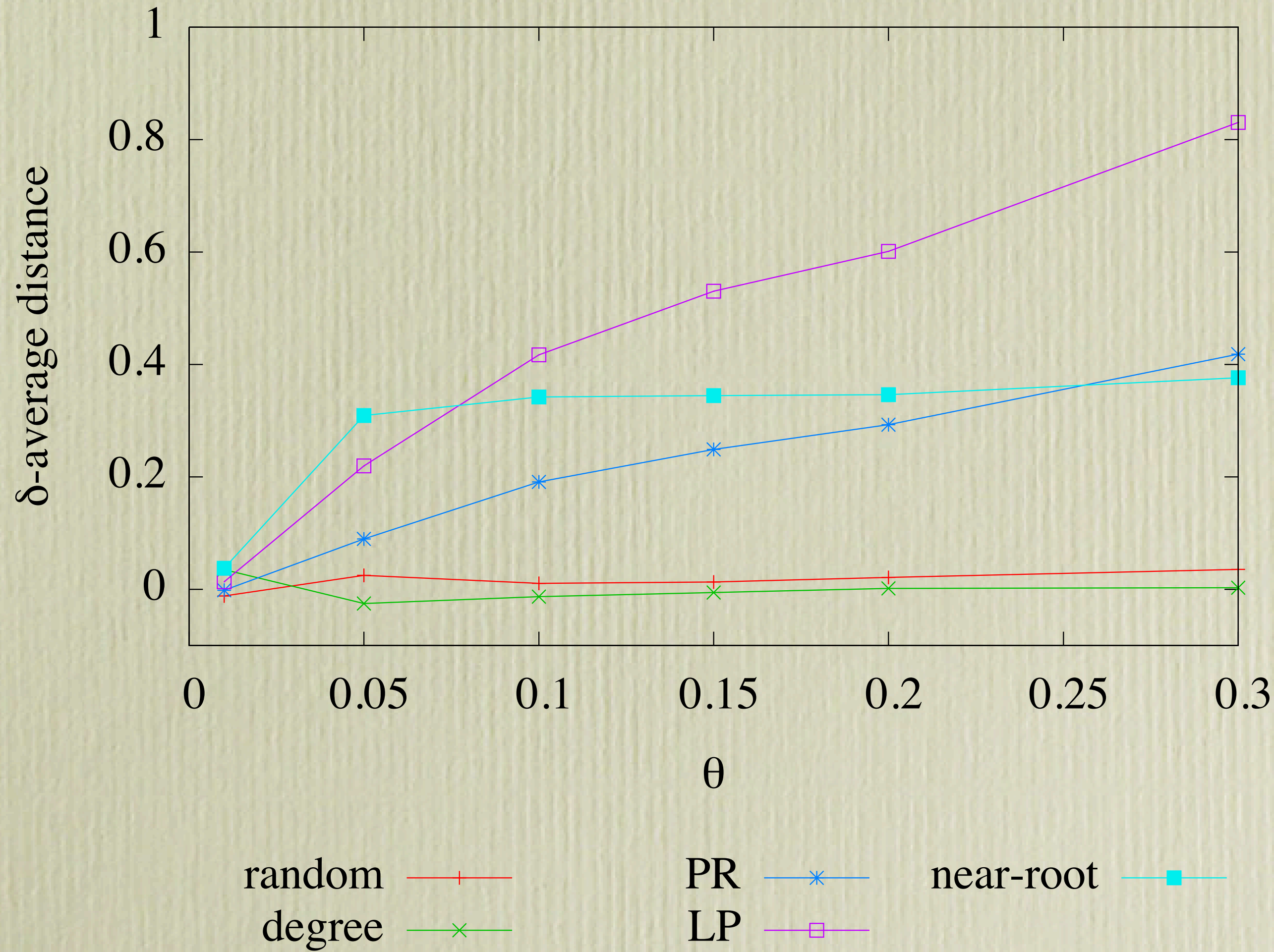
Deleting nodes in order of (syntactic) depth

Experiment (cont.)

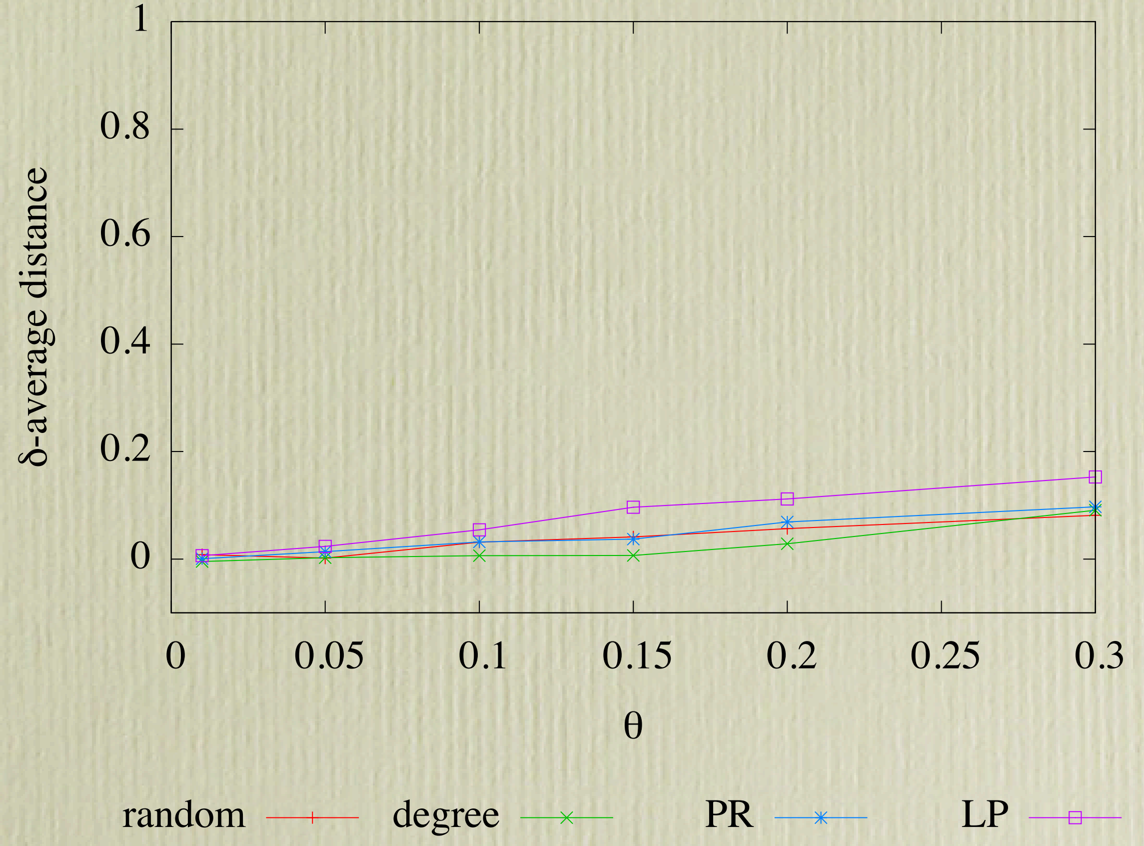


Distribution divergence (various measures)

Removal strategies compared



Removal in social networks



Findings

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- Proper social networks are much more robust, still being similar to web graphs under many respects

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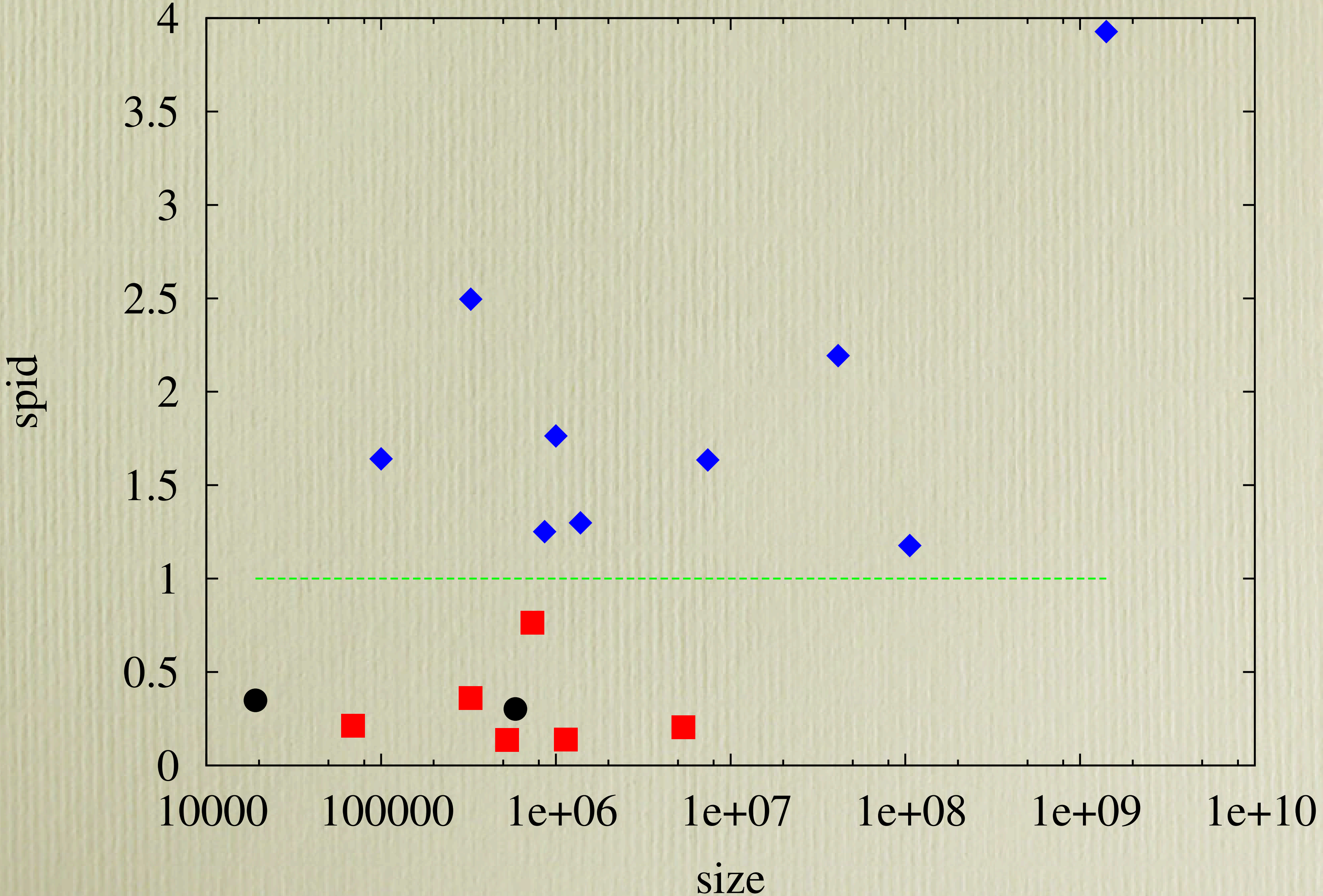
- We propose to use spid (*shortest-paths index of dispersion*), the ratio between variance and average in the distance distribution
- When the dispersion index is <1, the distribution is *subdispersed*; >1, is *superdispersed*
- Web graphs and social networks are **different** under this viewpoint!

Spid plot



4
3.5
3
2.5
2
1.5
1
0.5
0
spid
10000
100000
1e+06
1e+07
1e+08
1e+09
1e+10
size

Spid conjecture

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Spid conjecture

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[Answer: 0.093]

Average distance ∝ Effective diameter